RANGE RECURSIVE AND TAYLOR SERIES BASED SPACE-TIME ADAPTIVE PROCESSING FOR RANGE DEPENDENT CLUTTER REJECTION

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ABSTRACT

This paper presents a new space time adaptive processing (STAP) for the rejection of range dependent ground clutter in order to detect slow moving targets with an airborne radar. STAP usually requires the estimation of the clutter plus noise covariance matrix from secondary data neighboring the cell under test. However, in most radar antenna array architectures and/or configurations which are different from the conventional uniform linear antenna array and side-looking configuration the clutter is range dependent. We recently proposed the use of a Taylor series expansion of the clutter plus noise subspace in conjunction with the eigencanceler-based (EC) STAP in order to mitigate the range non-stationarity of the clutter. In this paper, the computationally costly EC is replaced by a range-recursive algorithm which is capable of tracking non stationarities with a reduced complexity compared to the EC. The performance of the proposed algorithm is satisfactorily tested and compared to other algorithms in the case of a bistatic configuration.

1. INTRODUCTION

A main issue in airborne radar signal processing is to detect and track targets slowly moving on the ground which may be masked by Doppler spread ground clutter generated by the radar platform motion. Space-time adaptive processing (STAP) consists in mitigating the ground clutter by filtering the radar echoes received on a multiple antenna array for different coherent time pulses [1]-[4]. A key issue is that the construction of the optimal STAP filter at each range requires the estimation of the clutter plus noise covariance matrix. This is usually done by the straight averaging of secondary snapshots at neighboring ranges. However in most applications, the snapshots statistics are range dependent. Indeed, in the radar antenna architectures and/or configurations which are different from the conventional uniform linear antenna array (ULA) and monostatic side-looking (SL) configuration the clutter is range dependent.

A number of methods in the literature have been devoted to the compensation of the clutter range dependency in STAP. Among the parametric methods are the Doppler Warping (DW) method [5][6], the Angle Doppler Compensation (ADC) method [7][8] and the Adaptive Angle Doppler Compensation (AADC) [8][9]. The derivative based updating (DBU) method [10], the Prediction of inverse covariance matrix (PICM) [11] and the Registration Based range dependence Compensation (RBC) [12][13][14] are non parametric methods in the sense that they do not require the knowledge of the radar configuration parameters.

Following the idea of [15], we recently proposed the use of a Taylor series expansion of the clutter subspace, associated with the clutter plus noise covariance matrix, in conjunction with the eigencanceler-based (EC) STAP [16] in order to mitigate the range non-stationarity of the clutter [17]. In the present paper, the computationally costly EC is replaced by a range-recursive algorithm which is capable of tracking non stationarities with a reduced complexity compared to the subspace expansion (SE) based EC (SE-EC). The performance of the proposed algorithm is satisfactorily tested and compared to the classical sample matrix inversion (SMI) method [18], the EC, SE-EC, DBU and also SMI with diagonal loading (DL) in the case of a bistatic configuration.

The problem is further formulated in the following section and section 3 is devoted to the Taylor series based expansion based STAP algorithms. First the DBU and the SE-EC algorithms are recalled. Then the proposed range recursive SE method is derived. Section 4 exhibits the simulation contexts and results and section 5 is the conclusion.

2. PRESENTATION OF THE PROBLEM

The aim of STAP is to mitigate the effects of ground clutter in order to detect an eventual slowly moving target. This is performed by a two dimensional filtering of the received data followed by a detector. At a given range from the radar, the received signal can be written as the sum of the target component $\mathbf{x}_\mathrm{t}$ (when it is present at this range), the noise $\mathbf{x}_\mathrm{n}$ and the clutter $\mathbf{x}_\mathrm{c}$ components (we here suppose the absence of jammer). The optimal STAP weight vector maximizing the signal to noise plus interference ratio (SINR) is given at range k by $\mathbf{w}_k^{opt} = \mathbf{R}_k^{-1}\mathbf{x}_\mathrm{t}$ where $\mathbf{R}_k = \mathrm{E}\left\{\mathbf{x}_k\mathbf{x}_k^H\right\}$ is the clutter plus noise covariance matrix and $\mathbf{x}_k = \mathbf{x}_{\mathrm{c},k} + \mathbf{x}_{\mathrm{n},k}$. We suppose that these two components are mutually uncorrelated such that $\mathbf{R}_k = \mathbf{R}_{\mathrm{c},k} + \sigma^2\mathbf{I}$ where $\mathbf{R}_{\mathrm{c},k}$ is the clutter covariance matrix at range k and σ^2 is the noise variance (the noise is assumed to be spatially and temporally white). The computation of this filter thus requires the estimation of the clutter plus noise covariance matrix. This is classically achieved using K snapshots at neighboring ranges

$$\hat{\mathbf{R}}_k = \frac{1}{K}\sum_{l=1,l\neq k}^{K}\mathbf{x}_l\mathbf{x}_l^H \tag{1}$$

yielding the SMI [18] algorithm for the STAP filter

$$\mathbf{w}_k^{SMI} = \hat{\mathbf{R}}_k^{-1}\mathbf{v}_\mathrm{t} \tag{2}$$

where $\mathbf{v}_t$ is the target steering vector. It is shown in [18] that an average performance loss of 3 dB compared to the optimum can be obtained with $K = 2MN$, where M and N are the number of pulses and the number of antenna elements, respectively, when the snapshots are independent and identically distributed (iid) over ranges. This happens in the very particular case of a ULA with a side looking (the platform velocity vector is collinear with the antenna axis) monostatic (the receiver and the transmitter are colocated on the same platform) configuration which is classically used in the literature. In this case only, the locus of the repartition of the power spectral density (PSD) of the clutter, namely, the locus of the clutter ridges, forms a straight line in the direction-Doppler (DD) (spatial frequency, Doppler frequency) plane [19]. These DD curves overlap for all ranges but their length is reduced when the range is reduced. However, in practice, whether in the presence of wind implying a crab angle between the platform direction and the ULA axis [5] or in the case of non ULA antenna arrays [20] or also in the case of bistatic radar (the transmitter and the receiver are not on the same platform) [21], the clutter ridges are no longer straight lines and, above all, become range dependent making the neighboring data non stationary in range and not iid. It then follows that the weight vector in (2) is range dependent.

3. TAYLOR SERIES EXPANSION BASED STAP ALGORITHMS

We here briefly recall the DBU and the SE-EC algorithms introduced in [10] and [17], respectively, and we present the new algorithm.

3.1 DBU

The DBU algorithm [10] is based on the first order development of the STAP weight vector at range k given by

$$\mathbf{w}_k \approx \mathbf{w}_o + \alpha_k \mathbf{\Delta w}_o \tag{3}$$

where α_k is a real scalar number chosen so that $\frac{1}{K}\sum_{k=1}^{K}\alpha_k = 0$ and $\frac{1}{K}\sum_{k=1}^{K}\alpha_k^2 = 1$. The output of the STAP filter is $\mathbf{w}_k^H\mathbf{x}_k \approx \tilde{\mathbf{w}}^H\tilde{\mathbf{x}}_k$, where the extended weight vector and the extended data vector are

$$\tilde{\mathbf{w}} = \begin{bmatrix} \mathbf{w}_o \\ \mathbf{\Delta w}_o \end{bmatrix} \tag{4}$$

and

$$\tilde{\mathbf{x}}_k = \begin{bmatrix} \mathbf{x}_k \\ \alpha_k \mathbf{x}_k \end{bmatrix} \tag{5}$$

respectively. The DBU range compensation method thus consists in finding $\mathbf{w}_o$ and $\mathbf{\Delta w}_o$ of (3) through the optimum extended weight vector

$$\tilde{\mathbf{w}} = \mathbf{R}_E^{-1}\begin{bmatrix} \mathbf{v}_t \\ \mathbf{0} \end{bmatrix} \tag{6}$$

where

$$\mathbf{R}_E = \mathrm{E}\left\{\tilde{\mathbf{x}}_k\tilde{\mathbf{x}}_k^H\right\} \tag{7}$$

is the extended covariance matrix and $\mathbf{0}$ is the NM-dimensional null vector.

The extended covariance matrix is estimated by

$$\hat{\mathbf{R}}_E = \begin{bmatrix} \frac{1}{K}\sum_{k=1}^{K}(\mathbf{x}_k\mathbf{x}_k^H) & \frac{1}{K}\sum_{k=1}^{K}(\alpha_k\mathbf{x}_k\mathbf{x}_k^H) \\ \frac{1}{K}\sum_{k=1}^{K}(\alpha_k\mathbf{x}_k\mathbf{x}_k^H) & \frac{1}{K}\sum_{k=1}^{K}(\alpha_k^2\mathbf{x}_k\mathbf{x}_k^H) \end{bmatrix} \tag{8}$$

The DBU is an extension of the SMI method with an extended data vector of dimension twice the dimension of the actual observation vector. Its convergence in term of the number of snapshots required in the estimation of $\hat{\mathbf{R}}_E$ (8) to achieve a performance loss of 3 dB compared to the optimum is twice that of the SMI, it is to say $4MN$. Moreover, as the derivation of the DBU STAP filter (6) requires the inversion of a $(2MN, 2MN)$-dimensional matrix, the main computational complexity of the DBU algorithm is in $O((2MN)^3)$ (where $O(.)$ means "of the magnitude order of").

3.2 Subspace Expansion based eigencanceler (SE-EC)

The EC introduced in [16] relies on the eigendecomposition

$$\mathbf{R}_k = \mathbf{U}_k\mathbf{\Delta}_k\mathbf{U}_k^H = \mathbf{U}_{c,k}\mathbf{\Delta}_{c,k}\mathbf{U}_{c,k}^H + \mathbf{U}_{n,k}\mathbf{\Delta}_{n,k}\mathbf{U}_{n,k}^H \tag{9}$$

where $\mathbf{\Delta}_{c,k}$ and $\mathbf{\Delta}_{n,k}$ are the diagonal matrices $diag\{\lambda_1\cdots\lambda_r\}$ and $diag\{\lambda_{r+1}\cdots\lambda_{NM}\}$ containing the eigenvalues of the clutter plus noise covariance matrix $\mathbf{R}_k$ such as $\lambda_1 \geq \cdots \geq \lambda_r > \lambda_{r+1} = \lambda_{NM} = \sigma^2$ and $\mathbf{U}_{c,k}$ and $\mathbf{U}_{n,k}$ are the associated eigenvectors. r is the rank of the clutter only covariance matrix (in the absence of noise) and is given by Brennan's rule [1][1]. The subspaces spanned by $\mathbf{U}_{c,k}$ and $\mathbf{U}_{n,k}$ are refered to as the clutter and noise subspaces at range k, respectively. A STAP filter can be defined using the eigencanceler method [16], as

$$\mathbf{w_k} = (\mathbf{I} - \mathbf{U}_{c,k}\mathbf{U}_{c,k}^H)\cdot\mathbf{v}_t \tag{10}$$

In [17], this approach was combined with a Taylor series expansion of the interference subspace. Let $\tilde{\mathbf{u}}_i = \begin{bmatrix} \mathbf{u}_{o_i} \\ \mathbf{\Delta u}_{o_i} \end{bmatrix}$ be the i^{th} extended eigenvector of $\mathbf{R}_E$ of (7). According to (8) and the conditions on α_k given in the DBU subsection above, one can write

$$\mathbf{R}_E = \mathrm{E}\{\hat{\mathbf{R}}_E\} = \begin{bmatrix} \mathbf{R}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_k \end{bmatrix} \tag{11}$$

It is worth noting that the extended matrix $\mathbf{R}_E$ has the same r eigenvalues greater than σ^2 as $\mathbf{R}_k$ but with multiplicity 2. Note that if $\mathbf{u}$ is an eigenvector of $\mathbf{R}_k$ associated with a given eigenvalue λ, the two linearly independent $2MN$-dimensional vectors $[\mathbf{u},\mathbf{0}]^{\mathrm{T}}$ and $[\mathbf{0},\mathbf{u}]^{\mathrm{T}}$ are eigenvectors of $\mathbf{R}_E$ associated with λ. It follows that in the absence of noise, the rank of $\mathbf{R}_E$ is $2r$ and that the extended clutter subspace is of dimension $2r$. Conversely, if a vector $\mathbf{u} = \left[\mathbf{u}_x^{\mathrm{T}}, \mathbf{u}_y^{\mathrm{T}}\right]^{\mathrm{T}}$ where $\mathbf{u}_x$ and $\mathbf{u}_y$ are vectors of dimension MN, is an eigenvector of the extended matrix $\mathbf{R}_E$ associated with a given eigenvalue λ, then $\mathbf{u}_x$, $\mathbf{u}_y$ and any combination of $\mathbf{u}_x$ and $\mathbf{u}_y$ are eigenvectors of $\mathbf{R}_k$ associated with the same eigenvalue λ.

Thus, it follows that the clutter subspace associated with matrix $\mathbf{R}_k$ is, in particular, spanned by the vectors of the form

$$\hat{\mathbf{u}}_{i,k} = \mathbf{u}_{o,i} + \alpha_k\mathbf{\Delta u}_{o,i} \tag{12}$$

[1]The rank r of the clutter covariance matrix in the case of an ULA in SL configuration is given by $r = M + \beta(N-1)$ where $\beta = \frac{2v_a T_r}{\lambda}$, v_a is the platform velocity, T_r the pulse interval and λ the wavelength. In the other cases of antenna array geometries and configurations the rank is larger, it is usually approximated by twice the rank of the ULA-SL case.

where $\mathbf{u}_{o,i}$ and $\mathbf{\Delta u}_{o,i}$ enters the partition of $\tilde{\mathbf{u}}_i$, the eigenvectors of $\mathbf{R}_E$ corresponding to the $2r$ largest eigenvalues of $\mathbf{R}_E$. Note that among these $2r$ eigenvectors only r of them are linearly independent implying that $span\left\{\hat{\mathbf{u}}_{1,k},...,\hat{\mathbf{u}}_{2r,k}\right\}$ is a r-dimensional subspace.

The SE-EC STAP algorithm thus consists in computing the MN-dimensional STAP weight vector as for the EC method in (10) but with $\hat{\mathbf{U}}_{c,k} = [\hat{\mathbf{u}}_{1,k} \cdots \hat{\mathbf{u}}_{2r,k}]$ obtained in (12) instead of $\mathbf{U}_{c,k}$.

The SE-EC algorithm was found in [17] to converge faster than the DBU algorithm. Twice the dimension of the clutter subspace associated with $\hat{\mathbf{R}}_E$, it is to say about $4r$ range cells, should be sufficient to obtain a 3 dB SINR loss compared to the optimal STAP. However, the computational complexity which is dominated by the search of the eigenvectors of a ($2MN$, $2MN$)-dimensional matrix is still in $O((2MN)^3)$ as for DBU.

In order to reduce the computational complexity of the SE-EC algorithm, we propose a range recursive eigencanceler based STAP filter. This approach allows to avoid both the inversion and the eigendecomposition of the clutter plus noise covariance matrix while keeping the advantage of a reduced convergence rate over DBU.

3.3 Subspace Expansion based FAPI (SE-FAPI)

We here propose to use a range recursive subspace-based algorithm in conjunction with the Taylor series expansion suggested above, in order to construct the STAP filter. Originally used in spectral analysis and antenna processing as a time-recursive adaptive algorithm [22], the Fast Approximated Power Iteration (FAPI) subspace-based algorithm has then been used in STAP for airborne radar [23]. In this case, the recursion relates to range instead of time.

FAPI is based on the power iteration method [24] consisting of the following *compression* and *orthonormalization* steps :

$$\begin{aligned} \mathbf{R}'(i) &= \mathbf{R}\mathbf{W}(i-1) \\ \mathbf{W}(i)\mathbf{C}(i) &= \mathbf{R}'(i) \end{aligned} \tag{13}$$

where, under some assumptions [24], $\mathbf{W}(i)$ is expected to converge towards an orthonormal matrix the columns of which span the dominant eigensubspace of a data covariance matrix $\mathbf{R}$. In (13), $\mathbf{C}(i)$ is such that $\mathbf{C}^H(i)\mathbf{C}(i) = \mathbf{\Phi}(i)$ with $\mathbf{\Phi}(i) = \mathbf{R}'^H(i)\mathbf{R}'(i)$ and where i is the iteration index. In order to reduce the computational complexity of the power iteration method when the covariance matrix is recursively updated as

$$\mathbf{R}(k) = \beta\mathbf{R}(k-1) + \mathbf{x}(k)\mathbf{x}(k)^H \tag{14}$$

each time a new observation $\mathbf{x}(k)$ is received and where $0 < \beta < 1$ is a forgetting factor, Badeau et al. replaced index i by k and introduced the following hypothesis

$$\boldsymbol{W}(k)\boldsymbol{W}(k)^H \approx \boldsymbol{W}(k-1)\boldsymbol{W}(k-1)^H \tag{15}$$

which means that the projection on the dominant eigensubspace at range[2] k is approximated by the projection of the dominant eigensubspace at range $k-1$. The interested reader can refer to [22] for the long derivation of FAPI. The details of this algorithm adapted for our subspace expansion are given in Table 1 where the input $\mathbf{x}(k)$ is the $2MN$-dimensional vector $\tilde{\mathbf{x}}_k$ defined in (5) at range k. The corresponding STAP filter computed for each snapshot is then obtained through

[2]In the initial version of FAPI k is time whereas in the present paper k is the range index and the dominant eigensubspace will be the clutter subspace

Initialization : $\mathbf{W}(0) \leftarrow \mathbf{I}_{2MN\times 2r}$, $\mathbf{Z}(0) \leftarrow \mathbf{I}_{2r\times 2r}$, $\mathbf{x}(k) \leftarrow \tilde{\mathbf{x}}_k$

FOR $k = 1$ to K (number of snapshots)

$$\begin{aligned}
&\mathbf{y}(k) = \mathbf{W}(k-1)^H \cdot \mathbf{x}(k)\\
&\mathbf{h}(k) = \mathbf{Z}(k-1) \cdot \mathbf{y}(k)\\
&\mathbf{g}(k) = \frac{\mathbf{h}(k)}{\beta + \mathbf{y}^H(k)\cdot\mathbf{h}(k)}\\
&\mathbf{e}(k) = \mathbf{x}(k) - \mathbf{W}(k-1)\cdot\mathbf{y}(k)\\
&\varepsilon^2(k) = \|\mathbf{x}(k)\|^2 - \|\mathbf{y}(k)\|^2\\
&\tau(k) = \frac{\varepsilon^2(k)}{1+\varepsilon^2(k)\|\mathbf{g}(k)\|^2 + \sqrt{1+\varepsilon^2(k)\|\mathbf{g}(k)\|^2}}\\
&\eta(k) = 1 - \tau(k)\,\|\mathbf{g}(k)\|^2\\
&\mathbf{y}'(k) = \eta(k)\mathbf{y}(k) + \tau(k)\mathbf{g}(k)\\
&\mathbf{h}'(k) = \mathbf{Z}(k-1)^H\mathbf{y}'(k)\\
&\mathbf{d}(k) = \frac{\tau(k)}{\eta(k)}(\mathbf{Z}(k-1)\mathbf{g}(k) - (\mathbf{h}'(k)\mathbf{g}(k))\mathbf{g}(k))\\
&\mathbf{Z}(k) = \frac{1}{\beta}(\mathbf{Z}(k-1) - \mathbf{g}(k)\mathbf{h}'(k)^H + \mathbf{d}(k)\mathbf{g}(k)^H)\\
&\mathbf{e}'(k) = \eta(k)\mathbf{x}(k) - \mathbf{W}(t-1)\mathbf{y}'(k)\\
&\mathbf{W}(k) = \mathbf{W}(k-1) + \mathbf{e}'(k)\cdot\mathbf{g}(k)^H
\end{aligned}$$

ENDFOR

Table 1: SE-FAPI algorithm

$$\mathbf{w}_{SE-FAPI}(k) = \left(\mathbf{I}_{MN} - \mathbf{W}_{SE-FAPI}(k)\mathbf{W}_{SE-FAPI}(k)^H\right)\mathbf{v}_{\mathrm{t}} \tag{16}$$

where

$$\mathbf{W}_{SE-FAPI}(k) = \mathbf{W}_0(k) + \alpha_k\mathbf{\Delta W}_0(k) \tag{17}$$

and

$$\mathbf{W}(k) = \begin{bmatrix} \mathbf{W}_0(k) \\ \mathbf{\Delta W}_0(k) \end{bmatrix} \tag{18}$$

$\mathbf{W}_0(k)$ and $\mathbf{\Delta W}_0(k)$ contain the MN first rows and the MN last rows of $\mathbf{W}(k)$ obtained through algorithm FAPI of Table 1, respectively, and where α_k is chosen as in subsection 3.2 for SE-EC.

The so-called SE-FAPI algorithm, being an eigencanceller based STAP filter is expected to converge in twice the rank of the extended covariance matrix, it is to say, in $4r$ snapshots as for the SE-EC algorithm. The key advantage of the proposed SE-FAPI algorithm is that it requires neither the inversion nor the eigendecomposition of the clutter plus noise covariance matrix. The computational complexity of the algorithm is in $O(2MN)$, it is to say, linear with respect to the parameters of the STAP filter.

The performance of the proposed algorithm is presented in the following section.

4. SIMULATION RESULTS

In this section, simulation results illustrate the performance of the proposed SE-FAPI algorithm[3]. The analysis of the

[3]Here $\beta = 0.99$. The influence of β is not analyzed in this paper

x_R	y_R	z_R	α_R	δ_R	v_T	v_R	R_{test}
80	50	20	60	35	90	90	170

Table 2: Characteristics of the bistatic configurations. x_R, y_R, z_R and R_{test} are expressed in km, α_R and δ_R in degrees and v_T and v_R in m/s.

influence of this compared to the optimal STAP, the SMI, the diagonal loading SMI [1], the EC-SE [17] and the DBU [10] algorithms. This comparison concerns the performance in term of signal to interference plus noise ratio loss (SINR loss) at range k defined as the ratio of the SINR to the SNR (without clutter)

$$SINR_{loss_k} = \frac{\sigma^2 \cdot \left|\mathbf{w}_k^H \cdot \mathbf{v}(f_D, f_S)\right|^2}{NM \cdot \mathbf{w_k}^H \cdot \mathbf{R}_k \cdot \mathbf{w}_k} \qquad (19)$$

where $\mathbf{w}_k$ is the weight vector of the clutter rejection filter calculated at range k according to each algorithm and where $\mathbf{v}(f_D, f_S)$ is a steering vector of dimension MN computed for a candidate couple of Doppler and spatial normalized frequencies (f_D, f_S) and which replaces the unknown actual target steering vector $\mathbf{v}_\mathrm{t}$ in (2), (6), (10), (16). It is written as:

$$\mathbf{v}(f_D, f_S) = \mathbf{b}(f_D) \otimes \mathbf{a}(f_S) \qquad (20)$$

where $\mathbf{b}(f_D)$, $\mathbf{a}(f_S)$ and $\otimes$ are the temporal and spatial steering vectors and the Kronecker product, respectively.

For this comparison, a pulsed Doppler airborne X-band radar transmits $M = 10$ pulses during a CPI. The bistatic configuration described in [13][14] is here considered. The transmitter T is at the center of an (x, y, z) coordinate system with a velocity vector $\mathbf{v}_T$ in the direction of the x-axis. The receiver R is located at (x_R, y_R, z_R). Its velocity vector $\mathbf{v}_R$ is in a plane parallel to (x, y) and makes an angle of α_R wrt $\mathbf{v}_T$. The receiving ULA array consisting of $N = 12$ elements is located in a plane parallel to (x, y) and makes an angle of δ_R wrt $\mathbf{v}_R$. The ground is assumed to be a horizontal plane at $z = -H$. The values of these bistatic parameters are given in Table 2 where R_{test} is the range of the cell under test and v_T and v_R are the amplitudes of the transmitter and receiver velocity vectors, respectively. The clutter to noise ratio (CNR) is assumed equal to 30 dB. The back lobe clutter is attenuated by 30 dB with respect to the front lobe clutter.

Figure 1 exhibits the range dependency of the 2D projection on (f_D, f_S) of the locus of the bistatic configuration clutter ridges (repartition of the power spectral density of the clutter) [19][14].

Figures 2 and 3 exhibit a slice of the $SINR_{loss}$ (19)for $f_S = 0$. In Figure 2 the number of snapshots for estimating the covariance matrices $\mathbf{R}_E$ and $\mathbf{R}_k$ with (8) and (1), respectively, is equal to $K = 480$. It is well known that, in the case of a ULA in a side-looking configuration, the SMI and the DBU converge to a SINRloss of 3 dB compared to the optimal STAP when the number of snapshots is equal to twice the dimension of the data vector. It follows that the required number of snapshots for the SMI and the DBU algorithms is at least $K = 2NM$ and $K = 4NM$, respectively. In our case of $M = 12$ and $N = 10$, $K = 480$ is then an adequate choice for the SMI and the DBU. Thus, one can see on Figure 2 that the loss of performance of the SMI compared to the optimal STAP is due to the range non stationarity involved by the bistatic configuration. This remark is also valid for the diagonal loading (DL) version of the SMI and the EC. At last, it is apparent that the proposed SE-FAPI algorithm succeeds as well as the SE-EC algorithm and even slightly better than the DBU to compensate the range non stationarity. Now by considering Figure 3 where the number of snapshots is $K = 240$ which is not enough for the DBU algorithm, one can see that the proposed SE-FAPI and the SE-EC algorithms are the only ones capable of mitigating the range non stationarity involved by the bistatic configuration.

5. CONCLUSION

A new STAP filter for the rejection of range dependent ground clutter in airborne radar has been proposed. It relies both on the Taylor series expansion of the clutter subspace associated with the clutter plus noise covariance matrix and on the range recursive estimation of this clutter subspace. The so-called SE-FAPI converges as fast as the SE-EC and then faster than the DBU with a computational complexity of $O(2NM)$ instead of $O((2NM)^3)$.

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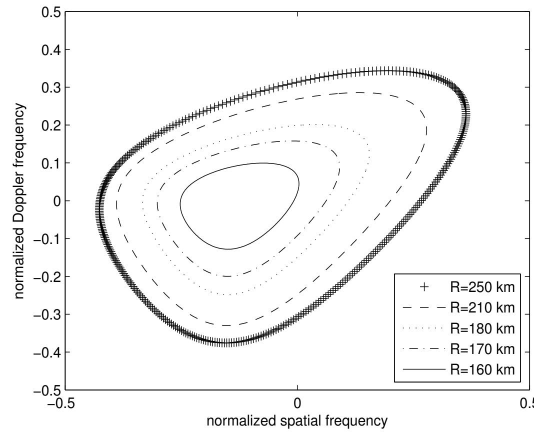

Figure 1: Clutter ridges for different ranges

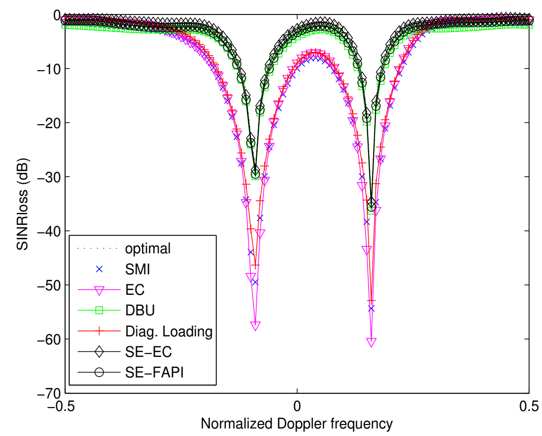

Figure 2: $SINR_{loss}$ in a bistatic configuration. $K = 480$

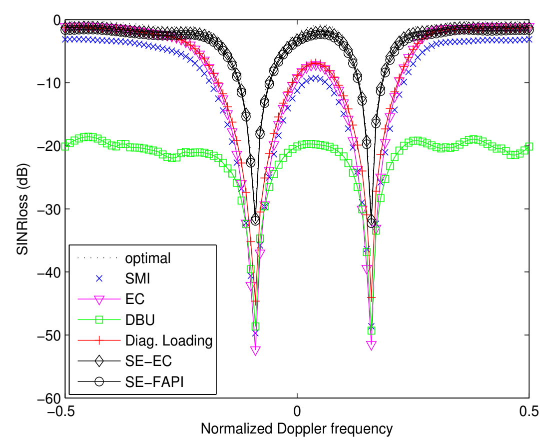

Figure 3: $SINR_{loss}$ in a bistatic configuration. $K = 240$