TWO DIFFERENT APPROACHES FOR WIDE BEAM SYNTHESIS:SECOND ORDER CONE PROGRAMMING AND SWARM INTELLIGENCE

Yasar Kemal Alp, Fatih Altiparmak, Gokhan Gok, Aydin Bayri, Aykut Arikan

Radar Electronic Warfare and Intelligence Systems Division, ASELSAN A.Ş.

ABSTRACT

Different beam patterns can be generated by imposing different weights and phases to the elements of a phased array. Wide beam patterns are essential for many applications from various fields such as military and telecommunications. In this work, we propose two different approaches for generating wide beam patterns having high power levels. In the first approach, the problem is modelled as a second order cone program (SOCP). In this model, the antenna weights are the optimization parameters. The global optimal solution to this model can be achieved by any SOCP solver. In the other approach, all the antenna elements are assumed to work at the same power level. By choosing only the element phases as the optimization parameters, a non-convex optimization problem is constructed. Particle swarm optimization (PSO) is employed for finding the best solution. Experiments show that, both methods have similar performances for generating wide beam patterns with high effective radiated powers (ERP). It is observed that, SOCP based method suppressed the beam pattern in the undesired azimuth interval more effectively, since its optimization parameters are chosen from a larger search space. However, along with the phase shifters, it requires inserting transducers into the system hardware.

Index Terms— beam pattern, SOCP, PSO, ERP

1. INTRODUCTION

Electronic beam shaping and steering properties of phased array antennas make them very popular for many transmitting/receiving systems [1]. By changing the element weights, beam patterns with arbitrary shapes can be generated [1, 2]. The desired beam pattern is application dependent. For instance, while a narrow beam width with reduced side lobe levels is essential for a tracking radar, a search radar may require wide beam width to increase the area of its range cell in azimuth or elevation.

For linear phased arrays, the literature is very dense for narrow beam pattern synthesis with reduced side lobe levels by tuning the antenna weights [1, 2, 3]. The simple relationship based on the Fourier transform between the antenna weights and the corresponding beam pattern enables using an FIR filter design approach for the problem [4]. For generating wide beam patterns with rectangular shapes, antenna weights can simply be chosen from samples of a $sinc$ function [2]. However, the resulting effective radiated power (ERP) in the desired azimuth interval becomes small.

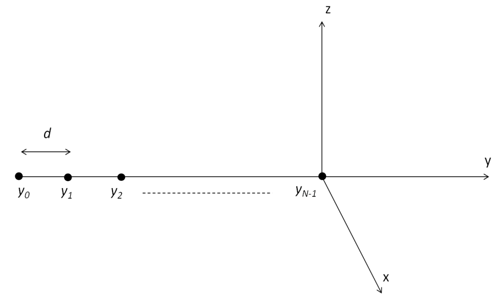

Fig. 1. Linear antenna array.

In this work, we propose two different approaches for generating wide beam patterns for linear antenna arrays. For a given azimuth interval $[\bar{\phi}_1, \bar{\phi}_2]$, we optimize the antenna element weights such the resulting pattern is localized in $[\bar{\phi}_1, \bar{\phi}_2]$ with uniform power distribution and suppressed outside $[\bar{\phi}_1, \bar{\phi}_2]$. In both approaches, our design criteria is: 1) The beam pattern should have as much energy as possible in $[\bar{\phi}_1, \bar{\phi}_2]$; 2) The beam pattern should have as minimum energy as possible outside$[\bar{\phi}_1, \bar{\phi}_2]$; 3) The power pattern should be as flat as possible in $[\bar{\phi}_1, \bar{\phi}_2]$. While the first and the second criteria impose localization of the beam energy in $[\bar{\phi}_1, \bar{\phi}_2]$, the third criteria forces the beam pattern to have a uniform power distribution in $[\bar{\phi}_1, \bar{\phi}_2]$. In the first approach, the problem is modelled as a quadratically constrained quadratic problem (QCQP), which has convex quadratic cost function and convex quadratic constraints. Then, it is converted to a second order cone problem (SOCP) in order to solve it at the global optimum point [5]. To satisfy high energy criteria, an iterative procedure is utilized. The resulting antenna weights are complex, which requires transducers and phase shifters in the system hardware.

In the second approach, in order to obtain a high power beam and simplify the antenna hardware by eliminating the need for transducers, the optimization parameters are chosen as the antenna phases. However, the constructed optimization problem is non-convex and has many local minimums. To solve this optimization problem, Particle Swarm Optimiza-

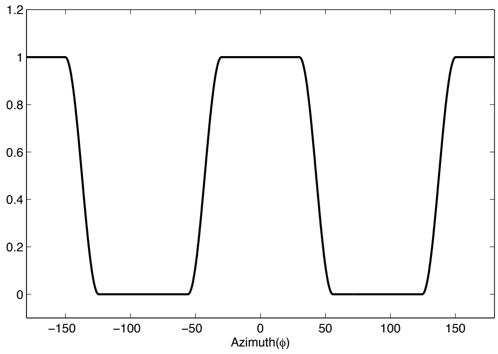

Fig. 2. Normalized magnitude of the spatial mask $M(\phi)$ for azimuth interval $[\bar{\phi}_1, \bar{\phi}_2] = [-30, 30]$ degrees.

tion (PSO) is utilized [6].

The organization of the paper is as follows. In Section-2, problem definition is given. In Section-3, the first approach is detailed. In Section-4, the second approach is explained. Results and comparisons are given in Section-5. Finally, concluding remarks are provided in Section-6.

Through out the paper, bold characters will be used for vectors/matrices. The operators $(.)^T$ and $(.)^H$ will denote vector/matrix transposition and hermitian operations, respectively. Complex numbers will be expressed by using the $j = \sqrt{-1}$ definition.

2. PROBLEM DEFINITION

Consider the linear antenna array of N elements shown in Fig-1. Here, the antenna element locations are given by $y_n = (n - (N-1))d,\ n = 0, 1, .., N-1$ where d is the element spacing. Let w_n denote the complex weight of the n^{th} antenna element, which can be implemented by a transducer and a phase shifter. Then, the beam pattern generated by the phased array is given by

$$B(\theta, \phi) = \sum_{n=0}^{N-1} w_n V_n(\theta, \phi)\ , \theta \in [o, \pi],\ \phi \in [\pi, \pi] \qquad (1)$$

where $V_n(\theta, \phi) = \exp\{j2\pi \frac{y_n}{\lambda} \sin(\theta) \sin(\phi)\}$, λ denotes the wavelength and ϕ, θ are the azimuth and elevation angles, respectively. For a certain elevation angle $\bar{\theta}$ and given azimuth interval $[\bar{\phi}_1, \bar{\phi}_2]$ $(-\pi/2 \leq \bar{\phi}_1 < \bar{\phi}_2 \leq \pi/2)$, a power pattern $|B(\theta, \phi)|^2$ that is localized inside $[\bar{\phi}_1, \bar{\phi}_2]$ with a uniform distribution and suppressed outside of it by using the complex antenna weights w_n, is to be generated . Also, high ERP in $[\bar{\phi}_1, \bar{\phi}_2]$ interval is desired. ERP is defined as

$$ERP(\theta, \phi) = P_T \frac{|B(\theta, \phi)|^2}{\frac{1}{4\pi} \int\limits_0^{\pi} \int\limits_{-\pi}^{\pi} |B(\theta, \phi)|^2 \sin(\theta) d\theta d\phi} \qquad (2)$$

where $P_T = \sum\limits_{n=0}^{N-1} |w_n|^2$ is the total antenna power. Through out this paper, it will be assumed that maximum allowed element power is 1 Watt, i.e. $|w_n|^2 \leq 1\ \forall n$. From now on, without loss of generality, the elevation angle will be set to $\bar{\theta} = \pi/2$ and the θ dependence in beam pattern will be dropped.

Algorithm 1 Optimal Antenna Weight Selection with Iterative Phase Adaptation

1: %**Initializations**
2: $i \leftarrow 0$
3: $\psi^i(\phi) = 0,\ \phi \in [-\pi, \pi]$
4: $\hat{\mathbf{w}}^i(n) = 1/\sqrt{2}\ n = 1, 2, .., 2N$
5: %**Phase Adaptation**
6: Find optimal coefficient vector $\hat{\mathbf{w}}^i$ by solving (6)
7: $B^i(\phi) = \mathbf{V}_p(\phi)\hat{\mathbf{w}}^i + j\mathbf{V}_q(\phi)\hat{\mathbf{w}}^i$
8: $\psi^{i+1}(\phi) = \angle B^i(\phi)$
9: $i \leftarrow i + 1$
10: **if** $\|\mathbf{w}^i - \mathbf{w}^{i-1}\|^2 \leq \zeta$ **then**
11: Terminate iterations
12: **else**
13: Return to 6
14: **end if**

3. WIDE BEAM SYNTHESIS BY USING SECOND ORDER CONE PROGRAMMING

To impose a shape to the beam pattern $B(\phi)$, consider the following cost function

$$J_1 = \int_{-\pi}^{+\pi} |B(\phi) - \alpha M(\phi) \exp\{j\psi(\phi)\}|^2 d\phi \qquad (3)$$

where $M(\phi)$ is the magnitude, $\psi(\phi)$ is the phase and $\alpha > 0$ is the amplitude of the spatial mask in azimuth. Forming this mask will be detailed in the next subsections. To ease the notation, the following definitions are made:
$\mathbf{w} = [w_0, w_1, .., w_{N-1}]^T$, $\mathbf{w}_R = \Re\{\mathbf{w}\}$, $\mathbf{w}_I = \Im\{\mathbf{w}\}$,
$\hat{\mathbf{w}} = [\mathbf{w}_R; \mathbf{w}_I]$, $\mathbf{V}(\phi) = [V_0(\phi), V_1(\phi), .., V_{N-1}(\phi)]$,
$\mathbf{V}_p(\phi) = [\Re\{\mathbf{V}(\phi)\}, -\Im\{\mathbf{V}(\phi)\}]$,
$\mathbf{V}_q(\phi) = [\Im\{\mathbf{V}(\phi)\}, Re\{\mathbf{V}(\phi)\}]$,
$M_R(\phi) = \Re\{\alpha M(\phi) \exp\{j\psi(\phi)\}\}$,
$M_I(\phi) = \Im\{\alpha M(\phi) \exp\{j\psi(\phi)\}\}$.
Here $\Re\{.\}$ and $\Im\{.\}$ are the operators which return the real and imaginer parts of their arguments, respectively. The cost function in (3) can be written by:

$$J_1 = \hat{\mathbf{w}}^T \mathbf{P} \hat{\mathbf{w}} + 2\mathbf{q}^T \hat{\mathbf{w}} \qquad (4)$$

where $\mathbf{P} = \int \left(\mathbf{V}_p^T(\phi)\mathbf{V}_p(\phi) + \mathbf{V}_q^T(\phi)\mathbf{V}_q(\phi)\right) d\phi$ and $\mathbf{q}^T = \int \left(\mathbf{V}_p(\phi) M_R(\phi) + \mathbf{V}_q(\phi) M_I(\phi)\right) d\phi$. The power constraint $|w_n|^2 \leq 1\ \forall n$ can be restated as $\hat{\mathbf{w}}(n+1)^2 + \hat{\mathbf{w}}(n+1+N)^2 \leq 1\ \forall n$ in terms of the new variable vector $\hat{\mathbf{w}}$. Then, the following optimization problem is constructed:

$$P1: \ \min_{\hat{\mathbf{w}}} \hat{\mathbf{w}}^T \mathbf{P} \hat{\mathbf{w}} + 2\mathbf{q}^T \hat{\mathbf{w}}$$
$$\text{s.t. } \hat{\mathbf{w}}(n+1)^2 + \hat{\mathbf{w}}(n+1+N)^2 \leq 1\ \forall n\ . \qquad (5)$$

This is a quadratically constrained quadratic problem (QCQP) and can be solved at the global optimum point by transforming it into a second order cone program (SOCP) [5]:

$$P2: \ \min_{\hat{\mathbf{w}}, t} t + 2\mathbf{q}^T \hat{\mathbf{w}}$$
$$\text{s.t: } \|\mathbf{A}\hat{\mathbf{w}}\|^2 \leq t$$
$$\|\mathbf{A}_n \hat{\mathbf{w}}\| \leq 1\ \forall n = 0, 1, .., N-1\ . \qquad (6)$$

where $\mathbf{A}$ is the matrix such that $\mathbf{A}\mathbf{A}^H = \mathbf{P}$ and $\mathbf{A}_n$ is a $2N \times 2N$ matrix composed of all zeros except $\mathbf{A}_n(n+1, n+1) = 1, \mathbf{A}_n(n+1+N, n+1+N) = 1$. This optimization problem has linear cost function, a rotated cone constraint and a second order cone constraint. It can be solved by any SOCP solver at the global optimum point. In this work we used SEDUMI [7].

3.1. Selection of Magnitude of the Spatial Mask

The desired normalized power pattern shape is 1 in $[\bar{\phi}_1, \bar{\phi}_2]$ interval and 0 elsewhere. Then the magnitude of the normalized spatial mask should be chosen as a rectangular function with its non-zero region $[\bar{\phi}_1, \bar{\phi}_2]$. Since zero-one transition of a rectangular window is very strict, a smoother window function can be used. In this work, we used a raised cosine window given by:

$$H(\phi) = \begin{cases} 1 & \text{if } |\phi| \leq \frac{1-\beta}{2T}, \\ \frac{1}{2}\left[1 + \cos\left(\frac{\pi T}{\beta}\left(|\phi| - \frac{1-\beta}{2T}\right)\right)\right] & \text{if } \frac{1-\beta}{2T} < \phi < \frac{1+\beta}{2T} \\ 0 & \text{else} \end{cases} \tag{7}$$

where, $T = \frac{1}{2(\bar{\phi}_2 - \bar{\phi}_1)}$, β is the smoothness parameter, which was typically chosen as $\beta = 0.3$ in this work. Since $B(\phi) = B(\pi - \phi)\ \forall\phi \in [0, \pi/2]$ and $B(\phi) = B(-\pi - \phi)\ \forall\phi \in [-\pi/2, 0]$, $M(\phi)$ can be chosen as

$$M(\phi) = H(\phi) + H(\pi - \phi) + H(-\pi - \phi)\,. \tag{8}$$

In Fig.2, the magnitude of the spatial mask $M(\phi)$ for $[\bar{\phi}_1, \bar{\phi}_2] = [-30, 30]$ degrees interval is shown.

3.2. Selection of Phase of the Spatial Mask

Although the magnitude of the mask is available as explained in the previous subsection, an appropriate phase $\psi(\phi)$ should be chosen to minimize the cost function given in (3). To find the most appropriate phase, an iterative phase adaptation procedure given in Algorithm-1 is used [8]. First zero phase is assigned to the mask and the optimization problem in (6) is solved. Once the optimal coefficient vector $\hat{\mathbf{w}}$ is found, the beam pattern $B(\phi)$ is calculated. Then, its phase is assigned to the mask and the optimization problem in (6) is resolved. Iterations are terminated when the normalized distance between the coefficient vectors found in two consecutive iterations, i.e., $\|\hat{\mathbf{w}}^i - \hat{\mathbf{w}}^{i-1}\|/(2N)$ is less than a predefined threshold ζ, which was typically chosen as $\zeta = 0.01$. This phase adaptation method is detailed in Algorithm-1.

3.3. Achieving High ERP

Since $P2$ doesn't include any constraint imposing high ERP to the antenna weights, for a low choice of spatial mask amplitude α, the optimal coefficient vector $\hat{\mathbf{w}}$ returned by Algorithm-1 provides a perfect fit to the spatial mask. However, the total antenna power $\|\hat{\mathbf{w}}\|^2$ is small. If a very large α is selected, the power pattern has high ripples in $[\bar{\phi}_1, \bar{\phi}_2]$ region. To achieve high ERP, while perceiving low ripples in $[\bar{\phi}_1, \bar{\phi}_2]$ region, we start with a low α value and iteratively increase it. When the maximum amplitude difference of $|B(\phi)|^2$ in $[\bar{\phi}_1, \bar{\phi}_2]$ is higher than a threshold η, which was typically chosen as $\eta = 4$dB, the iterations are terminated and the optimal coefficient vector of the previous iteration is returned. Details of this method is provided in Algorithm-2

Algorithm 2 Optimal Antenna Weight Selection with High ERP

1: %**Initializations**
2: $j \leftarrow 0$
3: $\alpha \leftarrow \alpha_0$
4: %**Iterative Antenna Weight Selection with High ERP**
5: Run Algorithm-1, find optimal antenna weight vector $\hat{\mathbf{w}}$
6: Calculate $|B(\phi)|^2$
7: **if** $|\max_{\bar{\phi}_1 \leq \phi \leq \bar{\phi}_2} \{|B(\phi)|^2\} - \min_{\bar{\phi}_1 \leq \phi \leq \bar{\phi}_2} \{|B(\phi)|^2\}| < \eta$ **then**
8: $\quad \alpha \leftarrow \alpha + \Delta\alpha$
9: $\quad$ Return to 5
10: **else**
11: $\quad$ Terminate iterations
12: **end if**

4. WIDE BEAM SYNTHESIS VIA SWARM INTELLIGENCE

In this approach, to achieve high ERP, we force the antenna elements to work with full power. The complex antenna weights are modelled as $w_n = \exp\{j\beta_n\}$ and the beam pattern is given by $B(\phi) = \sum_{n=0}^{N-1} \exp\{j\beta_n\} V_n(\phi)$. Note that, with this model only the antenna phases $\beta_n, n = 0, 1, .., N-1$ will be optimized. Consider the following cost function:

$$J_2 = \int_{-\pi}^{\pi} \left[|B(\phi)| - \alpha M(\phi)\right]^2 d \tag{9}$$

where $M(\phi)$ is the magnitude of the normalized spatial mask shown in Fig.2 and α is its amplitude level. By this cost function definition, the magnitude square of the difference between the magnitude of the beam pattern and the spatial mask will be minimized. The following optimization problem is constructed:

$$P3: \min_{\alpha;\boldsymbol{\beta}} \int_{-\pi}^{\pi} \left[|B(\phi)| - \alpha M(\phi)\right]^2 d\phi \tag{10}$$

where $\boldsymbol{\beta} = [\beta_0, \beta_1, .., \beta_{N-1}]$. Note that, the mask amplitude α is an optimization parameter. However, since α has linear relation with $|B(\phi)|$, its optimal value in least squares sense can be found as a function of $\boldsymbol{\beta}$:

$$\tilde{\alpha}(\boldsymbol{\beta}) = \int_{-\pi}^{\pi} |B(\phi)| M(\phi) d\phi \Big/ \int_{-\pi}^{\pi} M(\phi)^2 d\phi \tag{11}$$

Then the optimization problem $P3$ takes the following form:

$$P4: \min_{\boldsymbol{\beta}} \int_{-\pi}^{\pi} \left[|B(\phi)| - \tilde{\alpha}(\boldsymbol{\beta}) M(\phi)\right]^2 d\phi \tag{12}$$

The optimal solution of $P4$ provides the best fit to the spatial mask in terms of minimizing the magnitude square of the error $|B(\phi)| - \tilde{\alpha}(\boldsymbol{\beta}) M(\phi)$. Since the magnitudes of antenna weights are all 1, high ERP criteria is also achieved. However, to impose uniform power distribution to the beam pattern in $[\bar{\phi}_1, \bar{\phi}_2]$ interval, the maximum deviation of $|B(\phi)|^2$ in $[\bar{\phi}_1, \bar{\phi}_2]$ should be penalized. To overcome this problem, a penalty term is appended to $P4$ and a new optimization problem is constructed:

$$
\begin{aligned}
P5 : \min_{\boldsymbol{\beta}} & \int_{\Phi_p} [|B(\phi)| - \tilde{\alpha}(\boldsymbol{\beta}) M(\phi)]^2 \, d\phi / (\bar{\phi}_2 - \bar{\phi}_1) \\
& + \lambda \int_{\Phi_q} [|B(\phi)| - \tilde{\alpha}(\boldsymbol{\beta}) M(\phi)]^2 \, d\phi / (\bar{\phi}_2 - \bar{\phi}_1) \\
& + (1-\lambda) \left[\max_{\bar{\phi}_1 \leq \phi \leq \bar{\phi}_2} \{|B(\phi)|^2\} - \min_{\bar{\phi}_1 \leq \phi \leq \bar{\phi}_2} \{|B(\phi)|^2\} \right]
\end{aligned} \tag{13}
$$

where $\Phi_p = [-\pi/2, \bar{\phi}_1) \cup (\bar{\phi}_2, \pi/2]$, $\Phi_q = [\bar{\phi}_1, \bar{\phi}_2]$. While the first and second term in $P5$ controls the adaptation to the spatial mask, third term controls the maximum deviation of the power pattern in Φ_q region and λ is the trade off parameter between the second and the third term, which was typically chosen as $\lambda = 0.7$ in this work.

4.1. Particle Swarm Optimization

The cost function of $P5$ is non-convex, has a very complicated structure and includes multiple local minimums. As a result gradient-descent based solvers would not yield reliable solutions, since their performance highly depends on the provided initialization points. To solve $P5$, we used particle swarm optimization (PSO).

PSO is a very famous optimization technique used for minimizing non-convex functions [6]. Since the algorithm is very easy to implement, it has been used by researchers from various fields. In this algorithm, first, particles are randomly distributed to the solution space. Then, cost function is evaluated for each particle and location of each particle is updated such that all of the particles are moved towards the specific particle for which the cost function takes the lowest value. Cost function is reevaluate and location update is reperformed. Iterations are terminated when a predefined maximum allowed iteration number is achieved. Details of PSO algorithm adapted to our problem is given in Algorithm-3.

Algorithm 3 Particle Swarm Optimization for Antenna Phase Selection

%**Definitions**
%N_d: *dimension of the search space*
%N_p: *number of particles*
%$J(.)$: *cost function given in (13)*
%$\mathbf{x}_k^i \in \mathbb{R}^{N_b \times 1}$: *Location of the k^{th} particle in the search space at the i^{th} iteration*
%$\mathbf{v}_k^i \in \mathbb{R}^{N_b \times 1}$: *Velocity of the k^{th} particle in the search space at the i^{th} iteration*
%$\mathbf{p}_k \in \mathbb{R}^{N_b \times 1}$:*Location of the k^{th} particle in the search space for which the cost function took its minimum value untill the current iteration*
%J_k^p: *Value of the cost function at* $\mathbf{p}_k$
%$\mathbf{g} \in \mathbb{R}^{N_b \times 1}$: *Particle location for which the cost function took its minimum value until the current iteration*
%J^g: *Value of the cost function at* $\mathbf{g}$
%N^i: *Iteration number*
%**Initialization**
$i \leftarrow 0$
$\mathbf{x}_k^i = \text{rand}(Np, 1) \times 2\pi - \pi$
$\mathbf{v}_k^i = \text{rand}(Np, 1)$
$J_k^p = \infty, k = 1, 2, .., N_p$
$J^g = \infty$
%**PSO iterations**
while $i < N_i$ **do**
    for $k = 1 : N_p$ **do**
        if $J(\mathbf{x}_k^i) < J_k^p \, \& -\pi \preceq \mathbf{x}_k^i \preceq \pi$ **then**
            $\mathbf{p}_k \leftarrow \mathbf{x}_k^i$
            $J_k^p \leftarrow J(\mathbf{x}_k^i)$
        end if
        if $J(\mathbf{p}_k) < J^g$ **then**
            $\mathbf{g} \leftarrow \mathbf{p}_k$
            $J^g \leftarrow J(\mathbf{p}_k)$
        end if
    end for
    for $k = 1 : N_p$ **do**
        $\mathbf{v}_k^{i+1} = 0.72984 \times [\mathbf{v}_k^i + 2.05 \times \text{rand}(N_b, 1)(\mathbf{p}_k - \mathbf{x}_k^i) + 2.05 \times \text{rand}(N_b, 1)(\mathbf{g} - \mathbf{x}_k^i)]$
        $\mathbf{x}_k^{i+1} = \mathbf{x}_k^i + \mathbf{v}_k^{i+1}$
    end for
    $i \leftarrow i + 1$
end while

5. RESULTS AND COMPARISONS

Performance of the proposed two methods were tested and compared by generating beam patterns with different beam widths. The simulations were performed for $N = 18$ linear antenna elements located on y axis as shown in Fig.1. The ratio of the element spacing to the wavelength was set to $d/\lambda = 0.479$. Once the optimal antenna weights were found, the corresponding ERPs given in (2) were computed.

For the first experiment, the azimuth interval was selected as $[\bar{\phi}_1, \bar{\phi}_2] = [-10, 10]$ degrees to generate a beam pattern with 20 degrees beam width in azimuth. The elevation angle $\bar{\theta} = \pi/2$ cut of the ERPs computed from the antenna weights returned by the first (SOCP based-blue) and the second (PSO based-red) method are plotted in Fig.3. Also, for comparison purposes, the $\bar{\theta} = \pi/2$ cut of the ERPs of the narrow beam (B1-black) generated by setting all the antenna weights to 1 and large beam(B2-green) generated by setting only the first two antenna weights to 1, remaining ones to 0 are provided. As observed, both methods performed similarly and successfully generated the desired beam patterns. It is reported that achieved total antenna power for the first method was $\|\mathbf{w}\|^2 = 17.9$ Watts (For the second method, $\|\mathbf{w}\|^2 = 18$ Watts, since it uses only the antenna phases).

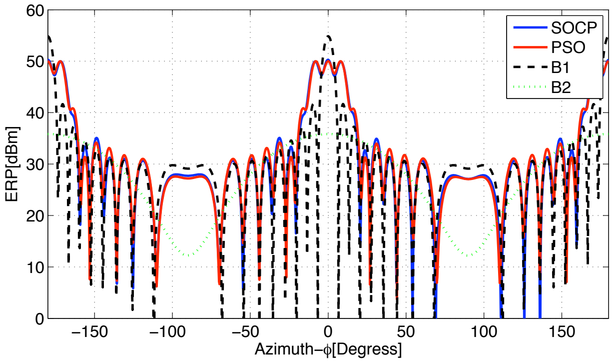

Fig. 3. $\bar{\theta} = \pi/2$ cut of ERP computed for the beam patterns generated by SOCP (blue) and PSO based (red) methods for $[\bar{\phi}_1, \bar{\phi}_2,] = [-10, 10]$ degrees interval. High ERP narrow beam (B1-black) and low ERP wide beam (B2-green) are also provided.

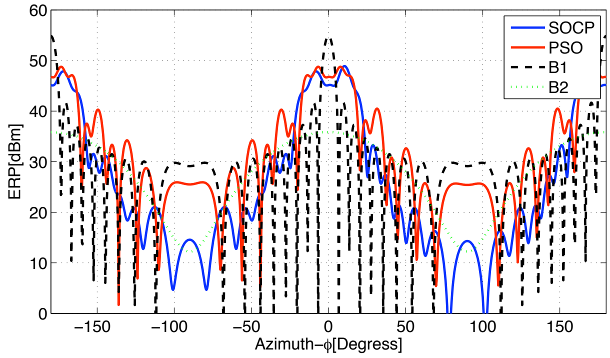

Fig. 4. $\bar{\theta} = \pi/2$ cut of ERP computed for the beam patterns generated by SOCP (blue) and PSO based (red) methods for $[\bar{\phi}_1, \bar{\phi}_2,] = [-15, 15]$ degrees interval.

The azimuth interval $[\bar{\phi}_1, \bar{\phi}_2] = [-15, 15]$ was used for the second design and the results are shown in Fig.4. Although both methods achieved the same power level in $[\bar{\phi}_1, \bar{\phi}_2]$ region, SOCP based method suppressed the pattern in $[-\pi/2, \bar{\phi}_1] \cup [\bar{\phi}_2, \pi/2]$ region more effectively and achieved a total antenna power of $\|\mathbf{w}\|^2 = 16.5$ Watts.

For the last experiment, we set the desired azimuth interval to $[\bar{\phi}_1, \bar{\phi}_2] = [-25, 25]$ degrees to generate a very wide beam pattern. As observed, SOCP based method performed better for suppressing the pattern in $[-\pi/2, \bar{\phi}_1] \cup [\bar{\phi}_2, \pi/2]$ region and achieved a total antenna power of $\|\mathbf{w}\|^2 = 17.5$ Watts.

6. CONCLUSIONS

In this work, we proposed two different approaches for generating wide beam patterns having high power levels. In the first approach, the problem is modelled as a SOCP where the optimization parameters are the complex antenna weights. In the other method, only antenna phases are optimized by utilizing PSO. Although both methods satisfy the high ERP criteria, SOCP based method performs better in terms of pattern suppression in the undesired azimuth interval. Since it optimizes

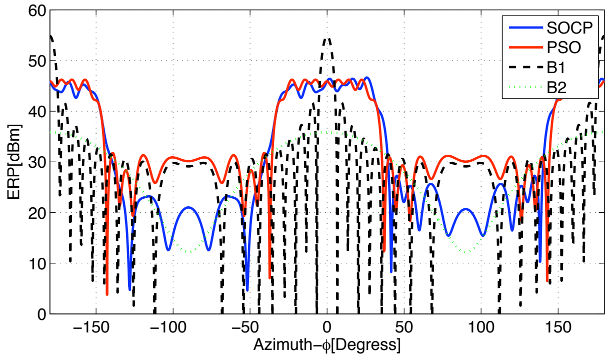

Fig. 5. $\bar{\theta} = \pi/2$ cut of ERP computed for the beam patterns generated by SOCP (blue) and PSO based (red) methods for $[\bar{\phi}_1, \bar{\phi}_2,] = [-25, 25]$ degrees interval.

the antenna element weights rather than phases, the resulting implementation requires inclusion of transducers in the system hardware. On the other hand, PSO based method optimizes only element phases, which makes this method more practical for system level implementations.

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