TIME SERIES ANALYSIS USING WAVELETS AND GJR-GARCH MODELS

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ABSTRACT

The aim of this paper is to provide an improved alternative to the classical econometric tools in the financial markets prediction. The idea of forecasting stock market future prices with wavelet analysis is the central element of this paper. Additionally to a wavelet analysis, an econometric model has been used in order to improve the performance of prediction. An algorithm which makes use of wavelets together with an econometric model is implemented in order to prove the advantages of wavelet analysis in financial forecasting. On the analyzed data we proved that our forecasting algorithm has achieved better results compared with the approach which is not using the wavelet transform.

Index Terms — wavelets, time series, GARCH models, stock market prediction

1. INTRODUCTION

The signal processing techniques have many applications in real world, economics and finance areas being some of them. Signals prediction, as called in the signals processing applications or time series forecasting (often used in finance and economics) is very popular in many fields such as (macro) economics, statistics or empirical finances.

In the financial domain of stock market forecasting, the future is hard to predict. Stock markets prediction is required by many investors and it is extremely important for the researchers who activate in the domains of analysis and forecasting of financial time series. The accurate forecasting of financial prices is an important factor in investment decision making. Since many economic and financial time series exhibits changing frequencies over time, the wavelets could be a good approach in time series analysis [12]. For this kind of time series, explaining and inferring from observed (serially correlated) data involves the usage of non-stationary models of their second order structure [1]. Therefore, variance or equivalently the spectral structure, are often changing their values over time. The recent interest on applications of wavelets has focused on multiple research areas from economics and finance. Some of these research directions are exploratory analysis, density estimation, time scale decomposition, time series interdependencies identifications and forecasting [3].

Despite its suitability for this field, the first important applications of wavelets in economics emerged only in the last decade [16]. Recently, several studies which are focused on application of wavelet in finance have been introduced in the literature, but only a few of them are presenting the performances of a wavelet analysis together with an econometrical prediction scheme. A forecasting method based on the wavelet analysis and GJR-GARCH model is, from the best of our knowledge, not available in the literature. The GJR-GARCH model was proposed as an enhancement of classical GARCH (Generalized Autoregressive Conditional hetroskedasticty) model by [8]. Time series are often modeled with GARCH in order to capture the heteroscedastic behavior. This involves that the studied properties (e.g. variance or mean) of a time series are not constant over time [2].

The next section of this paper is presenting some previous work related to financial forecasting using wavelets and some econometrical prediction concepts. Then the methodology, the prediction method and the used data set are described. In the end we present comparative results between our proposed method and the prediction method which is not using the wavelet transform.

2. PREVIOUS WORK

The importance of wavelet in finance and especially in stock market prediction has been stressed through various approaches. One very popular application of wavelets is de-noising financial data which represents the transaction from foreign exchange market or intraday transactions from the stock markets [7]. On the other hand, some approaches improved the accuracy of prediction using the wavelet multiresolution analysis [14]. It has been proved that unifying multiresolution analysis with the autoregressive processes will increase the forecasting precision [4] and as matter of fact, the autoregressive processes together with wavelet analysis have been applied also in prediction of internet traffic and stock prices [17]. One step forward in improving this type of forecasting was done by researches that used the ARMA (Auto Regressive Moving Average) models together with wavelet analysis. They achieved good results, better than just using a simple autoregressive model [22], [13]. If the underlying process is modeled more complex, then the accuracy of prediction is even better. In previous works have been used some GARCH models and theirs performance was proven to be quite good [11]. An extended GARCH model is used also in this paper which captures better the asymmetries of stock returns distributions [8].

3. METHODOLOGY

This section presents the main theoretical and implementation concepts related to the algorithm and the method used to predict the stock prices.

3.1. Wavelet analysis

The wavelets are signal processing tools derived from applied mathematics domain and one of theirs applicability is to give a more complex and useful representation of a signal (function or time series) without knowing the underling functional form of the signal structures [20].

The wavelets analysis is a mature field and the wavelet transforms are considered to be an enhancement of the traditional Fourier transforms [9], [15], [5]. The wavelet transform uses scalable windows which are shifted across the whole signal and for each localized part from the signal is obtained the scale decomposition. This window function (i.e. basis or mother wavelet) is convoluted with the signal, in order to compute the wavelet transform (i.e. the wavelet coefficients). Although, there are several mother wavelets, only some wavelets are suitable for the financial time series analysis [17]. The Haar and Daubechies mother wavelets are leading to good results since they capture better the economic and financial time series characteristics, like the non-stationarity [18], [23].

Usefulness of wavelet analysis has to do with its flexibility in handling a variety of non-stationary signals. Wavelets are constructed over finite intervals of time and they are not necessarily homogeneous over time, but they are localized in both time and scale.

Thus, two interesting features of wavelet time scale analysis for economic variables worth to be mentioned. First, since the base scale includes any non-stationary components, the data does not need to be detrended or differenced. Second, the nonparametric nature of wavelets takes care of potential nonlinear relationships without losing details [21].

The wavelet transform could be computed as continuous transform or as discrete one. The discrete wavelet transform (DWT) is often used in practice [16]. The stationary version of the DWT has better properties and some strengthens very useful for a proper time series analysis [17].

3.2. GARCH models in time series analysis

If P_t is the stock price or the value of an index at time *t*, then r_t is called the continuously compounded daily returns (i.e. the return) of the underlying stock at time *t* and is expressed as:

$$r_t = \ln(P_t) - \ln(P_{t-1}) \tag{1}$$

The volatility which measures the variation of stock price over time is (sometimes) associated with the standard deviations of a stock returns time series. The property of volatility or variance of being variable over time is known also as heteroscedasticity. This property is modeled by the GARCH models, which capture the phenomenon of volatility clustering [2]. Empirically, most of financial and economic phenomena exhibit the clustering of volatilities. A GARCH(*p,q*) model is expressed by two equations, the mean and the variance equations. In order to deal with the symmetry and asymmetry for the effects of both positive and negative shocks which affect almost every financial time series, the GARCH model were adjusted to this property [8].

Therefore, the GJR-GARCH model is described by the following equations, where the variance equation is a modified expression of the initial GARCH variance equation:

$$r_t = \mu_t + \varepsilon_t \text{, where } \varepsilon_t = \sigma_t z_t \tag{2}$$

$$\sigma_t^2 = \omega + \sum_{k=1}^{q} \beta_k \sigma_{t-k}^2 + \sum_{j=1}^{p} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \gamma_j D_{t-j} \varepsilon_{t-j}^2 \tag{3}$$

where $\omega > 0$, $\alpha_j \geq 0$, $\beta_k \geq 0$, z is a sequence of independent identically distributed (i.i.d.) random variables with mean 0 and variance 1 and

$$D_{t-j} = \begin{cases} 1, & \varepsilon_{t-j} < 0 \\ 0, & otherwise \end{cases} \tag{4}$$

This process is weakly stationary (in econometric sense) if :

$$\sum_{j=1}^{p} \alpha_j + \sum_{k=1}^{q} \beta_k + \frac{1}{2} \sum_{j=1}^{p} \gamma_j < 1, \tag{5}$$

$$\sum_{j=1}^{p} \alpha_j + \sum_{j=1}^{p} \gamma_j > 0 \tag{6}$$

Thus, its one step ahead forecast can be obtained from (3) by replacing *t* with *t+1*. Therefore, GARCH models describe the time evolution of the average size of squared errors, or in other words, the evolution of the magnitude of uncertainty [2]. Therefore, this forecasting approach is considered to be successful in predicting volatility changes and the return future value. Although, the GJR-GARCH model has a good prediction power, the achieved performance differs from one period of time to another, depending also on the volatility structure of the underlying time series [6].

3.3. Proposed method in stock market prediction

Forecasting stock market prices is probably one of the most interesting topics in time-series prediction. A method which can predict with a very high accuracy the future price of a stock or another financial instrument has not been discovered yet. Each prediction method has its own advantages and statistically has been proved that these methods are giving good results with a certain probability. The prediction scheme is applied on the multiple series (approximation and details coefficients at each level of decomposition) from wavelet domain instead of just applying a forecasting algorithm directly on the raw data as many econometric models do. Thus, having different resolution scales and using an adequate model for financial time series modeling, the prediction accuracy is improved.

Based on this, bellow is presented a schema of the proposed algorithm, followed by a brief description of each block.

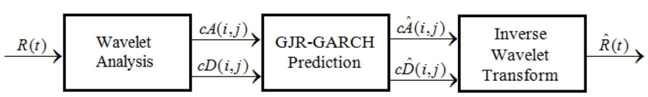

Figure 1. Prediction scheme using wavelets

First of all, the prices P_t for each index are transformed in returns r_t as presented in equation (1), since the GARCH models are using the returns instead of prices. Nevertheless, the wavelets can be applied on both prices series and return series.

We performed the prediction in the wavelet domain and for obtaining the wavelet coefficients from the return time series we used the stationary wavelet transform (SWT). The experiments were run with two mother wavelets, the Haar and the Daubechies. Although the Haar wavelet is considered to be the simplest mother wavelet function, the choice of this transform was motivated by the fact that its shape and analytical definition is similar to the financial time series patterns. The Daubechies is also a good alternative in this specific wavelet analysis since it captures the "spikes" and "shocks" from financial data. The choice for an undecimated wavelet transform, like SWT avoids aliasing problems [17].

The central element of the presented scheme is the prediction block. The SWT coefficients are modeled with the GJR-GARCH process. The mean and variance equations are presented in relations (2) and (3) respectively, where r_t is substituted by the wavelet approximation and details coefficients from each scale.

The residuals ε_t of GJR-GARCH model, from equation (2), are simulated using a Monte Carlo method in order to fulfill the definition requirements for these residuals [19]. For each time series we set the random generator seed to the same value for a proper reproducibly of experiments. When applying the wavelet transform and when making prediction only with the GJR-GARCH model, the simulation is performed in the same conditions.

In the end, the algorithm takes a random permutation sample from the simulated residuals [10]. The length of this sample is $M = N + hr$, where N is the initial size of analyzed data and hr is the length of forecasting horizon sample. Then, this sample is drawn from the residuals time series after a number of replications.

After the prediction is performed in the wavelet domain for all coefficient sets, the inverse SWT is applied in order to have the forecasted signal back in time domain. This block is followed by a transformation of returns into prices.

The efficiency of prediction could be estimated based on some quantitative measures. The used criteria are:

1) Root mean squared error (RMSE)

$$RMSE = \sqrt{\frac{1}{M}\sum_{t=1}^{M}(P_t - \hat{P}_t)^2} \qquad (7)$$

2) Mean absolute error (MAE)

$$MAE = \frac{1}{M}\sum_{t=1}^{M}\left|\frac{P_t - \hat{P}_t}{P_t}\right| \qquad (8)$$

In order to have a fair comparison, these measures are used when just applying the GJR-GARCH prediction and when applying this model together with wavelet transform. In the last equations M represents the number of observations used for analysis.

The proposed algorithm was implemented in Matlab and the experimental results are discussed in the experimental results section.

4. DATA SETS

The presented data analysis flow had been applied to stock indexes from two European countries. In order to prove to power of the presented method, one stock index is chosen from a very developed country and the other from an emerging country. Both of these kinds of economies are considered to be attractive for the investors. In the case of the developed market, the reason is related to stability, liquidity and a more stable risk structure. On the other hand, in the case of emerging market the reason for investing is strongly related to a very heterogeneous behavior of the market participants and to the possible return, which often is higher. That is, the names of the processed stock indexes are: FT100 (UK) and BET (Romania). The used data can be downloaded from each stock index web site or from the DataStream data provider.

For each country the data consists in one daily series which measures in a time domain framework the evolution of stock index values in points (sometimes denominated in the local currency). The time period for each data set is 10 years, starting from 3rd of January 2002 and ending in 30th December 2011.

Before presenting the results of the described analysis, we showed some statistical properties of each entirely time series in the following table:

	FT100	***BET***
Mean	0.000033	0.000716
Std. Dev.	0.01339	0.017853
Skewness	0.12093	-0.54927
Kurtosis	9.37484	10.08567

Table 1. Statistical properties of analyzed data sets

The time series characteristics presented in the previous table are most relevant in this study and they have a strong influence to prediction results.

5. RESULTS

In order to obtain the results for the prediction algorithm which use wavelet transform and GJR-GARCH one has to be sure that some prerequisites are fulfilled. The presented algorithm was applied for each of the two stock indexes (the BET and FT100). Since every stock index has its own statistical and mathematical characteristics, the prediction algorithm could be implemented slightly different in each case. We tried to avoid this kind of approach in order to provide a unitary processing methodology for each data set.

Therefore, the level of decomposition was set to be equal to 4. In the literature there is not a specific theory, proof, to show the appropriate level for decomposition when using SWT (or DWT) in financial time series analysis. Then, the horizon size used for prediction was set to be equal to 16 days.

In case of GJR-GARCH modeling, due to the specific volatility structure of each index (as shown also in Table 1), it is possible that models with higher order of p and q could give a better goodness of fit. Before taking the decision to use a GJR-GARCH(1,1) we applied specific tests and information criteria [20]. Therefore, this model is fitting well on each data set. The reduced number of parameter that have to be estimated and processed lead to a correct and compact model selection, which can be easily used for prediction.

The next table presents the summary of the experimental results. The first four rows are reflecting the prediction accuracy without wavelet transform. In these cases, the prediction algorithm is based only on the GARCH and GJR-GARCH model. In the last four rows are presented the results of prediction using the wavelets (Daubechies – rows 5-6 and Haar – rows 7-8) and GJR-GARCH model. The prediction measures described by equations (7) and (8) are computed and their values are listed in the table.

Nr. Crt.	Accuracy	***FT100***	***BET***
1	MAE(GARCH)	0.007903	0.029103
2	RSME(GARCH)	0.058442	0.087231
3	MAE(GJR)	0.007648	0.026452
4	RSME(GJR)	0.055971	0.080289
5	MAE(db2+GJR)	0.006294	0.028621
6	RSME(db2+GJR)	0.049203	0.071065
7	MAE(haar+GJR)	0.005817	0.019064
8	RSME(haar+GJR)	0.047037	0.065803

Table 2. Prediction results without wavelets (rows 1-4) and with wavelet analysis (rows 5-8). In the Accuracy column are the measures for the prediction accuracy, which depend on the used wavelet. GJR stands for GJR-GARCH

We presented also results using a simple GARCH(1,1) model (rows1-2) just for the purpose of having a fair comparison with the other methods. The GJR-GARCH is an enhancement of GARCH model, but the last sum from its variance equation – similar with equation (3) – is missing.

As shown in previous table the results are influenced, in a positive way, by the wavelet analysis. That is, the error of prediction when using the proposed algorithm is smaller than when just forecasting with GJR-GARCH or GARCH models. For the second time series, the BET index, when applying the Daubechies (db2) wavelet transform, the MAE measure is not better than the in the case of prediction only with GJR-GARCH model. This is due to the specific of this time series which exhibit a more dynamic structure as shown also in Table 1. Its statistical characteristics indicate that the BET index is more volatile and it will be harder to predict its future prices.

An interesting fact is related to the better results achieved for Haar wavelet compared with Daubechies wavelet. Thus, the “square-shaped” form of the Haar wavelet is more adequate to catch sudden changes encountered in stock market data.

6. CONCLUSIONS AND FUTURE WORK

We showed how wavelet analysis and GJR-GARCH prediction can be used as signal processing application for financial time series with only a few parameters to be estimated. Based on the experimental assessment, we demonstrate the powerfulness of this methodology.

Beside the fact that the volatility has a strongly asymmetric structure which influences the model selection the wavelet analysis is a useful tool in forecasting. Therefore, the proposed algorithm which uses the wavelet transform outperforms the traditional approaches in modeling and forecasting financial data.

The presented work is an approach in finding a better fitted model for stock market prediction. Thus, there are some aspects which can be improved. The econometric model used for modeling financial data could be enhanced and more complex model selection criteria can be used to achieve an even better goodness of fit. The wavelet analysis could be improved by using also other approaches like filtering or thresholding. The prediction method could be enhanced too, by using forecasting method based on lifting schemes and adaptive wavelets.

This study could be extended to a higher number of markets having different characteristics, in order to check how profit opportunities could occur in markets when using wavelet analysis. It could be also interesting to apply various technical analysis trading rules on these market indexes.

The results obtained in this paper strongly recommend the wavelet analysis and GJR-GARCH modeling as a good method in stock market prediction.

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