ENHANCEMENT OF MULTI-VALUED IMAGES USING PDE COUPLING

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ABSTRACT

In this paper, we present a new model for the enhancement of noisy and blurred multi-valued images. The proposed model is based on using single vectors of the gradient magnitude and the second derivatives as a manner to relate different colour components of the image. This model can be viewed as a generalization of Bettahar-Stambouli filter to multi-valued images. The proposed algorithm is more efficient than the mentioned filter and some previous works at colour images denoising and deblurring without creating false colours.

1. INTRODUCION

The need for efficient image restoration techniques has grown with the large introduction of digital images in our everyday life. These images are often sullied with different kinds of noise. A lot of methods have been developed to clean degradations and many restoration algorithms have been proposed: wavelet denoising [1], the total variation model [2], Bilateral Filtering [3] or the non local mean algorithm [4]. Among all these approaches, partial differential Equations based on diffusion methods [5,6] and shock filter [3,4] have received a lot of attention as a very good for noise elimination, image enhancement and edge detection. Many solutions have been proposed in the processing of gray level images by coupling diffusion to shock filter [9,10,11,12,13]. The extension of these methods to colour images can be achieved in two ways. The first one consists in using a marginal approach that enhances each colour component of the image separately using a scalar approach, that bases image processing on channel by channel filter. This technique is well known to produce false colours in the enhanced image, because each colour component is processed independently of the others. The second approach consists in using a single vector processing, where different components of the image are enhanced by considering correlation between them [14]. The main idea here is to find a mathematical framework to relate different colour values and variations, while conserving the self behaviour of each component.

2. RELATED WORKS

Originated from a well known physical heat transfer process, the PDE- based approaches consist in evolving in time the filtered image $u(t)$ under a PDE. When coupling diffusion and shock filter the PDE is a combination of three terms, so:

$$\frac{\partial u}{\partial t} = C_\eta \, u_{\eta\eta} + C_\xi \, u_{\xi\xi} - C_{sk} \, F\left(u_{\eta\eta}\right)\left|\nabla u\right| \tag{1}$$

where $u(t=0) = u_0$ is the input image, $|\nabla u|$ is the gradient magnitude, η is the gradient direction, ξ is the direction perpendicular to the gradient, and so $u_{\eta\eta}$ and $u_{\xi\xi}$ represent the diffusion terms in gradient and level set directions respectively. C_η and C_ξ are some flow control coefficients. The first kind of diffusion smoothes edges, while the second one smoothes parallel to the edge on both sides. The last term in (1) represents the contribution of the shock filter in the enhancement of the image. It is weighted by C_{sk} . The function $F(s)$ should satisfy the conditions $F(0)=0$ and $F(s).s \geq 0$. Hence, by considering adaptive weights C_η, C_ξ and C_{sk} as functions of the local contrast, we can favour smoothing process under diffusion terms in homogeneous parts of the image or enhancement operation under shock filter at edge locations. The first model of coupling diffusion and shock filter has been introduced by Alvarez and Mazorra [9], where the image is diffused only in the direction perpendicular to the gradient. The balance between diffusion process and shock filter has been more investigated by Kornprobst [10], it becomes a binary function of the local contrast. However, Gilboa developed a complex diffusion shock filter coupling model that smoothes the image with a weak edges enhancement [11]. The imaginary value of the solution, which is an approximated smoothed second derivative, is used as an edge detector. In the other hand, Fu developed a region-based shock-diffusion scheme [12], where directional diffusion and shock terms are factored by adaptive weights. In a more recent work, Bettahar and Stambouli proposed a new reliable and stable scheme, which is a kind of coupling diffusion to shock filter with reactive term [13]. All these methods have been proposed for the gray level images. Only a very few works tackle the shock diffusion coupling using an approach specifically dedicated to multi-valued images. So, to avoid the effect of the apparition of false colours, the processing applied to the image must be driven in a common and coherent manner for all image components. This type of approach is denoted as “vector processing”, in opposition to the marginal processing which is a multi-scalar processing. Thus, in order to describe vector-valued image variations and structures, Di Zenzo [15] and Lee [16] have proposed to use the local variation of a vector gradient norm that detects edges and corners when its value becomes high. It can be computed using the eigen-values of a symmetric and semi-positive matrix that can be computed using the concept described in [15,16]. Hence, by using multi-valued geometrical description of [15,16], Tschumperlé and Deriche proposed a new form of diffusion shock filter coupling especially for enhancement of colour images [17]

3. PROPOSED MODEL

The proposed method is based on the gray level model of Bettahar and Stambouli as an extension to multi-valued images, where each colour component u_p of the enhanced image u is considered with taking into account the correlation between the three components. This model is given by:

$$\begin{cases} \dfrac{\partial u_p}{\partial t} = |\nabla u|\, div\left(g\left(|\nabla u_\sigma|\right) \dfrac{\nabla u_p}{|\nabla u|} \right) - \alpha \dfrac{\left|\nabla\left(f\left(|\nabla u_\sigma|\right)\right)\right|^2}{1+\beta\left|\nabla\left(f\left(|\nabla u_\sigma|\right)\right)\right|^2 u_{\xi\xi}^2}\left(u_p - v_p\right) \\ \dfrac{\partial v_p}{\partial t} = -sign\left(G_\sigma * u_{\eta\eta}\right)\left|\nabla u_p\right| \end{cases} \quad (2)$$

where $u_\sigma(t)$ is the smoothed image of $u(t)$ using the Gaussian kernel G_σ, $v_p(t)$ is the last evolution of $u_p(t)$ and $v_p(0)=u_{p0}$. u_p (p=1,2,3) represents red, green and blue components of the multi-valued image u. In discrete time, $v_p(t)$ is the last value of $u_p(t)$. So first, we compute the second equation that gives the value of v_p, which will be injected in the first equation being , later, noted u_p. $g(|\nabla u_\sigma|)$ and $f(|\nabla u_\sigma|)$ are decreasing functions having the same form with free parameters respectively k_d (for g) and k_c (for f), so:

$$g(s) = \frac{1}{1+s^2/k_d^2} \quad (3)$$

α and β are constants.

In these equations, each component is processed separately, like in a marginal approach. However, the processing of each component takes into account the correlation between the different components of the multi-valued image by using the same gradient magnitudes (∇u and ∇u_σ) and the same derivatives ($u_{\xi\xi}$ and $u_{\eta\eta}$) obtained in vector way. This is the key point of the proposed approach which will avoid the generation of false colours, as it will be seen in the following behaviour of the proposed model.

The gradient magnitudes are computed by using the following norm like in Tschumperlé-Deriche model:

$$|\nabla u| = \sqrt{\sum_{l\in\{x,y\}} \sum_{p=1}^{3} u_{p\,l}^{\;2}} \quad (4)$$

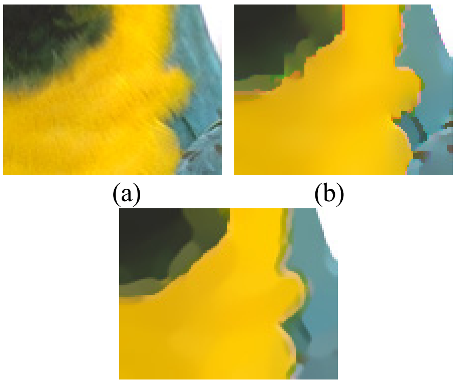
while we estimate the second derivatives by:

$$u_{\eta\eta} = \sum_{p=1}^{3} u_{p\,\eta\eta} \quad (5)$$

$$u_{\xi\xi} = \sum_{p=1}^{3} u_{p\,\xi\xi} \quad (6)$$

The first equation in the proposed filter behaves as a nonlinear reaction-curvature diffusion process like in the model of Bettahar-Stambouli for gray level case. In this equation, the first term $|\nabla u|\, div\left(g\left(|\nabla u_\sigma|\right) \dfrac{\nabla u_p}{|\nabla u|}\right)$ is used to assure a selective smoothing that reduces noise in homogeneous parts of the image without introducing new structures like false colours. The second term $\alpha \dfrac{\left|\nabla\left(f\left(|\nabla u_\sigma|\right)\right)\right|^2}{1+\beta\left|\nabla\left(f\left(|\nabla u_\sigma|\right)\right)\right|^2 u_{\xi\xi}^2}$ acts as a balance between smoothing of diffusion process and enhancement of shock filter. It is weak in smooth parts and strong at edge locations. Hence, in homogeneous regions, $|f(|\nabla u_\sigma|)| \to 1$, so $|\nabla f(|\nabla u_\sigma|)| \to 0$ and $u_{\xi\xi} \to 0$, while at edge locations $|f(|\nabla u_\sigma|)| \to 0$, so $|\nabla f(|\nabla u_\sigma|)| \to 1$. So, by choosing adequate values of α and β, the weight $\alpha \dfrac{\left|\nabla\left(f\left(|\nabla u_\sigma|\right)\right)\right|^2}{1+\beta\left|\nabla\left(f\left(|\nabla u_\sigma|\right)\right)\right|^2 u_{\xi\xi}^2}$ will be high at edge locations and low in smooth parts. Thus in the first case the process will operate as a diffusion-shock filter reaction that enhances edges, and in the second case the filter will behave as a linear diffusion that smoothes noise. The second equation in (2) is a simple shock filter and its result $v_p(t)$ is injected in the first equation as a reactive term. It can be noted again that this filter is not designed in a marginal way because of the use of the vector derivative $u_{\xi\xi}$.

To well understand the proposed filter behaviour, it is necessary to give more details about the reasons which have decided of the form of the equation (2). First, the choice of the balance diffusion-shock filter with the form $\alpha \dfrac{\left|\nabla\left(f\left(|\nabla u_\sigma|\right)\right)\right|^2}{1+\beta\left|\nabla\left(f\left(|\nabla u_\sigma|\right)\right)\right|^2 u_{\xi\xi}^2}$ has a critical impact on the behaviour of the proposed model at non creation of false colours. This choice has been done by considering some practical observations on different cases in order to take the best choice of this balance as following. So, by using the marginal method, we have computed the balance $\alpha|f\nabla(|\nabla u_{p\sigma}|)|^2$ of Bettahar-Stambouli model with processing each image component separately. We can observe in this case, after an excessive smoothing, false colours (red and green) at edge locations as an expected result (fig. 1-b), while in fig. 1-c using vector method, so the balance $\alpha \frac{\left|\nabla\left(f\left(\left|\nabla u_{\sigma}\right|\right)\right)\right|^{2}}{1+\beta\left|\nabla\left(f\left(\left|\nabla u_{\sigma}\right|\right)\right)\right|^{2} u_{\xi\xi}^{2}}$, we can see that the image has been enhanced without creating any false colours.



(a) (b) (c)

Figure 1- Enhancement of Textured image; (a) Original image; (b) Using the balance $\alpha \left| \nabla f(|\nabla u_{p\sigma}|) \right|^2$ ($\beta = 0$ in eq. (2)); (c) Using the balance $\alpha \frac{\left|\nabla\left(f\left(\left|\nabla u_{\sigma}\right|\right)\right)\right|^{2}}{1+\beta\left|\nabla\left(f\left(\left|\nabla u_{\sigma}\right|\right)\right)\right|^{2} u_{\xi\xi}^{2}}$.

4. RESULTS

We evaluate performances of our model comparing to marginal channel by channel methods of Alvarez-Mazorra, Kornprobst, Gilboa, Fu and Bettahar-Stambouli, while we consider vector regularization of Tschumperlé-Deriche. For this comparison, we choose the parameters that give better results for each filter, except for the number of iterations which must be the same for an objective comparison. All models are applied to blurry and noised images. In the production of artificially blurry images, we use the Gaussian convolution of original test images (σ=1). Noised images are produced by adding random Gaussian noise to blurred images (σ=18). As objective criterions, we use is the SNR and the MSSIM [18]. Figure 3 shows restoration results of Sunglasses image blurred and noised with SNR=9.93 dB. All results are obtained with 1500 iterations. We can see how our model has efficiently removed noise in smoother regions with edges sharpening referring to Alvarez-Mazorra, Kornprobst, Gilboa, Bettahar-Stambouli and Tschumperlé-Deriche filters. Only the last two models have successfully smoothed noise in homogeneous parts of the images. The other models have produced false piecewise constant images with residual noise, where artificial blobs have been created. Furthermore, we notice here how Alvarez-Mazorra, Kornprobst, Gilboa and Bettahar-Stambouli filters have introduced false colours as it appears on the different parts of the enhanced image. Only Tschumperlé-Deriche model presents some satisfactory results, with, however, a fine production of false colours at localized edges. On the contrary, in the proposed filter, edges can well be distinguished without any false colours. Our algorithm doesn't create false colours referring to other models with an effective selective smoothing of the sunglass according to the other parts of the image. In table 1, we present the SNR and the MSSIM values for the different approaches. It can be noted that the SNR of our solution is bigger than the SNR of other models. This can be deep-rooted by the MSSIM values which confirm the best visual quality of our solution. The second comparison is performed after region segmentation. The segmentation which is used is a classical region growing technique: blob-colouring using a 4 neighborhood [19]. This technique is a quick and simple one which is known to give over-segmentation, providing a lot of small regions in noisy situation. In our case, this drawback will be used to extract a measure of performance. Nevertheless, to limit over-segmentation, the region growing is realized iteratively, the aggregation threshold being incremented at each iteration. Figure 3 gives the results of the segmentation on Face image. All the different segmentations have been realized using the same parameter values (initial threshold = 3, threshold increment = 2, iteration number = 30). On this figure, it can be seen that the simplest segmentation is the one obtained with the proposed method, despite the largest number of regions. Tschumperlé-Deriche and our method excepted, all the segmentations contain false regions due to false colours. Alvarez-Mazorra and Gilboa gives an over-segmentation in the background, on the face and on the glasses. Kornprobst, Fu and Bettahar-Stambouli provides some quite good segmentations, but with false colour regions. With Tschumperlé-Deriche filter, there are thin regions along some edges, like the borders of the glasses. To characterize these results in a more precise way, we have focused our attention on a part of the border of the glasses (see figure 4) where there are two main regions, the glass and the face. Then, we have computed three numerical indicators: the number of regions, the average size of the small regions (by small regions, we mean regions which are not the two main regions, i.e. the face and the glass.), the sum of the colour distance (denoted D_{colour}) of these small regions to the two main regions, defined by:

$$D_{Colour} = \sum_{k} \min\left(dist(C_{R1}, C_{Rk}), dist(C_{R2}, C_{Rk})\right) \times \left| R_k \right| \quad (7)$$

where C_{R1} and C_{R2} denote the colour of the two main regions R_1 and R_2, C_{Rk} is the colour of a small region R_k, k is the small region index , $dist(.)$ is the Euclidean distance in the RGB space and $|R_k|$ is the size of the region R_k.

This last indicator is a measure of the false colour importance. Indeed, if a small region has a colour which is far from the colour of R_1 and R_2, it will have a large contribution to D_{colour}. Table 2 presents the results.

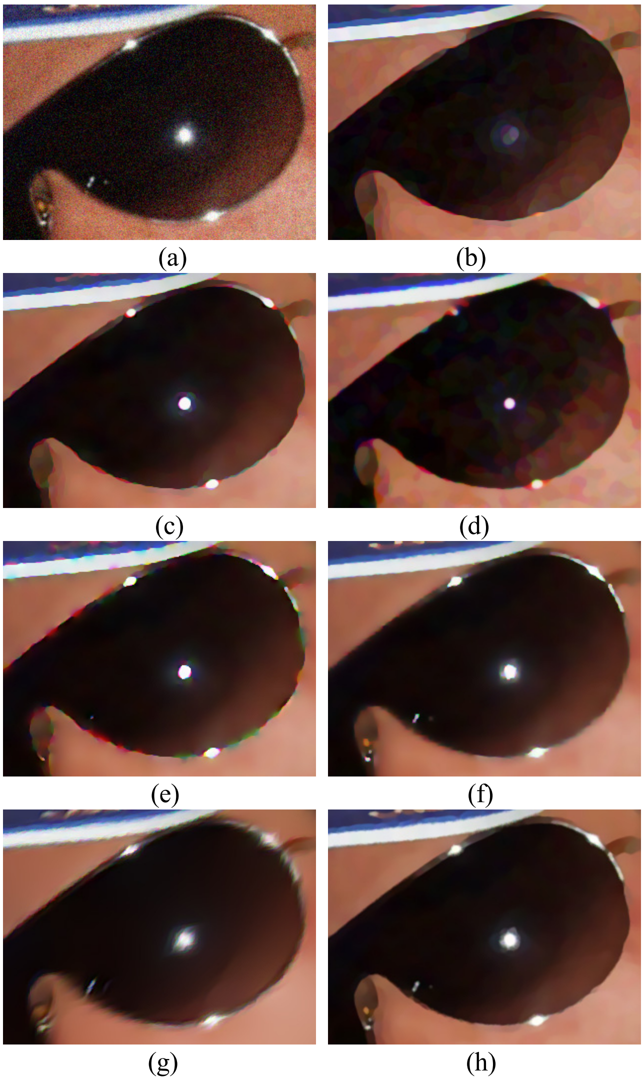


Figure 2 - Enhancement of Sunglasses image: Blurry and noised image; (b) Alvarez-Mazorra filter; (c) Kornprobst filter; (d) Gilboa filter; (e) Fu filter; (f) Bettahar-Stambouli filter; (g) Tschumperlé-Deriche filter; (f) Proposed filter.

Considering the proposed method, it can be seen that, despite the number of regions is large, these non wanted regions are small (average size is only one pixel) and their colours are close to the colour of one of the two the main regions (D_{colour} is three times lower than the smaller value provided by other approaches).

Method	SNR	MSSIM
Alvarez-Mazorra	12.870	0.7579
Kornprobst	14.611	0.7913
Gilboa	13.292	0.7488
Fu	14.117	0.7871
Bettahar-Stambouli	19.063	0.8209
Tschumperlé-Deriche	16.464	0.8037
Proposed method	19.175	0.8229

Table 1- The SNR and the MSSIM values of the different approaches.

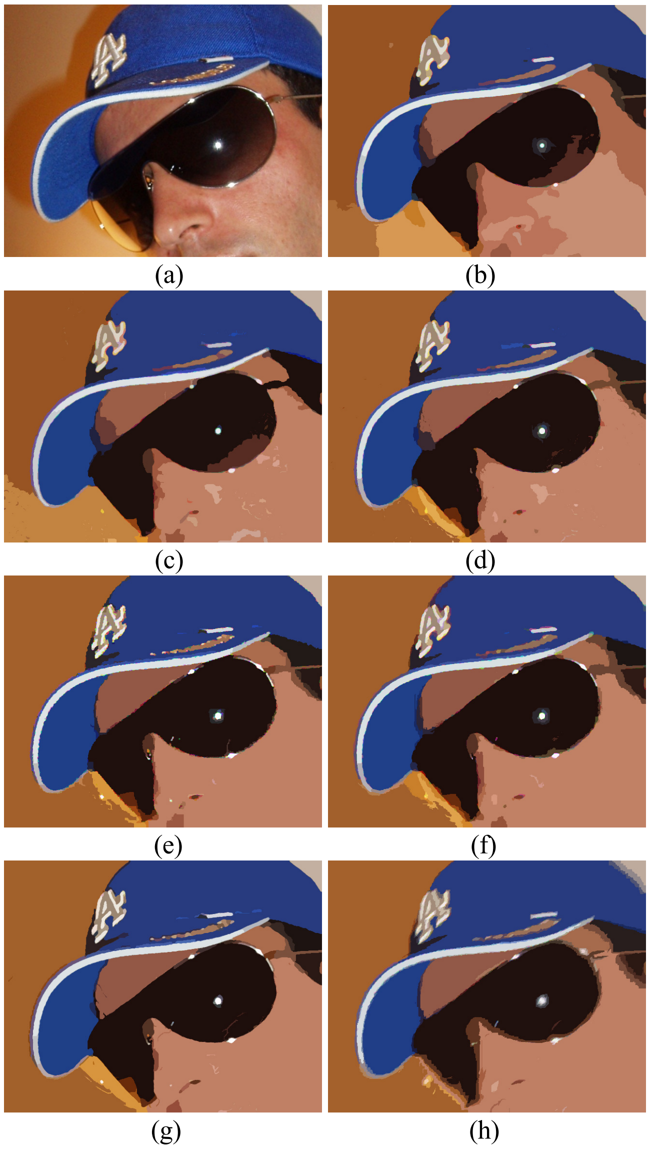
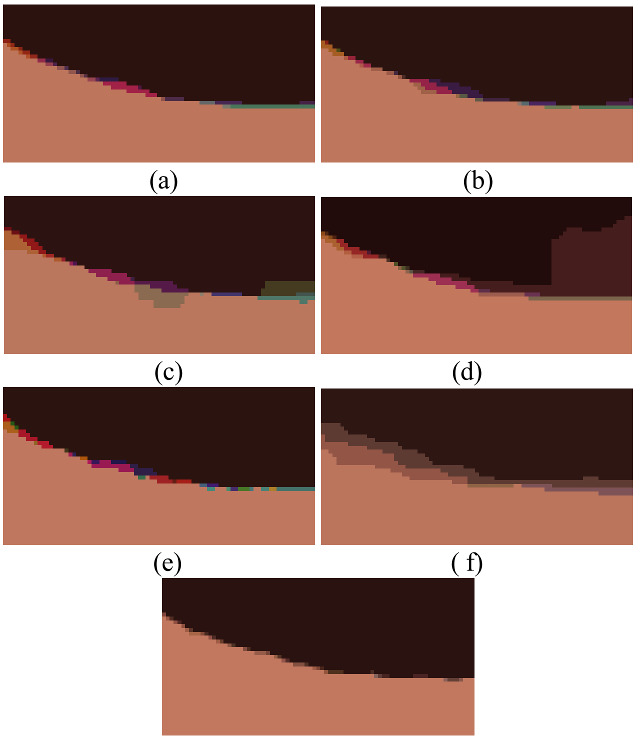


Figure 3- Segmentation of Face image: (a) Non noisy initial image; (b) Alvarez-Mazorra filter 2442 regions; (c) Kornprobst filter 2385 regions; (d) Gilboa filter 2198 regions; (e) Fu filter 2071 regions ; (f) Bettahar-Stambouli filter 2198 regions; (g) Tschumperlé-Deriche filter 1434 regions; (h) Proposed filter 2959 regions.

Method	Region number	Average size of small regions	D_{colour} ($\times 10^{-3}$)
Alvarez-Mazorra	22	3	8.9
Kornprobst	57	5	9.58
Gilboa	26	23	27.8
Fu	54	12	15.8
Bettahar-Stambouli	68	4	10.9
Tschumperlé-Deriche	6	5	25.0
Proposed method	42	1	3.3

Table 2 - Comparison of region segmentation performance on a small area of Face image.



(a) (b) (c) (d) (e) (f) (g)

Figure 4 - Enlargement of Segmentation of Face image: (a) Alvarez-Mazorra filter; (b) Kornprobst filter; (c) Gilboa filter; (d) Fu filter; (e) Bettahar-Stambouli filter; (f) Tschumperlé-Deriche; (g) Proposed filter.

5. CONCLUSION

We have proposed a new filter of coupling shock filter to curvature diffusion for multi-valued image enhancement, which is based on using single vectors for all components of the image. This filter produces a selective smoothing reducing efficiently noise and sharpens edges. Our analysis shows that the proposed method is more efficient than Alvarez-Mazorra, Kornprobst, Gilboa, Fu, Bettahar-Stambouli and Tschumperlé-Deriche models at multi-valued image restoration in presence of blur and noise simultaneously. In that it denoises homogeneous parts of the multi-valued image, while it keeps edges enhanced. However, due to the fact of using single vectors with the specific reaction, our filter doesn't create false colours that can appear when each component of the image is enhanced separately.

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