RESOURCE ALLOCATION BETWEEN FEEDBACK AND FORWARD LINKS: IMPACT ON SYSTEM PERFORMANCE AND CSI

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ABSTRACT

The performance of multiple-input multiple-output (MIMO) communication systems is greatly increased by exploiting channel state information (CSI) for the transmitter design. In systems where channel reciprocity does not hold, a limited feedback link can be used to send the CSI from the receiver to the transmitter. However, the resources for the feedback link come at the expense of resources for the forward link. This paper studies the trade-off between the accuracy of the feedback information and system performance. The optimum resource allocation is presented as the solution to an analytic equation for different beamforming transmission schemes to maximize the worst-case performance, in a framework including both time-division duplexing (TDD) and frequency-division duplexing (FDD).

1. INTRODUCTION

Multiple-input multiple-output (MIMO) communication channels are known to provide significant gains in system capacity [1], [2] and resilience to fading [3], [4]. These gains depend strongly on the quantity and quality of the channel state information (CSI) which is available during the design [5]. Obviously, the best performance is achieved when such CSI is complete and perfect, but this is not a realistic assumption, specially at the transmitter. In scenarios where channel reciprocity does not hold, a feedback link with limited capacity can be used to send the CSI from the receiver to the transmitter. In this sense, there has been extensive research on feedback techniques and quantization procedures to be applied to the channel estimates at the receiver, such as [6, 7, 8] and references therein.

In most cases the performance analysis is evaluated without taking into account the cost of using feedback. If this cost is taken into account explicitly it turns out that, while using a large amount of feedback improves the quality of the CSI available at the transmitter, it might not be optimum, since the remaining radio resources available for the forward data link are lower. Furthermore, the differential performance gain obtained by each additional feedback bit is a decreasing function and, eventually, it becomes smaller than the cost of dedicating an additional bit to feedback. The work in [9] presents an preliminary study of such trade-off and a purely numerical optimization of the radio resources. In this paper we go one step further and evaluate analytically the tradeoff and the associated radio resource allocation optimization problem.

Other works such as [10, 11] and [12] also deal with this issue, but differ in the following ways. In [10], random codebooks are used to quantize and transmit the CSI through the feedback link. The authors present a bound on the ergodic capacity loss as a function of the number of feedback bits. The work in [11] considers the achievable rate tradeoff analysis in two-way beamforming frequency-division duplexing (FDD) systems using Grassmannian codebooks and random vector quantization codebooks for the feedback, and features a resource optimization based on the allocation of power among the training, feedback and data transmission phases while keeping the length of each phase constant. The work from [12] considers multiple-input single-output (MISO) and MIMO systems with random vector quantization (RVQ) feedback and studies the performance when both the coherence block length and the number of transmit antennas become large.

In this paper we consider a framework which is valid for both time-division duplexing (TDD) and FDD, and study the tradeoff in a one way MIMO system with feedback of the channel Gram matrix, which contains the smallest amount of CSI required to perform the optimum linear transceiver design for all the usual design criteria [13, 14]. The tradeoff is presented for a general quantization and feedback scheme, and the particular case of uniform quantization of the elements of the channel Gram matrix exploiting the Hermitian property of the matrix is derived analytically as an illustrative example. The transmit energy is kept constant for each frame and the optimization is done over the radio resources (time in TDD, or frequency in FDD systems) devoted to feedback and data transmission. We present a general framework and derive, as an illustrative example, the optimum allocation of radio resources that maximizes the worst-case achievable data rate and the optimum allocation of radio resources that maximizes the worst-case fixed-length packet rate. The worst-case optimization regards the combination of the worst possible propagation channel and the realization of the CSI quantization error that minimizes the performance in such channel. These quality criteria based on worst-case scenarios are associated to systems designed to guarantee a minimum performance. However, note that other design criteria such as the maximization of the average achievable rate are also possible within this framework.

The paper is organized as follows. The system and signal models are given in section 2. The model for the CSI at the transmitter and the feedback quantization error are described in section 3. Section 4 presents the optimization of the tradeoff between the CSI accuracy at the transmitter and the performance in the data transmission, considering resource allocation. Simulations of such performance are shown in section 5, while section 6 concludes the paper.

2. SYSTEM AND SIGNAL MODELS

A single-user flat fading MIMO channel model is considered, with n_T and n_R transmit and receive antennas, respectively, represented by $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$, as depicted in Fig. 1. The system is described by the following equation:

$$\mathbf{y} = \mathbf{H}\mathbf{B}\mathbf{x} + \mathbf{w} \in \mathbb{C}^{n_R}, \quad (1)$$

where $\mathbf{x} \in \mathbb{C}^{n_S}$ represents the n_S streams of symbols to be transmitted with $\mathbb{E}\left\{\mathbf{x}\mathbf{x}^H\right\} = \mathbf{I}$, $\mathbf{y} \in \mathbb{C}^{n_R}$ represents the n_R received samples, and $\mathbf{B} \in \mathbb{C}^{n_T \times n_S}$ is the linear transmitter matrix that must satisfy the

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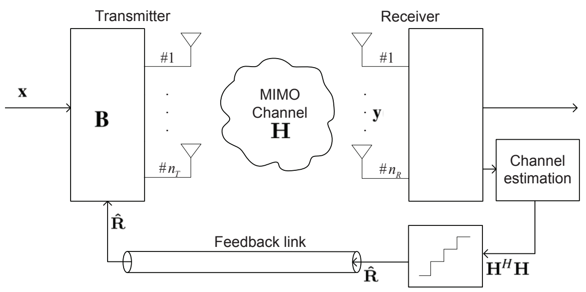

Figure 1: System model.

mean transmit power constraint $\|\mathbf{B}\|_F^2 \leq P_T$ ($\|\cdot\|_F$ stands for the Frobenius norm). As proved in [13], [14], the optimum matrix $\mathbf{B}$ depends on the channel Gram matrix $\mathbf{H}^H\mathbf{H}$ for all the usual design criteria. Additive white Gaussian noise (AWGN) is considered at the receiver, and is represented by $\mathbf{w} \in \mathbb{C}^{n_R}$, with $\mathbb{E}\{\mathbf{w}\mathbf{w}^H\} = \sigma_w^2\mathbf{I}$.

The performance associated to the data transmission in the forward link is represented by a function f that depends on the channel matrix $\mathbf{H}$, the error in the CSI at the transmitter $\boldsymbol{\Delta}$, and the allocation of radio resources (time or frequency) dedicated to the transmission of feedback and data. In this paper the optimum radio resource allocation is derived analytically for two different design criteria based on the worst-case scenario regarding the channel and CSI error realizations: the data rate in the forward link and the average number of error-free packets received per transmission frame.

In the next subsections a variable associated to the radio resource allocation between the forward and feedback links will be introduced in the system model, in order to study the tradeoff between the achievable communication performance in the forward link and the feedback load. In systems where the control information and data streams share the same physical communications link, the available radio resources have to be shared. In the case considered in this paper the transmission of data and CSI share the same pool of radio resources. For the practical implementation, two duplexing schemes are considered: dividing the time axis in different time slots and assigning each slot to the transmission of either data or feedback information (TDD), and dividing the frequency band into different subchannels corresponding to feedback or data transmission (FDD). It will be shown that, for resource allocation purposes, both schemes are dual, and the resource allocation will be optimized for the general case. Note that in order to provide a more realistic analysis a total energy constraint at the transmitter will be considered, instead of a total power constraint. This ensures that the same energy will be consumed for the transmission of data regardless of the duration or bandwidth of the feedback link transmission. For the equations describing these schemes the following notation is used: W_t stands for the total available bandwidth, and W_d represents bandwidth dedicated to data transmission. The total duration of a time frame is given by T_t, while T_d is the time dedicated to the transmission of data. E_t represents the total available energy for the transmission of data in a frame, and N_0 is the noise power spectral density (AWGN).

A single beamforming transceiver is considered, and the radio resource allocation is optimized to maximize the worst-case performance. Note that in the single beamforming transceiver design the precoder $\mathbf{B} \in \mathbb{C}^{n_T \times n_S}$ is a column vector, and will be denoted as the transmit beamforming vector $\mathbf{b} \in \mathbb{C}^{n_T}$, which satisfies $\|\mathbf{b}\|_F^2 \leq 1$. The next subsections describe the two particular cost functions that are considered in this paper. Note, however, that the same process can also be applied to other design criteria.

2.1 Transmit rate

2.1.1 Frequency-division duplexing (FDD):

In the FDD scheme, the total available bandwidth W_t is divided among the data and the feedback links, as depicted in Fig. 2.

In such a system, the maximum achievable data rate R_d is given by the following expression, which is an increasing function of the available bandwidth W_d:

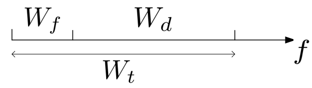

Figure 2: Frequency sharing in FDD systems.

$$R_d^{\text{FDD}} = W_d \log_2\left(1 + \frac{\frac{E_t}{T_t}}{W_d N_0}\mathbf{b}^H\mathbf{H}^H\mathbf{H}\mathbf{b}\right) \text{ (bits/s).} \quad (2)$$

2.1.2 Time-division duplexing (TDD):

On the other hand, the TDD scheme makes use of the complete bandwidth to transmit either data or feedback information[1]. The scheduling is performed in the time domain, i.e., there are time slots in which all the bandwidth is devoted to data transmission and in the other time slots all the bandwidth is dedicated to the feedback link, as depicted in Fig. 3.

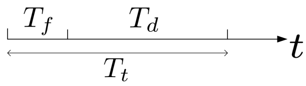

Figure 3: Time sharing in TDD systems.

In a TDD system, the maximum achievable data rate is an increasing function of the time devoted to transmitting data, and is given by the following expression:

$$R_d^{\text{TDD}} = \frac{T_d}{T_t} W_t \log_2\left(1 + \frac{\frac{E_t}{T_d}}{W_t N_0}\mathbf{b}^H\mathbf{H}^H\mathbf{H}\mathbf{b}\right) \text{ (bits/s).} \quad (3)$$

2.1.3 General expression (TDD & FDD):

As observed in (2) and (3), the expressions of the data rate for both TDD and FDD are dual, and they behave exactly the same as a function of variables T_d and W_d, respectively. It is possible to jointly formulate this dependance (based on (2) and (3)) as:

$$R_d = \frac{T_d}{T_t}\frac{W_d}{W_t} W_t \log_2\left(1 + \frac{E_t}{T_t W_t N_0}\frac{1}{\frac{T_d}{T_t}\frac{W_d}{W_t}}\mathbf{b}^H\mathbf{H}^H\mathbf{H}\mathbf{b}\right) \text{ (bits/s).} \quad (4)$$

The case where $T_d = T_t$ corresponds to FDD, and $W_d = W_t$ corresponds to TDD. Normalizing the bandwidth, (4) can also be written as:

$$f_1(\alpha) = \alpha \log_2\left(1 + \frac{\xi}{\alpha}\mathbf{b}^H\mathbf{H}^H\mathbf{H}\mathbf{b}\right) \text{ (bits/s),} \quad (5)$$

where $\alpha = \frac{T_d}{T_t}\frac{W_d}{W_t}$ $(0 \leq \alpha \leq 1)$ and $\xi = \frac{E_t}{T_t W_t N_0}$.

Following from (5), the optimum allocation of resources for data and feedback transmissions according this criterion can be performed using parameter α, as shown in section 4.1.

2.2 Practical fixed-length packet rate

In this subsection a more practical design scenario is also considered, where the transmission is structured in frames, each consisting of L bits, and data packets of L_p bits. From the L total bits, n_{b_T} bits are dedicated to the transmission of CSI feedback while

[1] In the literature it is usually assumed that in TDD systems there is channel reciprocity and therefore feedback is not required. In practical systems, however, the radio frequency (RF) chains have a different response for transmission and for reception. There are two solutions to this issue: one option is to still perform feedback of the complete CSI (which includes obviously the effect of the RF chain) and the other option is to perform a calibration of the RF chains for transmission and for reception. In this paper only the feedback solution is considered.

the remaining $L-n_{b_T}$ bits are devoted to data transmission. The system quality criterion is the average number of error-free packets received per frame, R_p, and is given by

$$R_p = \frac{L-n_{b_T}}{L_p}(1-\mathrm{PER}) = \frac{L-n_{b_T}}{L_p}(1-\mathrm{BER})^{L_p}, \tag{6}$$

which, assuming a QPSK modulation for the data transmission results in the following cost function[2]

$$R_p = \frac{L-n_{b_T}}{L_p}\left(1-Q\left(\sqrt{\xi \mathbf{b}^H\mathbf{H}^H\mathbf{H}\mathbf{b}}\right)\right)^{L_p}. \tag{7}$$

Observe that the effect of the parameter T_d or W_d, (for TDD or FDD schemes, respectively) can be expressed through the parameter α, as in the previous section. In (7), α has impact on two terms of the equation. On one hand, the number of packets transmitted per frame, $\frac{L-n_{b_T}}{L_p}$, grows with α, while on the other hand the resemblance of $\mathbf{b}$ with the optimum precoding vector (and therefore the value of $\mathbf{b}^H\mathbf{H}^H\mathbf{H}\mathbf{b}$) decreases with α. The optimization of the resource allocation following this criterion is derived in section 4.2.

3. FEEDBACK MODEL AND CSI AT THE TRANSMITTER

In the model presented in section 2, the CSI at the transmitter is obtained through a limited feedback link from the receiver, that is assumed to have perfect knowledge of the propagation channel. Since the feedback link has limited capacity it is necessary to quantize the CSI prior to sending it through the link, which introduces a quantization error in the CSI available at the transmitter. In this paper the channel Gram matrix $\mathbf{H}^H\mathbf{H}$ is normalized, quantized, and sent through the feedback link. The motivation behind sending the channel Gram matrix through the feedback link is that it contains the minimum necessary information for the design of the optimum linear precoder design for the usual design criteria, as proved in [13, 14]. First, the channel matrix $\mathbf{H}$ is divided at the receiver by a normalizing factor β, i.e., $\widetilde{\mathbf{H}} = \frac{\mathbf{H}}{\beta}$; $\beta = \|\mathbf{H}\|_F$. The normalized channel Gram matrix $\mathbf{R} \equiv \widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}}$ is then quantized and fed back. This means that only a quantized version of $\mathbf{R}$, denoted by $\hat{\mathbf{R}}$, is available at the transmitter[3]. The quantization introduces a quantization error $\mathbf{\Delta}$, as expressed by the following equation:

$$\hat{\mathbf{R}} = \mathbf{R} + \mathbf{\Delta}. \tag{8}$$

In this paper we assume that the quantization of $\mathbf{R}$ is performed at the receiver using a uniform quantization of the real and imaginary parts of each element independently as is done for example in [16] (in that work, a multiuser scenario is considered, but the feedback scheme can be applied also to the single user case), i.e., since the matrix is Hermitian and of size $n_T \times n_T$, there are n_T^2 different real elements to be quantized (the real and imaginary parts of the i,jth element of $\mathbf{R}, \forall i<j$ and the real part of the n_T elements of the diagonal of $\mathbf{R}$). Consequently, the quantization error is bounded as: $-\frac{\varepsilon}{2} \le \mathfrak{Im}\{\Delta_{ij}\} \le \frac{\varepsilon}{2}, -\frac{\varepsilon}{2} \le \mathfrak{Re}\{\Delta_{ij}\} \le \frac{\varepsilon}{2}; \forall i \ne j$ and $-\frac{\varepsilon}{2} \le \Delta_{ii} \le \frac{\varepsilon}{2}$, where Δ_{ij} is the element i,j of $\mathbf{\Delta}$, ε is the quantization step given by $\varepsilon = \frac{\gamma}{2^{n_b}}$, γ is the dynamic range taken from the quantization, and n_b is the number of bits used to quantize each element. Since there are n_T^2 real elements to be quantized, the total number of required quantization bits is given by $n_{b_T} = n_b n_T^2$. Using the system model described in the previous section for the feedback link, the number of bits available to quantize each parameter is given by

$$n_b = (1-\alpha)W_t T_t R_f / n_T^2, \tag{9}$$

where R_f is the transmission rate of the feedback link, in bits/s/Hz, which provides a negligible feedback error rate.

4. OPTIMIZATION OF THE RADIO RESOURCE ALLOCATION

The precoder that maximizes the design criteria in sections 2.1 and 2.2 corresponds to the eigenvector associated to the largest eigenvalue of the channel Gram matrix [14]. Since at the transmitter the available estimation of the channel Gram matrix $\hat{\mathbf{R}}$ contains the quantization error $\mathbf{\Delta}$, the beamforming vector used for the transmission corresponds to $\mathbf{b} = \sqrt{P_t}\mathrm{u}_{\max}\left(\hat{\mathbf{R}}\right)$, where P_t is the transmit power and the operator $\mathrm{u}_{\max}(\cdot)$ computes the eigenvector associated to the largest eigenvalue of a matrix, denoted as $\lambda_{\max}(\cdot)$.

First, we define the signal to noise ratio (SNR) as a function of the channel realization and the error of the CSI at the transmitter $\mathbf{\Delta}$. We then obtain an expression for a lower bound on the performance (i.e., minimum SNR) for the worst case of both the CSI error $\mathbf{\Delta}$ and the channel realization (or equivalently, the channel Gram matrix $\mathbf{R}$), jointly with the worst-case β.

Since a worst-case scenario is considered in this paper, we assume that the CSI error is smaller than the norm of the nominal value of the Gram matrix (otherwise the worst case would be zero and the design would not be applicable).

Given these considerations, and in the described scenario where the beamforming vector $\mathbf{b}$ corresponds to the eigenvector associated to the largest eigenvalue of the channel Gram matrix estimation $\hat{\mathbf{R}}$, the $\mathrm{SNR}(\mathbf{R},\mathbf{\Delta})$ is expressed as:

$$\begin{aligned}
\mathrm{SNR} &= \xi \mathbf{b}^H\mathbf{H}^H\mathbf{H}\mathbf{b} = \xi\beta^2\mathbf{b}^H\mathbf{R}\mathbf{b} = \xi\beta^2\left(\mathbf{b}^H\hat{\mathbf{R}}\mathbf{b} - \mathbf{b}^H\mathbf{\Delta}\mathbf{b}\right)\\
&= \xi\beta^2\left(\lambda_{\max}(\hat{\mathbf{R}}) - \mathbf{b}^H\mathbf{\Delta}\mathbf{b}\right) \ge \xi\beta^2\left(\frac{\mathrm{Tr}(\hat{\mathbf{R}})}{n_T} - \mathbf{b}^H\mathbf{\Delta}\mathbf{b}\right)\\
&= \xi\beta^2\left(\frac{\mathrm{Tr}(\mathbf{R})}{n_T} + \frac{1}{n_T}\sum_{i=1}^{n_T}\Delta_{ii} - \sum_{i=1}^{n_T}\sum_{j=1}^{n_T} b_i^*\Delta_{ij}b_j\right)\\
&= \xi\beta^2\left(\frac{1}{n_T} + \sum_{i=1}^{n_T}\Delta_{ii}\left(\frac{1}{n_T} - b_i^*b_i\right) - \sum_{i=1}^{n_T}\sum_{j\ne i}^{n_T} b_i^*\Delta_{ij}b_j\right)\\
&\ge \xi\beta^2\left(\frac{1}{n_T} - \frac{\varepsilon}{2}\sum_{i=1}^{n_T}\left|\frac{1}{n_T} - |b_i|^2\right| - \left|\sum_{i=1}^{n_T}\sum_{j\ne i}^{n_T} b_i^*\Delta_{ij}b_j\right|\right)\\
&\ge \xi\beta^2\left(\frac{1}{n_T} - \varepsilon\frac{n_T-1}{n_T} - \sum_{i=1}^{n_T}|b_i|\sum_{j\ne i}^{n_T}|\Delta_{ij}b_j|\right)\\
&\ge \xi\beta^2\left(\frac{1}{n_T} - \varepsilon\frac{n_T-1}{n_T} - \sum_{i=1}^{n_T}|b_i|\sqrt{\sum_{j\ne i}^{n_T}|\Delta_{ij}|^2}\sqrt{\sum_{j\ne i}^{n_T}|b_j|^2}\right) \quad (10)\\
&\ge \xi\beta^2\left(\frac{1}{n_T} - \varepsilon\frac{n_T-1}{n_T} - \sum_{i=1}^{n_T}|b_i|\,\varepsilon\sqrt{\frac{n_T-1}{2}}\right)\\
&\ge \xi\beta_{\mathrm{wc}}^2\left(\frac{1}{n_T} - \varepsilon\frac{n_T-1}{n_T} - \varepsilon\sqrt{\frac{n_T-1}{2}}\sqrt{\sum_{i=1}^{n_T}|b_i|^2}\sqrt{\sum_{i=1}^{n_T}1}\right) \quad (11)\\
&= \xi\beta_{\mathrm{wc}}^2\left(\frac{1}{n_T} - \varepsilon\left(\frac{n_T-1}{n_T} + \sqrt{\frac{n_T(n_T-1)}{2}}\right)\right) \equiv \mathrm{SNR}_{\mathrm{wc}}, \quad (12)
\end{aligned}$$

where β_{wc} is the worst-case normalization factor (i.e., $\beta_{\mathrm{wc}} \le \beta$), * denotes the conjugate operator, b_i is the ith element of vector $\mathbf{b}$, and the Cauchy-Schwarz inequality was used in (10) and (11).

[2] Note that $\mathrm{BER} = Q\left(\sqrt{2\frac{E_b}{N_0}}\right)$, where E_b is the transmit energy per bit, as shown in [15].

[3] Note that the normalization factor β does not impact the optimum single-beamformer transceiver design considered in this paper. For multi-stream systems that require knowledge of β at the transmitter, the quantization algorithms could be used to feed back the scalar parameter β without adding significant overhead.

Finally, developing the expression of ε in (12), results in

$$\mathrm{SNR}_{\mathrm{WC}} = \frac{\xi\beta_{\mathrm{wc}}^2}{n_T} - \frac{\beta_{\mathrm{wc}}^2\xi\gamma\left(\frac{n_T-1}{n_T}+\sqrt{\frac{n_T(n_T-1)}{2}}\right)}{2^{(1-\alpha)\frac{W_tT_tR_f}{n_T^2}}}. \quad (13)$$

For the sake of clarity, the following notation is used through this section: $C_1 = \frac{\xi\beta_{\mathrm{wc}}^2}{n_T}$, $C_2 = \xi\gamma\beta_{\mathrm{wc}}^2\left(\frac{n_T-1}{n_T}+\sqrt{\frac{n_T(n_T-1)}{2}}\right)$, and $C_3 = \frac{W_tT_tR_f}{n_T^2}$. This results in

$$\mathrm{SNR}_{\mathrm{WC}} = C_1 - \frac{C_2}{2^{(1-\alpha)C_3}}. \quad (14)$$

4.1 Maximization of the worst-case transmit rate

As described in section 2.1, the cost function for the maximization of the minimum achievable rate is given by

$$f_1(\alpha) = \alpha\log_2\left(1+\frac{1}{\alpha}\mathrm{SNR}_{\mathrm{WC}}\right), \quad (15)$$

where $\mathrm{SNR}_{\mathrm{WC}}$ is the lower bound on the SNR given by (13).

Then, the optimization problem to compute the optimum resource allocation α_{opt_1} can be expressed as

$$\alpha_{opt_1} = \arg\max_{\alpha}\alpha\log_2\left(1+\alpha^{-1}\left(C_1-\frac{C_2}{2^{(1-\alpha)C_3}}\right)\right), \quad (16)$$

with $0 \le \alpha \le 1$. Since $\left.\frac{df_1(\alpha)}{d\alpha}\right|_{\alpha=0} > 0$, $\left.\frac{df_1(\alpha)}{d\alpha}\right|_{\alpha=1} < 0$, and there is only a single value of α that makes the derivative $\frac{df_1(\alpha)}{d\alpha}$ equal to zero, the optimum value of α corresponds to this value. The following expression to determine α_{opt_1} is then obtained:

$$\log_2\left(1+\alpha_{opt_1}^{-1}\left(C_1-C_2 2^{-(1-\alpha_{opt_1})C_3}\right)\right) + \frac{\left(\frac{C_2}{\ln 2}-C_2C_3\alpha_{opt_1}\right)2^{-(1-\alpha_{d_1})C_3}-\frac{C_1}{\ln 2}}{\alpha_{opt_1}-C_2 2^{-(1-\alpha_{opt_1})C_3}+C_1} = 0. \quad (17)$$

4.2 Maximization of the worst-case packet error rate

Using the model described in section 2.2, the cost function for the maximization of the worst-case average number of error-free packets received per frame is:

$$f_2(\alpha) = \frac{L-n_{b_T}}{L_p}\left(1-Q\left(\sqrt{\mathrm{SNR}_{\mathrm{WC}}}\right)\right)^{L_p} \quad (18)$$

$$= \frac{L-(1-\alpha)C_3n_T^2}{L_p}\left(1-Q\left(\sqrt{C_1-\frac{C_2}{2^{(1-\alpha)C_3}}}\right)\right)^{L_p}. \quad (19)$$

Consequently, the optimization of the resource allocation α_{opt_2} can be expressed as:

$$\alpha_{opt_2} = \arg\max_{\alpha}\frac{L-(1-\alpha)C_3n_T^2}{L_p}\left(1-Q\left(\sqrt{C_1-\frac{C_2}{2^{(1-\alpha)C_3}}}\right)\right)^{L_p}. \quad (20)$$

It can be shown straightforwardly that $f_2(\alpha)$ is an increasing fuction at $\alpha = 0$, a decreasing function at $\alpha = 1$, and there is only a single value of α that makes the derivative $\frac{df_2(\alpha)}{d\alpha}$ equal to zero. Therefore, the optimum value α_{opt_2} corresponds to the value that

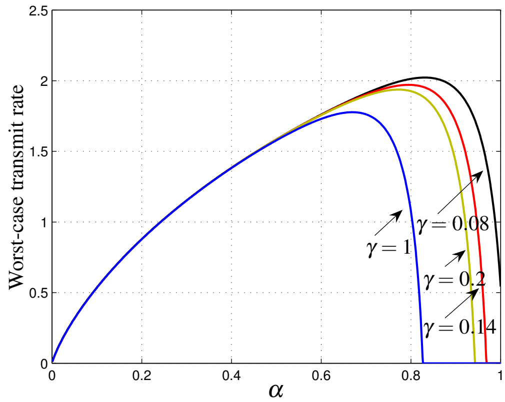

Figure 4: Worst-case transmit rate versus α for different values of the dynamic range γ.

nulls the first derivative. The value of α_{opt_2} is then obtained from

$$\frac{C_3n_T^2}{L_p}\left(\frac{1}{2}+\frac{1}{2}\mathrm{erf}\left(\sqrt{\frac{C_1}{2}-\frac{C_2}{2^{(1-\alpha_{opt_2})C_3+1}}}\right)\right)^{L_p} + \frac{L-(1-\alpha_{opt_2})C_3n_T^2}{2\sqrt{2\pi}}C_2C_3 2^{-(1-\alpha_{opt_2})C_3}\ln 2$$
$$\times\left(\frac{1}{2}+\frac{1}{2}\mathrm{erf}\left(\sqrt{\frac{C_1}{2}-\frac{C_2}{2^{(1-\alpha_{opt_2})C_3+1}}}\right)\right)^{L_p-1}\exp\left(-\frac{C_1}{2}\right)$$
$$\times\exp\left(\frac{C_2}{2^{(1-\alpha_{opt_2})C_3+1}}\right)\left(C_1-\frac{C_2}{2^{(1-\alpha_{opt_2})C_3}}\right)^{-1/2} = 0, \quad (21)$$

where we used the error function, $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\mathrm{d}t$.

5. SIMULATION RESULTS

In this section the results of the analysis performed in this paper are shown graphically. First, the optimization for the maximization of the worst-case transmit rate studied in section 4.1 is considered. The scenario used for these simulations is as follows: $E_t = 20, W_t = 1, T_t = 1, N_0 = 1, \beta_{\mathrm{wc}} = 1, R_f = 500$ bits/s/Hz, and $n_T = 5$ antennas. Fig. 4 shows the worst-case achievable rate as a function of the radio resource allocation parameter α for different values of the CSI error variance, which is related to the dynamic range γ of the quantizer used at the receiver. Fig. 5 shows the optimum resource allocation as a function of ξ, which is related to the transmit energy, for different values of γ. Observe that the allocation of resources to the transmission of data, represented by α, that maximizes the worst-case transmit rate increases as γ decreases, which means that if the error in the CSI is smaller, the proposed scheme allocates less resources to the feedback link and more to the data transmission. If the error is larger (higher γ), more resources are required for the feedback link in order to have a useful CSI at the transmitter, and α is lower.

The optimization for the maximization of the average number of error-free packets received per frame, studied in section 4.2 is considered next. For these simulations, the following scenario is considered: $L = 100$ bits, $L_p = 50$ bits, $E_t = 20, W_t = 1, T_t = 1, N_0 = 1, \beta_{\mathrm{wc}} = 1, R_f = 80$ bits/s/Hz, and $n_T = 5$ antennas. Fig. 6 shows the worst-case average number of error-free packets received per frame as a function of the radio resource allocation parameter α for different values of γ, while Fig. 7 shows the optimum resource allocation as a function of ξ, for different values of γ. As in the previous case, α_{opt_2} decreases for higher CSI error γ. Also note that in every case the optimum value of α is lower than 1, which means that the feedback improves the performance in such cases.

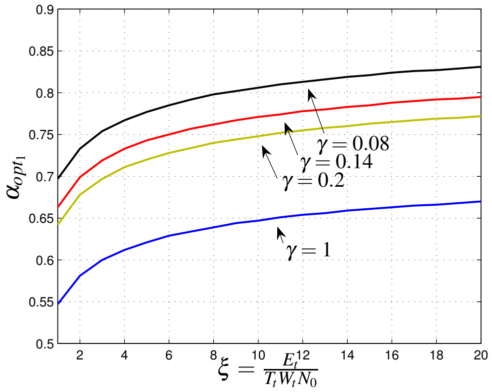

Figure 5: Optimum value of α for the maximization of the worst-case transmit rate versus ξ for different error scenarios.

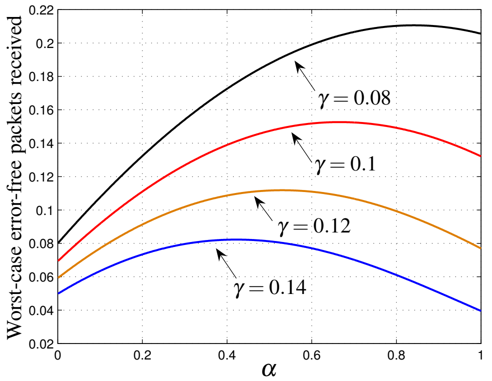

Figure 6: Worst-case number of error-free packets packets received per frame versus α for different values of the dynamic range γ.

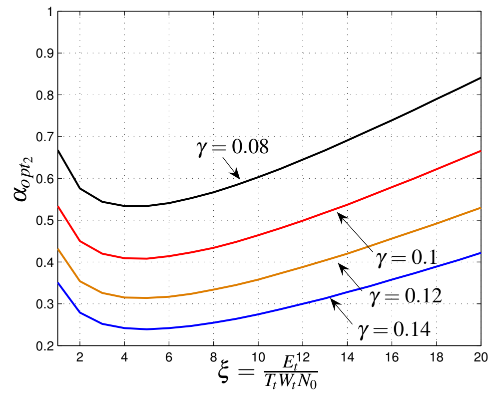

Figure 7: Optimum value of α for the maximization of the worst-case number of error-free packets received per frame versus ξ.

6. CONCLUSIONS

In this paper an analysis of the resource allocation (time and bandwidth) between the data and the feedback links of a MIMO communication system is presented and the optimum allocation strategy is derived. The tradeoff between the accuracy of the CSI at the transmitter and the radio resources for the forward link is based on the principle that resources for the feedback transmission come at a cost of resources for the data transmission. There is an optimum resource allocation strategy that maximizes system performance for any given quality criterion. The particular cases of worst-case data rate and average fixed-length packet rate are optimized analytically and the result is evaluated numerically for different scenarios. It is shown that, even when the cost of feedback transmission is taken into account, the benefits of CSI at the transmitter outweigh this cost, which results in better performance in the considered scenarios where the resource allocation is performed properly.

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