PERFORMANCE ANALYSIS OF THE DISTRIBUTED ZF BEAMFORMER IN THE PRESENCE OF CARRIER FREQUENCY OFFSET

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ABSTRACT

We study the impact of residual carrier frequency offset (CFO) on the performance of the distributed zero-forcing (ZF) beamformer in the interference channel, i.e., where multiple nodes simultaneously transmit the same data towards multiple receivers. Distributed transmissions have been proposed to mitigate the co-channel interference inherent to such scenarios and provide the gains of multi-antenna systems. However, frequency synchronization among the cooperating transmitters is required. Even when the transmitters perform frequency synchronization before transmission, a residual CFO is to be expected that degrades the performance of the system due to the in-phase misalignment of the incoming streams. This paper presents the signal-to-noise ratio analytically and the diversity semi-numerically of the distributed ZF beamformer for the ideal case and in presence of a residual CFO. We illustrate our results and their accuracy through simulations.

1. INTRODUCTION

Coordinated transmissions, where multiple cells cooperate to simultaneously transmit towards one or multiple receivers, have gained much attention recently as a means to provide the spectral efficiency and data rate targeted by emerging standards [1]. They have the potential to improve the per-user capacity or the performance of the users at the cell edge at a low cost, i.e., no need for new infrastructures or expensive devices.

In coordinated transmissions, the joint computation of the beamforming weights achieves optimal performance [2]. However, its implementation in a distributed network is challenging due to the complexity of the joint beamforming and the extensive sharing of information between the transmit cells. Conversely, distributed (yet coordinated) beamforming schemes where each cell exploits the knowledge of the information data but only a limited knowledge about the channels are a more practical alternative [3]. Besides the difficult exchange of the data and CSI between the transmit cells, coordinated systems rely on perfect synchronization between the different cells which is also challenging to achieve.

Carrier frequency offset (CFO) is caused by the mobility of the wireless devices (Doppler effect) and by the non-ideality of the local oscillator embedded in each wireless transceiver. CFO is a major source of impairment in orthogonal frequency division modulation (OFDM) schemes [4]. In coordinated communications, each stream originates from a distinct source, each with a different frequency error. As a result, the receiver must cope with multiple carrier frequency offsets. Because of the additive nature of the channel, the

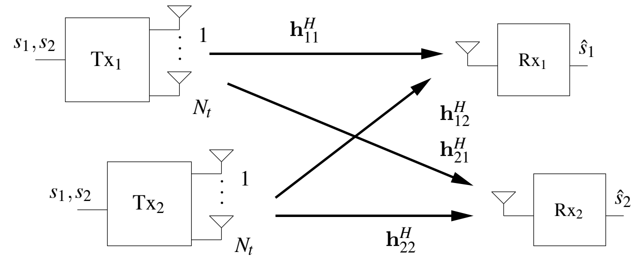

Figure 1: System model of a coordinated scheme in flat fading channels where both transmitters communicate simultaneously toward both receivers.

receiver cannot compensate for all the CFOs separately and the possibly destructive combination of the incoming streams is then impossible to correct. The most logical method to mitigate the effects of CFO consists in compensating the frequency offset prior to transmission

In practical scenarios, the perfect synchronization of the wireless devices is challenging and a residual CFO remains after synchronization. It is therefore of interest to understand its impact on coordinated communications. In multi-user systems the CFOs has been shown to degrade the accuracy of the beamformer hence decreasing the capacity [5]. Works also include the study of the phase mismatch in distributed schemes, i.e., phase offset errors [6] and of the time-varying phase drift on the performance [7]. These results are complementary to the results presented here, i.e., we consider the time- and CFO-dependent phase mismatch.

The SNR gain and diversity order are commonly used to measure the performance of multiple antenna systems. They are well known for single-user (SU) scenarios [8] and have also been proposed for amplify-and-forward relay scenarios. However, the literature does not evaluate the effects of residual CFO on those gains achieved with coordinated schemes, where the transmitters share the same time and frequency resources for transmitting a common data towards

We study the effects of a residual CFO on the performance of the distributed zero-forcing (ZF) beamforming scheme. We first introduce the system model (Section 2) and derive in Section 3 the SNR gain analytically and the diversity gain numerically assuming perfect synchronization. Next, we define the system model for multiple CFOs and propose the derivations of the SNR and diversity gains when residual CFO is present (Section 4). We show that the performance decreases with time as the residual CFO introduces a misalignment of the incoming streams. Section 5 shows the performance of the cooperative scheme for both the ideal

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case and when residual CFO is present and discuss the proposed derivations. Section 6 concludes our work.

The following notations are used: the vectors and matrices are in boldface letters, vectors are denoted by lower-case and matrices by capital letters. The superscript $(\cdot)^H$ denotes the Hermitian transpose operator, and $(\cdot)^\dagger$ denotes the pseudo inverse. $E[\cdot]$ is the expectation operator, $\mathbf{I}_N$ is an identity matrix of size $(N \times N)$ and $\mathbb{C}^{N\times 1}$ denotes the set of complex vectors of size $(N \times 1)$. The definition $\mathbf{x} \sim \mathbb{C}N(\mathbf{0}, \sigma^2 \mathbf{I}_N)$ means that the vector $\mathbf{x}$ of size $N \times 1$ has zero-mean Gaussian distributed independent complex elements with variance σ^2. We define $|\mathbf{a}^n|$ as the norm of the n^{th} element of the vector $\mathbf{a}$.

2. SYSTEM MODEL

We consider a distributed beamforming system where two independent nodes transmit simultaneously to two receivers. Fig. 1 shows the system model. We assume that the transmitters share information about the data to transmit. Each transmitter is equipped with $N_t \geq 2$ transmit antennas while the receiver has a single antenna. We assume flat fading channels and present the derivations for the single carrier case. We consider that a prior-to transmission frequency synchronization is performed so that only a residual CFO is present at the receivers.

The channel vector is composed of independent and identically distributed (i.i.d.) Rayleigh fading elements of unit variance, i.e., $\mathbf{h}_{ik} \sim \mathbb{C}N(\mathbf{0}, \mathbf{I}_{N_t})$. It models the N_t channels between receiver i and transmitter k with $i,k = 1,2$. We denote by $s_i \in \mathbb{C}^{1\times 1}$ the transmitted symbol to the receiver Rx_i where $E_{s_i}[|s_i|^2] = 1$. Each transmitter exploits only the knowledge of the channels from its own antennas to both receivers, i.e., Tx_1 has the channel knowledge of $\mathbf{h}_{11}^H$ and $\mathbf{h}_{21}^H$ and Tx_2 has the channel knowledge of $\mathbf{h}_{22}^H$ and $\mathbf{h}_{12}^H$. As a result, only the local computation of the beamforming weights is achievable. At the channel output, the received signals at Rx_i are denoted by $y_i \in \mathbb{C}^{1\times 1}$ and can be expressed as

$$
\begin{aligned}
y_1 &= \left(\mathbf{h}_{11}^H \mathbf{w}_{11} + \mathbf{h}_{12}^H \mathbf{w}_{21}\right) s_1 + \left(\mathbf{h}_{11}^H \mathbf{w}_{12} + \mathbf{h}_{12}^H \mathbf{w}_{22}\right) s_2 + n \\
y_2 &= \left(\mathbf{h}_{21}^H \mathbf{w}_{12} + \mathbf{h}_{22}^H \mathbf{w}_{22}\right) s_2 + \left(\mathbf{h}_{21}^H \mathbf{w}_{11} + \mathbf{h}_{22}^H \mathbf{w}_{21}\right) s_1 + n
\end{aligned} \tag{1}
$$

where $\mathbf{w}_{il} \in \mathbb{C}^{N_t \times 1}$ denotes the beamforming vector from the transmitter i towards the l^{th} receiver. The beamforming vectors satisfy the following power constraint $\mathbf{w}_{il}^H \mathbf{w}_{il} \leq P_i$ for $i = 1,2 \quad l = 1,2$. P_i denotes the transmit power dedicated to each receiver at Tx_i (a given transmitter allocates the transmit power evenly to both receivers). The term $n \in \mathbb{C}^{1\times 1}$ is the zero-mean circularly symmetric complex additive white Gaussian noise (AWGN) with variance σ_n^2. We consider a ZF beamformer. Such a beamformer exploits the knowledge of the channels from its own antennas to choose the beamforming vector that places the nulls in the direction of the non-targeted user while maximizing the energy towards the desired user. The computation of the beamforming weights can be decomposed into two steps: null beamforming and maximal energy beamforming. We focus on the computation of the weights for Tx_1, a similar approach can be done for Tx_2.

To cancel the interference towards the non-targeted user, the matrix $\mathbf{Z}_{ij} \in \mathbb{C}^{N_t \times N_t}$ is used as the orthogonal projection onto the orthogonal complement of the column space of $\mathbf{h}_{ij}$, e.g., from Tx_1 to cancel interference at Rx_1 and Rx_2

$$
\begin{aligned}
\mathbf{Z}_{11} &= \mathbf{I}_{N_t} - \mathbf{h}_{11}\left(\mathbf{h}_{11}^H \mathbf{h}_{11}\right)^{-1} \mathbf{h}_{11}^H \\
\mathbf{Z}_{21} &= \mathbf{I}_{N_t} - \mathbf{h}_{21}\left(\mathbf{h}_{21}^H \mathbf{h}_{21}\right)^{-1} \mathbf{h}_{21}^H.
\end{aligned} \tag{2}
$$

Next, the transmit maximum-ratio combining (MRC) beamformer is then applied towards the targeted-user [8]. The weights are chosen from the complementary space of the projection matrix to maximize the energy towards the receivers

$$
\mathbf{w}_{11} = \sqrt{P_1}\frac{\mathbf{Z}_{21}\mathbf{h}_{11}}{||\mathbf{Z}_{21}\mathbf{h}_{11}||} \quad \text{and} \quad \mathbf{w}_{12} = \sqrt{P_1}\frac{\mathbf{Z}_{11}\mathbf{h}_{21}}{||\mathbf{Z}_{11}\mathbf{h}_{21}||} \tag{3}
$$

which fulfills the power constraint. Since the ZF beamforming weights lay in the null space of the non-targeted user, the received signal is interference free, Equations in (1) become

$$
\begin{aligned}
y_1 &= \left(\mathbf{h}_{11}^H \mathbf{w}_{11} + \mathbf{h}_{12}^H \mathbf{w}_{21}\right) s_1 + n \\
y_2 &= \left(\mathbf{h}_{21}^H \mathbf{w}_{12} + \mathbf{h}_{22}^H \mathbf{w}_{22}\right) s_2 + n.
\end{aligned} \tag{4}
$$

3. SNR AND DIVERSITY GAINS

Section 3.1 proposes the derivations of the average SNR and compare it to the SNR gain in single-user scenarios. The derivation of the diversity is proposed in Section 3.2

3.1 Average SNR Gain

For the sake of clarity, the derivations are performed for Rx_1 only. In the following derivations, we assume a zero-forcing equalizer at the receiver, i.e., the complex scalar inversion of the equivalent channel. From Eq. (4), after processing at the receiver, we obtain the instantaneous output SNR from Rx_1 (γ_1), for one channel realization, by taking the expectations over the noise and the symbols. It is given as

$$
\gamma_1 = \frac{1}{\sigma_n^2}\left(\mathbf{h}_{11}^H \mathbf{w}_{11} + \mathbf{h}_{12}^H \mathbf{w}_{21}\right)^2. \tag{5}
$$

Next, we average γ_1 over the channel realizations to obtain the average SNR

$$
\begin{aligned}
E[\gamma_1] &= \frac{1}{\sigma_n^2} E\left[\left(\mathbf{h}_{11}^H \mathbf{w}_{11} + \mathbf{h}_{12}^H \mathbf{w}_{21}\right)^2\right] \\
&= \frac{1}{\sigma_n^2}\left(E\left[\left(\mathbf{h}_{11}^H \mathbf{w}_{11}\right)^2\right] + E\left[\left(\mathbf{h}_{12}^H \mathbf{w}_{21}\right)^2\right]\right. \\
&\qquad \left. + 2E\left[\mathbf{h}_{11}^H \mathbf{w}_{11}\right] E\left[\mathbf{h}_{12}^H \mathbf{w}_{21}\right]\right).
\end{aligned} \tag{6}
$$

If the matrix $\mathbf{Z}$ is a projection matrix, it is idempotent: $\mathbf{Z} = \mathbf{Z}^2$ [9]. We can then write $\mathbf{h}_{11}^H \mathbf{Z}_{21}^H \mathbf{Z}_{21} \mathbf{h}_{11} = \mathbf{h}_{11}^H \mathbf{Z}_{21} \mathbf{h}_{11}$. The combination of the precoder with the channel gives then $\mathbf{h}_{11}^H \mathbf{w}_{11} = \sqrt{P_1\left(\mathbf{h}_{11}^H \mathbf{Z}_{21} \mathbf{h}_{11}\right)}$. Next, applying the eigenvalue decomposition to the matrix $\mathbf{Z}_{21}$ we obtain $\mathbf{h}_{11}^H \mathbf{Z}_{21} \mathbf{h}_{11} = \mathbf{h}_{11}^H \mathbf{U}_{21} \mathbf{\Lambda}_{21} \mathbf{U}_{21}^H \mathbf{h}_{11}$. The matrix $\mathbf{U}_{21}$ is a unitary matrix of eigenvectors and $\mathbf{\Lambda}_{21}$ is a diagonal matrix containing the eigenvalues. Because the properties of a zero-mean complex Gaussian vector do not change when multiplied with a unitary matrix, we have $\mathbf{h}_{11}^H \mathbf{U} \sim \mathbf{h}_{11}^H$. Therefore, we obtain $E\left[\mathbf{h}_{11}^H \mathbf{w}_{11}\right] = E\left[\sqrt{P_1\left(\mathbf{h}_{11}^H \mathbf{\Lambda}_{21} \mathbf{h}_{11}\right)}\right]$. Again, the matrix $\mathbf{Z}_{21}$ being idempotent, its eigenvalues are either 1 or

0 [9] and the rank of $\mathbf{Z}_{21}$ equals its trace, i.e., $rank(\mathbf{Z}_{ij}) = tr(\mathbf{I}_{N_t}) - tr\left(\mathbf{h}_{ij}\left(\mathbf{h}_{ij}^H\mathbf{h}_{ij}\right)^{-1}\mathbf{h}_{ij}^H\right) = N_t - 1$. As a result,

$$E[\gamma_1] = \frac{1}{\sigma_n^2}\left(P_1 E\left[\sum_{n=1}^{N_t-1}|\mathbf{h}_{11}^n|^2\right] + P_2 E\left[\sum_{n=1}^{N_t-1}|\mathbf{h}_{12}^n|^2\right] + 2E\left[\sqrt{P_1\sum_{n=1}^{N_t-1}\left|\mathbf{h}_{11}^n\right|^2}\right] \times E\left[\sqrt{P_2\sum_{n=1}^{N_t-1}\left|\mathbf{h}_{12}^n\right|^2}\right]\right)$$

From this equation, $\sqrt{\sum_{n=1}^{N_t-1}\left|\mathbf{h}_{11}^n\right|^2}$ is a Rayleigh distributed random variable and $|\mathbf{h}_{11}^n|^2$ follows a chi-square distribution [10], we hence obtain

$$E\left[\sqrt{\sum_{n=1}^{N_t-1}\left|\mathbf{h}_{11}^n\right|^2}\right] = \frac{\Gamma(N_t - 0.5)}{(N_t-2)!} \tag{7}$$

$$E\left[\sum_{n=1}^{N_t-1}|\mathbf{h}_{11}^n|^2\right] = \frac{\Gamma(N_t)}{(N_t-2)!} = N_t - 1. \tag{8}$$

where Γ denotes the Gamma function and $(N)!$ the factorial of N. Finally, the average SNR gain (in dB) for the distributed ZF scheme over the single-user single-input single-output (SISO) scenario, assuming perfect synchronization can be expressed as G equals

$$10\log_{10}\left((P_1+P_2)(N_t-1) + 2\sqrt{P_1P_2}\left(\frac{\Gamma(N_t-0.5)}{(N_t-2)!}\right)^2\right) \tag{9}$$

For comparison, the SNR for the single-user case, with a transmit-MRC beamformer, is $G_{MRC} = 10\log_{10}(PN_t)$, while for the equal-gain combining (EGC) beamformer [8], it is given as $G_{EGC} = 10\log_{10}\left(P\left(1+(N_t-1)\frac{\pi}{4}\right)\right)$.

3.2 Diversity gain

The diversity gain is obtained by combining the multiple replicas of the signal collected at the receiver. The diversity order is calculated by evaluating the resulting slope of the average bit error rate curve. The diversity order for the first receiver is given as

$$-d_1 = \lim_{\sigma_n^2\to\infty}\frac{\log_{10}P_e}{\log_{10}\sigma_n^2} \tag{10}$$

where P_e denotes the average bit error rate probability for the first receiver

$$P_e = \int_0^\infty P_c(e|\gamma_1)\, p_{\gamma_1}(\gamma_1)\, d\gamma_1. \tag{11}$$

We denote by $p_{\gamma_1}(\gamma_1)$ the probability density function (PDF) of the instantaneous SNR (γ_1) at the receiver 1 given in Section 3.1. The expression $P_c(e|\gamma_1)$ denotes the conditional bit error rate and can be expressed, for a BPSK modulation, as

$$P_c(e|\gamma_1) = Q\left(\sqrt{2\gamma_1}\right). \tag{12}$$

where $Q(x)$ denotes the alternative Gaussian Q-function representation [8] given as

$$Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} exp\left(-\frac{x^2}{2\sin^2\phi}\right)d\phi \tag{13}$$

hence $P_c(e|\gamma_1) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} exp\left(-\frac{2\gamma_1}{2\sin^2\phi}\right)d\phi$. We can write the average bit error rate probability as

$$P_e = \int_0^\infty \frac{1}{\pi}\int_0^{\frac{\pi}{2}} exp\left(-\frac{2\gamma_1}{2\sin^2\phi}\right)d\phi\, p_{\gamma_1}(\gamma_1)\, d\gamma_1. \tag{14}$$

Developing the equation of the instantaneous SNR gives

$$\gamma_1 = \frac{1}{\sigma_n^2}\left(\left(\mathbf{h}_{11}^H\mathbf{w}_{11}\right)^2 + \left(\mathbf{h}_{12}^H\mathbf{w}_{21}\right)^2 + 2\mathbf{h}_{11}^H\mathbf{w}_{11}\mathbf{h}_{12}^H\mathbf{w}_{21}\right). \tag{15}$$

Because the terms in (15) are not independent, obtaining the equivalent PDF (and hence *Pe*) in a general closed-form is difficult. Therefore, to compute the diversity order of the considered cooperative scheme we approximate numerically the error probability *Pe* for a varying number of antennas.

From the numerical analysis and simulation results in Section 5.1, we can observe in Fig. 2 a diversity order of the considered scheme is of $2(N_t-1)$. This result should be expected as one degree of freedom cancels the interference towards the non-intended receiver. With two antennas at each transmitter the diversity order is equivalent to a single-user EGC scenario with two antennas and provides then a diversity order of 2 [11]. A similar approach can be employed for the second receiver.

4. DISTRIBUTED ZF BEAMFORMING: IMPACT OF CFO

In this section, we discuss the effects of the residual CFO on the SNR and diversity gains. In 4.1, we extend the system model given in Section 2 to the case where CFO is present. Then, the average SNR and diversity gains are derived for the general case (N_t transmit antennas) in Sections 4.2 and 4.3.

4.1 System Model with CFOs

The combination of the channel with the CFO can be equivalently represented by the channel vector multiplied by the complex component $c_i(t, f_{\Delta_i}) = e^{i2\pi f_{\Delta_i}t}$, where t is the time index and f_{Δ_i} denotes the CFO at the transmitter i to that of the receiver's carrier frequency. As introduced above, we assume that the frequency offset is pre-compensated at the transmitters prior to transmission, i.e., only the residual CFOs f_{Δ_1} and f_{Δ_2} are left. The instantaneous SNR $\xi_1(t, f_{\Delta_1}, f_{\Delta_2})$ is equal to

$$\frac{1}{\sigma_n^2}\left(c_1(t,f_{\Delta_1})\mathbf{h}_{11}^H\mathbf{w}_{11} + c_2(t,f_{\Delta_2})\mathbf{h}_{12}^H\mathbf{w}_{21}\right)^2. \tag{16}$$

We now average this instantaneous SNR over the channel realizations, i.e., $E\left[\xi_1(t, f_{\Delta_1}, f_{\Delta_2})\right]$

$$= \frac{1}{\sigma_n^2}\left(E\left[\left(\mathbf{h}_{11}^H\mathbf{w}_{11}\right)^2\right] + E\left[\left(\mathbf{h}_{12}^H\mathbf{w}_{21}\right)^2\right] + E\left[\mathbf{h}_{11}^H\mathbf{w}_{11}\right] \times E\left[\mathbf{h}_{12}^H\mathbf{w}_{21}\right] 2\cos(2\pi f_\Delta t)\right) \tag{17}$$

$$E\left[\xi_1(T_p,f_\Delta)\right]=\frac{1}{\sigma_n^2}\left(P_1E\left[\sum_{n=1}^{N_t-1}|\mathbf{h}_{11}^n|^2\right]+P_2E\left[\sum_{n=1}^{N_t-1}|\mathbf{h}_{12}^n|^2\right]+2\sqrt{P_1P_2}\,\mathrm{sinc}(2\pi f_\Delta T_p)E\left[\sqrt{\sum_{n=1}^{N_t-1}\left|\mathbf{h}_{11}^n\right|^2}\right]E\left[\sqrt{\sum_{n=1}^{N_t-1}\left|\mathbf{h}_{12}^n\right|^2}\right]\right) \tag{17}$$

where $c_1(t,f_{\Delta_1})c_2(t,f_{\Delta_2})^H+c_1(t,f_{\Delta_1})^Hc_2(t,f_{\Delta_2})$ is equal to $2\cos(2\pi f_\Delta t)$, with $f_\Delta=f_{\Delta_1}-f_{\Delta_2}$. The notation f_Δ refers to the relative CFO between the two transmitters.

4.2 Average SNR Gain with CFO

The Eq. (17) shows that the average SNR is time-dependent and varies over the transmission period T_p. We then compute the time-averaged result to obtain the exact average SNR

$$E_T\left[\cos(2\pi f_\Delta t)\right]=\frac{1}{T_p}\int_0^{T_p}\cos(2\pi f_\Delta t)\,dt=\mathrm{sinc}\left(2\pi f_\Delta T_p\right).$$

Following the procedure in Section (3.1) and from Eq. (17) we obtain the average SNR $E\left[\xi_1(T_p,f_\Delta)\right]$ given in Eq. (17). To conclude the average SNR gain, over the SU-SISO scenario, with residual CFO is

$$G_{CFO}=10\log_{10}\Bigg((P_1+P_2)(N_t-1)+2\mathrm{sinc}(2\pi f_\Delta T_p)\sqrt{P_1P_2}\left(\frac{\Gamma(N_t-0.5)}{(N_t-2)!}\right)^2\Bigg) \tag{18}$$

As a result, the average SNR gain degrades, following a sinc function with the parameters f_Δ and T_p. Hence, if f_Δ or T_p are too large, no SNR gain is obtained.

4.3 Diversity gain with CFO

The CFO introduces a phase rotation. As a result, the previous approximations or numerical analyses of the diversity for the considered scheme do not hold anymore.

As expressed in Section 3.2, the average error probability P_e is required to compute the diversity gain d. However, a residual CFO makes the error probability P_e time- and CFO-dependent. For $\xi_1(t,f_\Delta)$ denotes the equivalent signal at the time index $T_p=t$ and, from Eq.(16), it can be expressed as

$$\xi_1(t,f_\Delta)=\frac{1}{\sigma_n^2}\Big(\left(\mathbf{h}_{11}^H\mathbf{w}_{11}\right)^2+\left(\mathbf{h}_{12}^H\mathbf{w}_{21}\right)^2+2\cos(2\pi f_\Delta t)\mathbf{h}_{11}^H\mathbf{w}_{11}\mathbf{h}_{12}^H\mathbf{w}_{21}\Big). \tag{19}$$

Similarly to Section 3.2, the average BER $P_e(t,f_\Delta)$ is

$$\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\int_0^{\infty}exp\left(-\frac{2\xi_1(t,f_\Delta)}{2\sin^2\phi}\right)p\left(\xi_1(t,f_\Delta)\right)d\xi_1(t,f_\Delta)\,d\phi.$$

Moreover, the average BER must be integrated over the time index t for a given transmission duration (T_p). Because its evaluation is not trivial, we instead approximate the PDF semi-numerically, for different transmission durations. We then obtain the average probability of error by integrating the different P_e, i.e., for a given residual CFO (f_Δ), over the desired transmissions duration, i.e., $P_{e|T_p}=\frac{1}{T_p}\int_0^{T_p}P_e(t)dt$. We then use these numerical approximations to compute the resulting diversity order. This is presented in the Section 5.2.

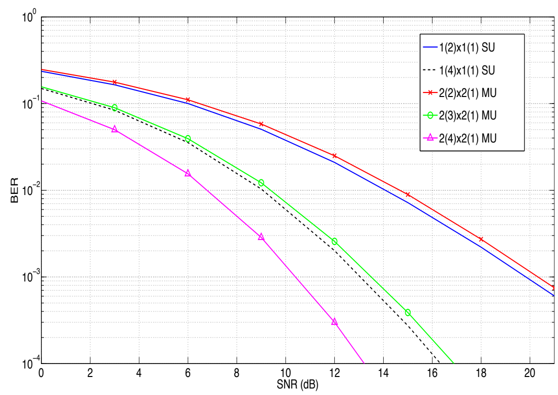

Figure 2: displays the BER curves versus the SNR of the considered ZF beamforming scheme, i.e., the multi-user (MU) scenario, with $N_t=2$, $N_t=3$ and $N_t=4$ assuming perfect frequency synchronization and for a 16QAM modulation scheme. The BER curves of a single-user (SU) system with transmit-MRC beamforming and $N_t=2$ and $N_t=4$ are also displayed.

5. SIMULATION RESULTS

This section aims at comparing the SNR and diversity gains of the distributed ZF beamforming scheme without synchronization errors to that of the case with residual CFO and verifying the proposed derivations. The simulations are performed for the IEEE802.11n system, with a 5GHz carrier frequency and a 20MHz bandwidth. We consider an uncoded OFDM scheme with 64 subcarriers. A power of 1 is allocated to each receiver from each transmitter. The multiple CFOs are assumed known at the receiver where a zero-forcing frequency domain equalizer is applied for synchronization. Presynchronization of the frequency offset is performed at the transmitters so that only residual CFO f_Δ is left. The f_Δ is expressed in part per million (ppm) with respect to the system carrier frequency.

Each scenario can be described as $N_{TX}(N_t)\times N_{RX}(N_r)$, where N_{TX} denotes the number of transmitters and N_{RX} the number of receivers, N_t is the number of transmit antennas at each transmitter, N_r the number of antennas at each receiver.

5.1 Performance of Cooperative Beamforming: Ideal Case

Fig. 2 shows that the diversity order (d) of the considered scheme with $N_t=3$ results in a diversity gain for the ZF

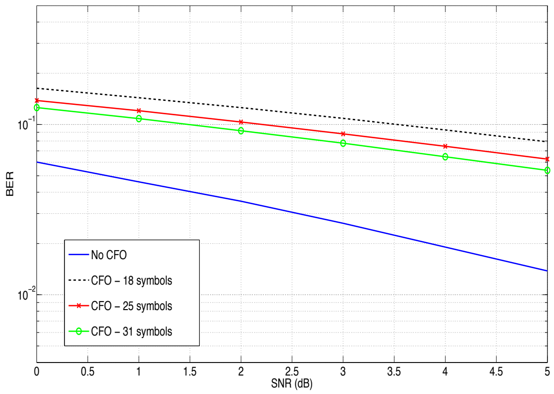

Figure 3: displays the BER curves obtained for a BPSK modulation scheme, $f_\Delta = 2$ppm and for $18, 25$ and 31 transmit symbols (T_p). These values represent the minimum, a zero and a local maximum of the $\text{sinc}(2\pi f_\Delta T_p)$ function. The curve of the SNR gain without CFO is also displayed.

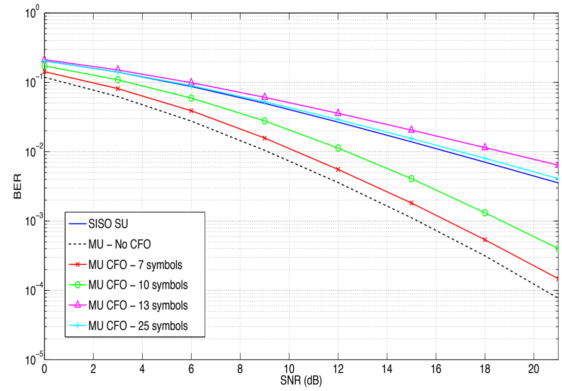

Figure 4: shows the BER curves for various transmission durations based on the derivations in Section 4.3. The simulations are for $f_\Delta = 2$ppm and a 4-QAM modulation scheme

beamformer of 4. Similarly, the ZF beamforming scheme with $N_t = 2$ provides the same diversity gain as the SU scheme with two transmit antennas. The figure also shows that the SNR gain of the ZF beamforming scheme with $N_t = 2$ is of approximately 0.5dB less than that of the SU scheme with two transmit antennas and that the SNR gain of the ZF beamformer with $N_t = 3$ is of approximately 0.2dB less than that of the SU transmit-MRC scheme with 4 transmit antennas. These results confirm the derivations proposed in Section 2.

5.2 Performance of Coordinated Beamforming with Frequency Offset

Here, we study the effects of residual CFO on the SNR and diversity gains. In the simulations, we assume 2 transmitters each equipped with two antennas, i.e., $N_t = 2$.

From the derivations in Section 3.1, a residual CFO introduces a SNR loss that follows a sinc function. Fig. 3 shows that the CFO degrades the SNR gain and that the curve from $T_p = 31$ achieves a SNR gain higher than that of $T_p = 25$. Similarly, the SNR gain for $T_p = 31$ and 25 outperforms that of $T_p = 18$, as expected from Eq. (18).

The Fig. 4 shows that the diversity order decreases quickly with the number of transmit symbols to finally approach the SU-SISO curve for a long transmit period, e.g., $T_p > 25$ transmit symbols. Moreover, for $T_p = 13$ the diversity order is lower than 1 ($d < 1$), i.e., worse than the SU-SISO case; confirming the results from Section 4.3.

6. CONCLUSIONS

We propose the derivations of the SNR gain analytically and of the diversity order numerically for the ZF distributed beamformer when a residual CFO is present. Results show that the residual CFO degrades significantly the SNR and diversity gains. As a result, additional efforts for the estimation and correction of the frequency offset are necessary to achieve the gain promised by coordinated transmissions.

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