A CLASS OF ROBUST ADAPTIVE BEAMFORMING ALGORITHMS FOR COHERENT INTERFERENCE SUPPRESSION

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Abstract. Based on the uniform linear array (ULA) and the spatial-smoothing technique, a robust beamforming problem for the reception of coherent signals is addressed and a forward-only (FO) beamformer is proposed based on worst-case optimization by considering both the steering vector and the correlation matrix errors. It is then extended to the forward-backward (FB) case with an improved performance. By introducing a preprocessing matrix, real-valued closed-form solutions are then derived with the same performance as in the FB case, but with much lower computational complexity. Simulations verified the effectiveness of the proposed algorithms.

1. INTRODUCTION

In the past decades, various algorithms have been proposed for adaptive beamforming, such as the well-known linearly constrained minimum variance (LCMV) beamformer [1, 2]. One key assumption for most of the methods is that the interferences are not correlated with the desired signal. In the presence of coherent interferences, the traditional algorithms will not work effectively since the desired signal will be canceled at the output. The spatial-smoothing method was proposed by Shan et al. with a detailed analysis to deal with this problem by separating the whole array into several subarrays [3]. However, this method is based on special array structures such as uniform linear arrays (ULAs), whose correlation matrix is of Toeplitz. Moreover, all sensors are assumed to have identical response. In practice, performance of this method will degrade when the special structure is destroyed due to model perturbations, such as array mutual coupling, sensor position errors, discrepancies in sensor responses, etc. Therefore, a robust beamformer is required for the scenario with coherent signals.

Various robust algorithms for uncorrelated signals have been proposed in the past decades. Based on a model for steering vector mismatches, the worst-case optimization based robust beamformer was proposed in [4, 5]. By estimating the real steering vector through maximizing the beamformer's output power, the robust Capon beamformer was proposed in [6] and it was then implemented in a recursive form in [7]. By generalizing the signal covariance matrix into a higher rank (non-point) one and transforming the optimization into a generalized eigenvector problem (GEP), a robust approach for general-rank signal models was proposed in [8], which was further developed in [9] with positive semidefinite constraints. However, all of the proposed robust beamformers are based on uncorrelated signals with significantly increased computational complexity for most cases.

For coherent interfering signals, in [10], a robust Capon beamformer was derived in the presence of coherent interference with the assumption that all interfering DOA angles are known. Without knowing the interfering DOA angles, in this paper, a robust forward-only (FO) worst-case optimization based beamformer is first proposed for coherent signals by considering both the array structural error and the steering vector error. The solution is then extended to the forward-backward (FB) case with an improved output signal-to-interference-plus-noise ratio (SINR). By further introducing a preprocessing matrix, we transform the complex-valued FB robust beamfomer into a real-valued one, with a computational complexity reduction of $50\% - 75\%$, depending on values of the parameters.

2. SIGNAL MODELS

Consider an array system with M sensors. The nth snapshot vector $\mathbf{x}[n]$ of the received array signals can be expressed as

$$\mathbf{x}[n] = \mathbf{s}_0[n] + \sum_{i=1}^{L-1} \mathbf{s}_i[n] + \mathbf{n}[n] \ , \qquad (1)$$

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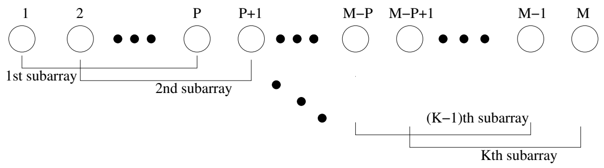

Fig. 1: The beamforming structure with spatial smoothing.

where $\mathbf{s}_0[n]$, $\mathbf{s}_i[n]$, $(i = 1, \cdots, L-1)$ and $\mathbf{n}[n]$ are the desired signal, interference and noise vectors, respectively, and L is the total number of impinging signals. By applying a set of coefficients w_i, $(i = 0, \ldots, M-1)$ to $\mathbf{x}[n]$, we obtain the beamformer output $y[n]$ as

$$y[n] = \mathbf{w}^H \mathbf{x}[n] \,, \tag{2}$$

where $\{\cdot\}^H$ denotes the Hermitian transpose operation and $\mathbf{w}$ is the weight vector. Given the steering vector of the desired signal, a traditional solution to the interference suppression problem is the LCMV beamformer. However, when the interferences are coherent with the desired signal, performance of the traditional LCMV beamformer will be degraded significantly. Spatial smoothing is an effective way to decorrelate the signals based on special array structures such as ULA [11]. As shown in Fig. 1, a full-length ULA with M sensors is divided into K overlapped subarrays with each subarray composed of P sensors. The first subarray is formed by sensors $\{1, 2, \cdots, P\}$ and the second subarray formed by sensors $\{2, 3, \cdots, P+1\}$, etc. As a result, the full array is divided into $K = M - P + 1$ subarrays. Each set of subarray data can be expressed as

$$\mathbf{x}_i[n] = [x_i[n], x_{i+1}[n], \cdots, x_{i+P-1}[n]],\ i = 1, 2, \cdots, K \,, \tag{3}$$

where $x_i[n]$ $(i = 1, \cdots, M)$ is the ith sensor output at time index n.

The forward-only optimum Capon weight vector with spatial smoothing is given by [3]

$$\hat{\mathbf{w}}_{\text{c,fo}} = \frac{\hat{\mathbf{R}}_{ss}^{-1}\hat{\mathbf{a}}_0}{\hat{\mathbf{a}}_0^H \hat{\mathbf{R}}_{ss}^{-1}\hat{\mathbf{a}}_0} \,. \tag{4}$$

$\hat{\mathbf{a}}_0$ is the estimated steering vector of the desired signal and $\hat{\mathbf{R}}_{ss}$ is the correlation matrix of the subarrays, defined as

$$\hat{\mathbf{R}}_{ss} = \frac{1}{NK}\sum_{i=1}^{K}\sum_{n=1}^{N}\mathbf{x}_i[n]\mathbf{x}_i^H[n] = \frac{1}{NK}\sum_{i=1}^{K}\hat{\mathbf{X}}_i\hat{\mathbf{X}}_i^H, \tag{5}$$

where $\hat{\mathbf{X}}_i = [\mathbf{x}_i[1], \mathbf{x}_i[2], \cdots, \mathbf{x}_i[N]]$.

The use of $\hat{\mathbf{R}}_{ss}$ will introduce the finite-sample effect and $\hat{\mathbf{a}}_0$ can cause DOA angle mismatch error. Moreover, the spatial-smoothing method is based on a special array structure. Since array position errors, sensor response discrepancies and the finite-sample effect will destroy the Toeplitz structure of the correlation matrix, the algorithm with spatial smoothing will not converge to the optimum solution any more, causing new errors to the system. Therefore, a robust beamforming algorithm for coherent signals is required.

3. PROPOSED ROBUST BEAMFORMING ALGORITHMS

The relationship between the estimated and the real steering vectors and real correlation matrices can be written as

$$\mathbf{a}_0 = \hat{\mathbf{a}}_0 + \Delta\hat{\mathbf{a}}_0,\ \ \mathbf{R}_{ss} = \hat{\mathbf{R}}_{ss} + \boldsymbol{\Delta}\hat{\mathbf{R}}_{ss} \,, \tag{6}$$

where $\Delta\hat{\mathbf{a}}_0$ and $\Delta\hat{\mathbf{R}}_{ss}$ represent the corresponding estimation errors and $\Delta\hat{\mathbf{R}}_{ss}$ is assumed to be Hermitian. Each of them is composed of two parts, with the first part due to traditional errors such as DOA angle mismatch and finite-sample error, and the second part due to the spatial-smoothing operation.

3.1. Forward-only Implementation

We can solve the robust beamforming problem against arbitrary errors $\boldsymbol{\Delta}\hat{\mathbf{a}}_0$ and $\Delta\hat{\mathbf{R}}_{ss}$ in the following way

$$\begin{aligned}
&\min_{\hat{\mathbf{w}}}\ \hat{\mathbf{w}}^H(\hat{\mathbf{R}}_{ss} + \boldsymbol{\Delta}\hat{\mathbf{R}}_{ss})\hat{\mathbf{w}} \\
&\text{subject to } |\hat{\mathbf{w}}^H(\hat{\mathbf{a}}_{\mathbf{0}} + \Delta\hat{\mathbf{a}}_0)| \geq 1 \\
&\text{for all } \quad \|\Delta\hat{\mathbf{a}}_0\| \leq \varepsilon,\ \|\boldsymbol{\Delta}\hat{\mathbf{R}}_{ss}\| \leq \gamma \,,
\end{aligned} \tag{7}$$

where $\hat{\mathbf{w}}$ is the weight vector, $\|\cdot\|$ is the Frobenius norm, and γ and ε are two positive constants.

We first consider the correlation error matrix $\boldsymbol{\Delta}\hat{\mathbf{R}}_{ss}$. The worst-case extension of (7) can be written as

$$\begin{aligned}
&\min_{\hat{\mathbf{w}}}\ \max_{\|\Delta\hat{\mathbf{R}}_{ss}\| \leq \gamma}\ \hat{\mathbf{w}}^H(\hat{\mathbf{R}}_{ss} + \boldsymbol{\Delta}\hat{\mathbf{R}}_{ss})\hat{\mathbf{w}} \\
&\text{subject to } |\hat{\mathbf{w}}^H(\hat{\mathbf{a}}_{\mathbf{0}} + \Delta\hat{\mathbf{a}}_0)| \geq 1 \\
&\text{for all } \|\boldsymbol{\Delta}\hat{\mathbf{a}}_0\| \leq \varepsilon \,.
\end{aligned} \tag{8}$$

For the Hermitian error matrix $\boldsymbol{\Delta}\hat{\mathbf{R}}_{ss}$, given an arbitrary $\hat{\mathbf{w}}$, the maximum of $\hat{\mathbf{w}}^H(\hat{\mathbf{R}}_{ss} + \boldsymbol{\Delta}\hat{\mathbf{R}}_{ss})\hat{\mathbf{w}}$ occurs on the boundary $\|\boldsymbol{\Delta}\hat{\mathbf{R}}_{ss}\| = \gamma$. Then the maximization in (8) can be simplified to

$$\begin{aligned}
&\min_{\Delta\hat{\mathbf{R}}_{ss}}\ -\hat{\mathbf{w}}^H(\hat{\mathbf{R}}_{ss} + \boldsymbol{\Delta}\hat{\mathbf{R}}_{ss})\hat{\mathbf{w}} \\
&\text{for all } \ \|\Delta\hat{\mathbf{R}}_{ss}\| = \gamma \,.
\end{aligned} \tag{9}$$

We can form the following Lagrange function

$$Q = -\hat{\mathbf{w}}^H(\hat{\mathbf{R}}_{ss} + \boldsymbol{\Delta}\hat{\mathbf{R}}_{ss})\hat{\mathbf{w}} + \lambda_1(\|\Delta\hat{\mathbf{R}}_{ss}\|^2 - \gamma^2) \,, \tag{10}$$

where λ_1 is the Lagrange multiplier. Taking the gradient of Q with respect to $\Delta\hat{\mathbf{R}}_{ss}$ and setting it to zero, we have $\Delta\hat{\mathbf{R}}_{ss} = \hat{\mathbf{w}}\hat{\mathbf{w}}^H/(2\lambda_1)$. With $\|\Delta\hat{\mathbf{R}}_{ss}\|^2 = \gamma^2$, we get

$$\Delta\hat{\mathbf{R}}_{ss} = \gamma\frac{\hat{\mathbf{w}}\hat{\mathbf{w}}^H}{\hat{\mathbf{w}}^H\hat{\mathbf{w}}} . \tag{11}$$

Substituting it into (8) and notice that $|\hat{\mathbf{w}}^H(\hat{\mathbf{a}}_\mathbf{0} + \Delta\hat{\mathbf{a}}_0)| \geq |\hat{\mathbf{w}}^H\hat{\mathbf{a}}_\mathbf{0}| - \varepsilon\|\hat{\mathbf{w}}\|$. Then we have

$$\begin{aligned}&\min_{\hat{\mathbf{w}}}\ \hat{\mathbf{w}}^H(\hat{\mathbf{R}}_{ss} + \gamma\mathbf{I})\hat{\mathbf{w}}\\ &\text{subject to } |\hat{\mathbf{w}}^H\hat{\mathbf{a}}_\mathbf{0}| - \varepsilon\|\hat{\mathbf{w}}\| \geq 1 .\end{aligned} \tag{12}$$

The imaginary part of $\hat{\mathbf{w}}^H\hat{\mathbf{a}}_\mathbf{0}$ can be zero by rotating the optimum solution. Moreover, the inequality constraint $|\hat{\mathbf{w}}^H\hat{\mathbf{a}}_\mathbf{0}|-\varepsilon\|\hat{\mathbf{w}}\| \geq 1$ can be changed to $|\hat{\mathbf{w}}^H\hat{\mathbf{a}}_\mathbf{0}|-\varepsilon\|\hat{\mathbf{w}}\| = 1$ by scaling the optimum weight vector without affecting the output SINR [4]. Then (12) can be transformed to

$$\begin{aligned}&\min_{\hat{\mathbf{w}}}\ \hat{\mathbf{w}}^H(\hat{\mathbf{R}}_{ss} + \gamma\mathbf{I})\hat{\mathbf{w}}\\ &\text{subject to } |\hat{\mathbf{w}}^H\hat{\mathbf{a}}_\mathbf{0} - 1|^2 = \varepsilon^2\hat{\mathbf{w}}^H\hat{\mathbf{w}} .\end{aligned} \tag{13}$$

Using the method of Lagrange multipliers, the optimum solution, $\hat{\mathbf{w}}_{\text{o,fo}}$, is obtained by

$$\hat{\mathbf{w}}_{o,fo} = \frac{\lambda_2(\hat{\mathbf{R}}_{ss} + \gamma\mathbf{I} + \lambda_2\varepsilon^2\mathbf{I})^{-1}\hat{\mathbf{a}}_0}{\lambda_2\hat{\mathbf{a}}_0^H(\hat{\mathbf{R}}_{ss} + \gamma\mathbf{I} + \lambda_2\varepsilon^2\mathbf{I})^{-1}\hat{\mathbf{a}}_0 - 1} , \tag{14}$$

where the matrix inverse lemma has been used and λ_2 is an unknown Lagrange multiplier.

$\hat{\mathbf{R}}_{ss}$ can be decomposed into $\hat{\mathbf{R}}_{ss} = \mathbf{U}\Lambda\mathbf{U}^H$ with $\mathbf{U}$ and $\Lambda = \text{diag}[\delta_1, \delta_2 \cdots, \delta_M]$ being the eigenvector matrix and diagonal eigenvalue matrix of $\hat{\mathbf{R}}_{ss}$, respectively. Then equation (14) is further simplified to

$$\hat{\mathbf{w}}_{\text{o,fo}} = \frac{\lambda_2\mathbf{U}^H(\Lambda + \gamma\mathbf{I} + \lambda_2\varepsilon^2\mathbf{I})^{-1}\mathbf{U}\hat{\mathbf{a}}_0}{\lambda_2\hat{\mathbf{a}}_0^H\mathbf{U}^H(\Lambda + \gamma\mathbf{I} + \lambda_2\varepsilon^2\mathbf{I})^{-1}\mathbf{U}\hat{\mathbf{a}}_0 - 1} . \tag{15}$$

Since $\hat{\mathbf{w}}_{\text{o,fo}}$ satisfies the constraint equation in (13), substituting (15) into (13), we have

$$\lambda_2\varepsilon^2|\hat{\mathbf{a}}_0^H\mathbf{U}^H(\Lambda + \gamma\mathbf{I} + \lambda_2\varepsilon^2\mathbf{I})^{-1}\mathbf{U}\hat{\mathbf{a}}_0| = 1 . \tag{16}$$

Using the substitution $\mathbf{z} = \mathbf{U}^H\hat{\mathbf{a}}_0$ with z_i being the ith element of $\mathbf{z}$, (16) becomes

$$f(\lambda_2) = \lambda_2\varepsilon^2\sum_{m=1}^{M}\frac{|z_m|^2}{\delta_m + \gamma + \lambda_2\varepsilon^2} = 1 , \tag{17}$$

where $f(\lambda_2)$ is a function of λ_2 and denotes the left side of (16). λ_2 can be obtained by solving (17). Since $f(\lambda_2)$ is a monotonically increasing function of λ_2, $lim_{\lambda_2\to\infty}f(\lambda_2) > 1$ and $f(0) = 0 < 1$, we can see that the solution to (17) is unique.

The result of (14) is based on the coherent signal model and the spatial-smoothing technique. If the correlation matrix error is zero and the spatial-smoothing operation is removed, (14) will be reduced to the original worst-case optimization solution in [4].

3.2. FB Implementations

The solution (14) is based on the FO sample correlation matrix, which requires at least $2L$ sensors to eliminate the correlation between the desired signal and interferences. Based on the same ULA structure, FB processing can be employed to improve performance of the FO-based algorithms and reduce the number of sensors required [11, 12].

For the ULA structure, we have known that the correlation matrix $\mathbf{R}_{ss}$ is centrohermitian, i.e. $\mathbf{R}_{ss} = \mathbf{J}\mathbf{R}_{ss}^*\mathbf{J}$, where $\mathbf{J}$ is the exchange matrix defined as

$$\mathbf{J} = \begin{bmatrix} 0 & \dots & 0 & 1\\ 0 & \dots & 1 & 0\\ \vdots & \ddots & \vdots & \vdots\\ 1 & \dots & 0 & 0\end{bmatrix} . \tag{18}$$

Moreover, its steering vector satisfies $\mathbf{a}_0 = \mathbf{J}\mathbf{a}_0^*$ when the zero phase position is chosen to be the center of the array. Then we can use the FB estimations $\tilde{\mathbf{R}}_{ss}$ and $\tilde{\mathbf{a}}_0$ instead of the FO ones as follows

$$\tilde{\mathbf{R}}_{ss} = \frac{\hat{\mathbf{R}}_{ss} + \mathbf{J}(\hat{\mathbf{R}}_{ss})^*\mathbf{J}}{2} \text{ and } \tilde{\mathbf{a}}_0 = \frac{\hat{\mathbf{a}}_0 + \mathbf{J}\hat{\mathbf{a}}_0^*}{2} , \tag{19}$$

with the following relationship

$$\mathbf{a}_0 = \tilde{\mathbf{a}}_0 + \boldsymbol{\Delta}\tilde{\mathbf{a}}_0,\ \ \mathbf{R}_{ss} = \tilde{\mathbf{R}}_{ss} + \boldsymbol{\Delta}\tilde{\mathbf{R}}_{ss} , \tag{20}$$

where $\boldsymbol{\Delta}\tilde{\mathbf{a}}_0$ and $\boldsymbol{\Delta}\tilde{\mathbf{R}}_{ss}$ are the corresponding estimation errors, which now also include the new errors caused by the FB operation due to structural errors. In order to improve robustness of the beamformer against the new set of errors, we can formulate the problem into:

$$\begin{aligned}&\min_{\hat{\mathbf{w}}}\ \hat{\mathbf{w}}^H(\tilde{\mathbf{R}}_{ss} + \boldsymbol{\Delta}\tilde{\mathbf{R}}_{ss})\hat{\mathbf{w}}\\ &\text{subject to } |\hat{\mathbf{w}}^H(\tilde{\mathbf{a}}_\mathbf{0} + \Delta\tilde{\mathbf{a}}_0)| \geq 1\\ &\text{for all} \qquad \|\Delta\tilde{\mathbf{a}}_0\| \leq \tilde{\varepsilon},\ \|\boldsymbol{\Delta}\tilde{\mathbf{R}}_{ss}\| \leq \tilde{\gamma} .\end{aligned} \tag{21}$$

where $\tilde{\varepsilon}$ and $\tilde{\gamma}$ are two positive constants. For simplification, we will still use ε and γ in the following derivation. With the same process as in the FO case, we can obtain the following optimum solution $\hat{\mathbf{w}}_{\text{o,fb}}$ to the above FB-based beamformer as

$$\hat{\mathbf{w}}_{\text{o,fb}} = \frac{\tilde{\lambda}_2(\tilde{\mathbf{R}}_{ss} + \gamma\mathbf{I} + \tilde{\lambda}_2\varepsilon^2\mathbf{I})^{-1}\tilde{\mathbf{a}}_0}{\tilde{\lambda}_2\tilde{\mathbf{a}}_0^H(\tilde{\mathbf{R}}_{ss} + \gamma\mathbf{I} + \tilde{\lambda}_2\varepsilon^2\mathbf{I})^{-1}\tilde{\mathbf{a}}_0 - 1} , \tag{22}$$

which has exactly the same form as (14) except that the FO estimations are now replaced by the FB ones. $\tilde{\lambda}_2$ is the Lagrange multiplier and can be solved in a similar way as in the FO case.

3.3. Real-valued Implementations

Let us first introduce a unitary transformation matrix $\mathbf{T}$ [13, 12]

$$\mathbf{T} = \begin{cases} \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I} & \mathbf{J} \\ j\mathbf{J} & -j\mathbf{I} \end{bmatrix} & \text{for even } P \\ \\ \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{J} \\ \mathbf{0} & \sqrt{2} & \mathbf{0} \\ j\mathbf{J} & \mathbf{0} & -j\mathbf{I} \end{bmatrix} & \text{for odd } P \end{cases} \tag{23}$$

By using this $\mathbf{T}$ to transform $\hat{\mathbf{X}}_i$ to a new output $\mathbf{T}\hat{\mathbf{X}}_i$ and keeping the beamformer output the same, the original weight vector $\hat{\mathbf{w}}$ will be changed to $\mathbf{T}\hat{\mathbf{w}}$ since

$$\mathbf{y} = \hat{\mathbf{w}}^H\hat{\mathbf{X}}_i = \hat{\mathbf{w}}^H\mathbf{T}^H\mathbf{T}\hat{\mathbf{X}}_i = (\mathbf{T}\hat{\mathbf{w}})^H(\mathbf{T}\hat{\mathbf{X}}_i) \ . \tag{24}$$

We can prove that after such a transformation, the optimum FB solution $\hat{\mathbf{w}}_{\text{o,real}} = \mathbf{T}\hat{\mathbf{w}}_{\text{o,fb}}$ will be real-valued. i.e.

$$\hat{\mathbf{w}}_{\text{o,real}} \in \mathbb{R} \ , \tag{25}$$

Proof: Let us first define

$$\bar{\mathbf{X}}_i = \mathbf{T}\hat{\mathbf{X}}_i, \ \ \bar{\mathbf{R}}_{ss} = \mathbf{T}\tilde{\mathbf{R}}_{ss}\mathbf{T}^H, \ \text{ and } \ \bar{\mathbf{a}}_0 = \mathbf{T}\tilde{\mathbf{a}}_0 \ . \tag{26}$$

Using (22) and noticing $\mathbf{T}^H = \mathbf{T}^{-1}$, we have

$$\begin{aligned} \hat{\mathbf{w}}_{\text{o,real}} = \mathbf{T}\hat{\mathbf{w}}_{\text{o,fb}} &= \frac{\tilde{\lambda}_2\mathbf{T}(\tilde{\mathbf{R}}_{ss} + (\gamma + \tilde{\lambda}_2\varepsilon^2)\mathbf{I})^{-1}\tilde{\mathbf{a}}_0}{\tilde{\lambda}_2\tilde{\mathbf{a}}_0^H(\tilde{\mathbf{R}}_{ss} + (\gamma + \tilde{\lambda}_2\varepsilon^2)\mathbf{I})^{-1}\tilde{\mathbf{a}}_0 - 1} \\ &= \frac{\tilde{\lambda}_2(\mathbf{T}^H\tilde{\mathbf{R}}_{ss}\mathbf{T} + (\gamma + \tilde{\lambda}_2\varepsilon^2)\mathbf{I})^{-1}\mathbf{T}\tilde{\mathbf{a}}_0}{\tilde{\lambda}_2\tilde{\mathbf{a}}_0^H\mathbf{T}^H(\mathbf{T}^H\tilde{\mathbf{R}}_{ss}\mathbf{T} + (\gamma + \tilde{\lambda}_2\varepsilon^2)\mathbf{I})^{-1}\mathbf{T}\tilde{\mathbf{a}}_0 - 1} \\ &= \frac{\tilde{\lambda}_2(\bar{\mathbf{R}}_{ss} + b\mathbf{I})^{-1}\bar{\mathbf{a}}_0}{\tilde{\lambda}_2\bar{\mathbf{a}}_0^H(\bar{\mathbf{R}}_{ss} + b\mathbf{I})^{-1}\bar{\mathbf{a}}_0 - 1} \ , \end{aligned} \tag{27}$$

where $b = \gamma + \tilde{\lambda}_2\varepsilon^2$. Using (19), and with $\mathbf{TJ} = \mathbf{T}^*$ and $\mathbf{JT}^H = \mathbf{T}^T$, where $\{\cdot\}^T$ denotes the transpose operation, we have

$$\begin{aligned} \bar{\mathbf{R}}_{ss} &= \mathbf{T}\tilde{\mathbf{R}}_{ss}\mathbf{T}^H = \mathbf{T}\frac{\hat{\mathbf{R}}_{ss} + \mathbf{J}\hat{\mathbf{R}}_{ss}^*\mathbf{J}}{2}\mathbf{T}^H \\ &= \frac{\mathbf{T}\hat{\mathbf{R}}_{ss}\mathbf{T}^H + (\mathbf{T}\hat{\mathbf{R}}_{ss}\mathbf{T}^H)^*}{2} = \text{Real}(\mathbf{T}\hat{\mathbf{R}}_{ss}\mathbf{T}^H) \\ &= \frac{1}{NK}\sum_{i=1}^{K}\text{Real}(\mathbf{T}\hat{\mathbf{X}}_i\hat{\mathbf{X}}_i^H\mathbf{T}^H) \ , \end{aligned} \tag{28}$$

where Real$\{\cdot\}$ denotes the operation of taking the real part. We then have $\bar{\mathbf{R}}_{ss} \in \mathbb{R}$. For $\bar{\mathbf{a}}_0$, we can follow the same method to prove that $\bar{\mathbf{a}}_0 = \text{Real}(\mathbf{T}\hat{\mathbf{a}}_0) \in \mathbb{R}$. So all of the parameters in (27) are real-valued, we then have $\hat{\mathbf{w}}_{\text{o,real}} \in \mathbb{R}$, which completes the proof.

We can further separate $\hat{\mathbf{X}}_i$ into the sum of its real part $\hat{\mathbf{X}}_{i,R}$ and imaginary part $\hat{\mathbf{X}}_{i,I}$ as

$$\hat{\mathbf{X}}_i = \hat{\mathbf{X}}_{i,R} + j\hat{\mathbf{X}}_{i,I} \ . \tag{29}$$

Then $\text{Real}(\mathbf{T}\hat{\mathbf{X}}_i\hat{\mathbf{X}}_i^H\mathbf{T}^H) = \bar{\mathbf{X}}_{i,R}\bar{\mathbf{X}}_{i,R}^T + \bar{\mathbf{X}}_{i,I}\bar{\mathbf{X}}_{i,I}^T$. Substituting it into (28), we have

$$\bar{\mathbf{R}}_{ss} = \frac{1}{NK}\sum_{i=1}^{K}(\bar{\mathbf{X}}_{i,R}\bar{\mathbf{X}}_{i,R}^T + \bar{\mathbf{X}}_{i,I}\bar{\mathbf{X}}_{i,I}^T) \ . \tag{30}$$

Substituting (30) into (27), we can simplify $\hat{\mathbf{w}}_{\text{o,real}}$ to

$$\hat{\mathbf{w}}_{\text{o,real}} = \frac{\tilde{\lambda}_2(\sum_{i=1}^{K}(\bar{\mathbf{X}}_{i,R}\bar{\mathbf{X}}_{i,R}^T + \bar{\mathbf{X}}_{i,I}\bar{\mathbf{X}}_{i,I}^T) + bNK\mathbf{I})^{-1}\bar{\mathbf{a}}_0}{\tilde{\lambda}_2\bar{\mathbf{a}}_0^H(\sum_{i=1}^{K}(\bar{\mathbf{X}}_{i,R}\bar{\mathbf{X}}_{i,R}^T + \bar{\mathbf{X}}_{i,I}\bar{\mathbf{X}}_{i,I}^T) + bNK\mathbf{I})^{-1}\bar{\mathbf{a}}_0 - NK} \tag{31}$$

If $\gamma = 0$, (31) is then reduced to the real-valued worst-case optimization beamformer without considering the correlation matrix error.

With the real-valued implementation in (31), the computational complexity of the system has been reduced by $50\% - 75\%$ compared with the FO algorithm in (15) and the FB based algorithm in (22).

3.4. Robust Algorithms for Uncorrelated Signals

If the subarray number is 1, i.e. $P = M$ and $K = 1$, then all the proposed algorithms are reduced to a normal worst-case optimization beamformer without spatial smoothing, which is robust against both the steering vector and the correlation matrix errors. So the robust worst-case optimization algorithm for correlated signals based on the spatial-smoothing technique is a general case of the uncorrelated ones.

4. SIMULATION RESULTS

Our simulations are based on a ULA with $M = 8$ and a sensor spacing $d = \lambda_0/2$, where λ_0 is the signal wavelength. The sensor number for each subarray is $P = 5$ and there are in total $K = M - P + 1 = 4$ subarrays. The data sample size $N = 20$. There are three signals with the same power arriving from DOA angles $\theta_0 = 10°$, $\theta_2 = 40°$ and $\theta_3 = -40°$, respectively, with the first signal being the desired one. We assume that the estimated DOA angle for the desired signal is $\theta_0 = 12°$ so that there is a 2° mismatch error. A Gaussian distributed random vector with zero mean and variance $\delta_e = 0.025$ will be added to the original steering vector. The parameters $\gamma = 3$, $\varepsilon = 3$ are used. Since all real-valued algorithms have the same performance as the FB-based ones except for their much lower computational complexity, in simulations we only consider the proposed real-valued algorithm.

Fig. 2 shows the output SINR performance of the original FO Capon (O-FOC) beamformer (4), the robust FO solution (R-FO) (14), and the robust real-valued solution (R-RV) (31), with respect to the signal-to-noise ratio (SNR) varied from 0dB to 20dB. The optimum SINR value shown

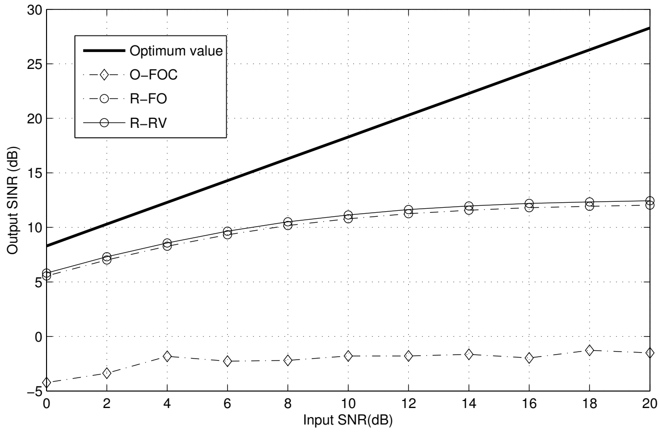

Fig. 2: Output SINR versus input SNR for coherent signals.

is for the ideal case without any errors. As we can see, the robust algorithm has achieved a much higher output SINR than the original ones (O-FOC), and the output SINR is increasing steadily with the increase of input SINR. The real-valued algorithm gives a little higher output SINR, which means that the gain due to FB processing is greater than the loss due to the structural error caused by the same processing.

5. CONCLUSIONS

A robust forward-only beamformer against both arbitrary steering vector errors and correlation matrix errors has been proposed based on worst-case optimization for coherent interfering signals. The forward-only solution is extended to forward-backward case with improved performance. By adding a preprocessing stage to the beamformer through a unitary transformation matrix, low-complexity robust algorithm with real-valued implementation was derived. Simulation results have shown that all the proposed algorithms can work effectively against the introduced steering vector and correlation matrix errors.

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