SPATIAL-TEMPORAL MODE TRANSMISSION BASED UPON THE QR DECOMPOSITION OF A POLYNOMIAL CHANNEL MATRIX WITH UNCERTAINTIES

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ABSTRACT

This paper presents an entirely time domain approach for communicating over frequency selective multiple-input multiple-output (MIMO) channels. The proposed system applies a paraunitary matrix to the received signals, which is specifically designed to transform the effective polynomial channel matrix for the system into an upper triangular polynomial matrix. This then enables the MIMO channel equalisation problem to be transformed into a set of single-input single-output (SISO) equalisation problems, by exploiting the upper triangular structure of the transformed channel matrix, which can then be individually solved using Turbo equalisation. In particular, this paper investigates how the system performs when there is error present in the channel matrix state information and discusses the possible errors that are encountered when formulating the QR decomposition (QRD) of a polynomial matrix.

Index Terms— Convolutive mixing, paraunitary matrix, polynomial matrix QR decomposition, MIMO channel equalisation

1. INTRODUCTION

Polynomial matrices arise, in the context of this paper, when a set of signals arrive at an array of sensors via multiple paths resulting in the received signals consisting of a sum of weighted and delayed versions of the transmitted signals. The mixing process in this situation can be characterised by a polynomial matrix where the indeterminate variable of each polynomial element of the matrix is z^{-1}, as this is often used to represent a unit delay. A $p \times q$ polynomial matrix of this form can be expressed as

$$\underline{\mathbf{A}}(z) = \sum_{\tau=t_1}^{t_2} \mathbf{A}(\tau) z^{-\tau} = \begin{bmatrix} \underline{a}_{11}(z) & \underline{a}_{12}(z) & \cdots & \underline{a}_{1q}(z) \\ \underline{a}_{21}(z) & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \underline{a}_{p1}(z) & \cdots & \cdots & \underline{a}_{pq}(z) \end{bmatrix} \quad (1)$$

where $\tau \in \mathbb{Z}$, $\mathbf{A}(\tau) \in \mathbb{C}^{p\times q}$ is the matrix of coefficients of $z^{-\tau}$ and $t_1 \leq t_2$, where the values of the parameters t_1 and t_2 are not necessarily positive. The $(j,k)^{th}$ element of this matrix can then be expressed as

$$\underline{a}_{jk}(z) = \sum_{\tau=t_1}^{t_2} a_{jk}(\tau) z^{-\tau}. \quad (2)$$

In terms of a communication channel, this will represent the channel from the k^{th} transmitter to the j^{th} sensor. The quantity $(t_2 - t_1)$ represents the temporal length of the channel over this path. For a polynomial matrix, as expressed in equation (1), this quantity is referred to as the order of the matrix. Throughout this paper a polynomial matrix, vector or scalar will use the qualifier (z) to denote it is a polynomial in the indeterminate variable z^{-1}. Furthermore, to avoid confusion with the notation used for the z-transform of a variable, polynomial quantities will use the additional underline notation, as demonstrated in equations (1) and (2).

Algorithms have been developed for calculating several different decompositions of a polynomial matrix [1, 2], in particular, the authors have previously proposed an algorithm for calculating the QR decomposition of a polynomial matrix (PQRD) in [3, 4]. This algorithm has been applied to the problem of MIMO channel equalisation in [5, 6], where the decomposition has been used to transform this problem into a set of single channel equalisation problems using a process of back substitution, which are each solved in turn using a iterative process of SISO channel equalisation and decoding. However, these papers assume that the receiver has perfect knowledge of the system channel matrix, which is clearly not a realistic assumption. This paper extends this work to demonstrate how the method performs when error is observed in the channel matrix state information and confirms the performance of the system for various levels of this error in terms of average bit error rate (BER). Note that the conventional approach to communicating over this type of channel is to use orthogonal frequency division multiplexing (OFDM), which firstly transforms the signals into the frequency domain, thus allowing the system to be transformed into a set of frequency flat problems. The approach outlined in this paper, however, offers an entirely time domain approach for communicating over MIMO frequency selective channels.

1.1. Notation and Definitions

Throughout this paper, matrices are denoted as bold upper case characters, vectors by bold lower case characters and scalars by regular lower case characters. $\mathbf{I}_p$ defines a $p \times p$ identity matrix. Let $\underline{\mathbb{C}}^{p\times q}$ define the set of polynomial matrices with p rows and q columns, where the series of coefficients of each of the polynomial elements are complex scalars. The Frobenius norm (F-norm) of a polynomial matrix is the quantity $\|\underline{\mathbf{A}}(z)\|_F = \sqrt{\sum_{\tau=t_1}^{t_2}\sum_{i=1}^{p}\sum_{j=1}^{q} |a_{ij}(\tau)|^2}$.

The paraconjugate of the polynomial matrix $\underline{\mathbf{A}}(z)$ is defined to be $\tilde{\underline{\mathbf{A}}}(z) = \underline{\mathbf{A}}_*^T(1/z)$ where $(\cdot)^T$ denotes matrix transposition and $(\cdot)_*$ the complex conjugation of each of the coefficients of the polynomial matrix. The tilde notation $\tilde{(\cdot)}$ will be used throughout this paper to denote paraconjugation.

2. THE QR DECOMPOSITION OF A POLYNOMIAL MATRIX

The PQRD by columns (PQRD-BC) algorithm is a technique for factorising a polynomial matrix into an upper triangular and a paraunitary polynomial matrix [3, 4]. Let $\underline{\mathbf{A}}(z) \in \underline{\mathbb{C}}^{p\times q}$, then the objective

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of the algorithm is to calculate a matrix $\underline{\mathbf{Q}}(z) \in \underline{\mathbb{C}}^{p\times p}$ such that

$$\underline{\mathbf{Q}}(z)\underline{\mathbf{A}}(z) = \underline{\mathbf{R}}(z) \tag{3}$$

where $\underline{\mathbf{R}}(z) \in \underline{\mathbb{C}}^{p\times q}$ is an approximately upper triangular polynomial matrix. The polynomial matrix $\underline{\mathbf{Q}}(z)$ must also be paraunitary, which means it will satisfy the following condition

$$\underline{\mathbf{Q}}(z)\underline{\widetilde{\mathbf{Q}}}(z) = \underline{\widetilde{\mathbf{Q}}}(z)\underline{\mathbf{Q}}(z) = \mathbf{I}_p. \tag{4}$$

This matrix represents a multichannel all-pass filter and therefore, if applied to a set of signals, will preserve the total signal power at every frequency. This matrix is calculated as a series of polynomial Givens rotations [3], which are applied to the matrix $\underline{\mathbf{A}}(z)$ to drive all polynomial elements beneath the diagonal of each column of the matrix approximately to zero in turn. The details of the algorithm will not be discussed here, but is outlined in detail, together with example decompositions, in [3].

2.1. Potential Errors in the Decomposition

There are three possible errors that can arise when using the polynomial matrix QR decomposition technique within a practical communication system. Firstly, the proposed system requires that the channel matrix must be estimated and so the channel matrix to be decomposed can therefore be expressed as

$$\underline{\mathbf{A}}(z) = \underline{\mathbf{A}}_T(z) + \underline{\mathbf{E}}(z), \tag{5}$$

where $\underline{\mathbf{A}}_T(z)$ is the true channel matrix and $\underline{\mathbf{E}}(z)$ is the matrix accounting for the errors present in the estimated channel matrix. Suppose the PQRD of this matrix is calculated according to equation (3), then the relative error of the observed decomposition can then be calculated as

$$E_{rel} = \left\| \underline{\mathbf{A}}_T(z) - \underline{\widetilde{\mathbf{Q}}}(z)\underline{\mathbf{R}}(z) \right\|_F / \|\underline{\mathbf{A}}(z)\|_F. \tag{6}$$

This measure can be used to determine the level of accuracy in the performed decomposition. Assuming that the only source of error in the decomposition is from the estimation process, then the relative error for the decomposition simplifies to $E_{rel} = \|\underline{\mathbf{E}}(z)\|_F / \|\underline{\mathbf{A}}(z)\|_F$.

Furthermore, as each element of $\underline{\mathbf{A}}(z)$ is an FIR filter, it is generally not possible to obtain an exactly upper triangular matrix. For this reason the algorithm stops once all coefficients of the elements beneath the diagonal of $\underline{\mathbf{R}}(z)$ are less than a specified value ε in magnitude. The relative error of the decomposition can then be calculated as

$$E_{rel} = \left\| \underline{\mathbf{A}}_T(z) - \underline{\widetilde{\mathbf{Q}}}(z)\underline{\mathbf{R}}'(z) \right\|_F / \|\underline{\mathbf{A}}(z)\|_F, \tag{7}$$

where the matrix $\underline{\mathbf{R}}'(z)$ is the approximately upper triangular matrix obtained from the decomposition with all elements beneath the diagonal of the matrix set equal to zero.

Finally, the third error arises due to truncating the orders of the polynomial matrices within the algorithm. With every application of a polynomial Givens rotation within the decomposition process, the orders of the polynomial matrices will increase. This will typically result in the final matrices $\underline{\mathbf{R}}(z)$ and $\underline{\mathbf{Q}}(z)$ of equation (3) both being of very large orders. In [3] it is shown that a truncation method can be applied to ensure that an accurate decomposition is still achieved, whilst also ensuring that the orders of the final polynomial matrices are as small as possible. This method will not be discussed here, but a detailed description can be found in [3] and references therein. This process is also advantageous for the equalisation performed at the receiver, where the computational complexity is proportional to the order of the matrix $\underline{\mathbf{R}}(z)$. It has previously been shown in [3] that a good approximation is achievable when using the PQRD-BC algorithm.

3. SYSTEM MODEL FOR SPATIAL-TEMPORAL MODE TRANSMISSION WITH THE PQRD

Assume that $\mathbf{x}(\tau) \in \mathbb{C}^{q\times 1}$, where $\tau = 0,\ldots,N_1-1$, denote the set of signals that are to be transmitted, to be received at an array of p sensors. The k^{th} element of this vector, $x_k(\tau)$, is the signal transmitted at time τ from the k^{th} antenna. The environment between the transmitters and receivers can be expressed by the polynomial channel matrix $\underline{\mathbf{A}}(z) \in \underline{\mathbb{C}}^{p\times q}$, where it is assumed that $p \geq q$, i.e., there are at least as many receivers as transmitters. The received signals from this system can therefore be expressed as

$$\mathbf{y}(t) = \sum_{k=0}^{L-1} \mathbf{A}(k)\mathbf{x}(t-k) + \mathbf{n}(t) \tag{8}$$

where $t = 0,\ldots,N_1+L-1$, the order of the channel matrix $\underline{\mathbf{A}}(z)$ is L and $\mathbf{n}(t) \in \mathbb{C}^{p\times 1}$ denotes an additive zero-mean circular complex Gaussian noise process. This could alternatively be expressed, using the polynomial vector or matrix notation, as

$$\underline{\mathbf{y}}(z) = \underline{\mathbf{A}}(z)\underline{\mathbf{x}}(z) + \underline{\mathbf{n}}(z) \tag{9}$$

where

$$\underline{\mathbf{y}}(z) = \sum_{t=0}^{N_1+L-1} \mathbf{y}(t) z^{-t} \in \underline{\mathbb{C}}^{p\times 1}, \tag{10}$$

$$\underline{\mathbf{x}}(z) = \sum_{t=0}^{N_1+L-1} \mathbf{x}(t) z^{-t} \in \underline{\mathbb{C}}^{q\times 1} \tag{11}$$

and

$$\underline{\mathbf{n}}(z) = \sum_{t=0}^{N_1+L-1} \mathbf{n}(t) z^{-t} \in \underline{\mathbb{C}}^{p\times 1} \tag{12}$$

denote respectively algebraic power series of the received, transmitted and noise terms.

The PQRD of the matrix $\underline{\mathbf{A}}(z)$ can then be calculated according to equation (3) to obtain an approximately upper triangular polynomial matrix $\underline{\mathbf{R}}(z) \in \underline{\mathbb{C}}^{p\times q}$ and a paraunitary polynomial matrix $\underline{\mathbf{Q}}(z) \in \underline{\mathbb{C}}^{p\times p}$. For this application, all elements beneath the diagonal of $\underline{\mathbf{R}}(z)$, which are approximately equal to zero, are now set equal to zero. The received signals are then filtered by the polynomial matrix $\underline{\mathbf{Q}}(z)$ to obtain

$$\underline{\mathbf{y}}'(z) = \underline{\mathbf{Q}}(z)\underline{\mathbf{y}}(z) \in \underline{\mathbb{C}}^{p\times 1}. \tag{13}$$

Then the equivalent system, from transmitter to receiver, can be expressed as

$$\underline{\mathbf{y}}'(z) = \underline{\mathbf{R}}(z)\underline{\mathbf{x}}(z) + \underline{\mathbf{n}}'(z) \tag{14}$$

where $\underline{\mathbf{n}}'(z) = \underline{\mathbf{Q}}(z)\underline{\mathbf{n}}(z) \in \underline{\mathbb{C}}^{p\times 1}$, which remains an additive zero-mean circular complex Gaussian noise process with identical spectral properties due to the paraunitary nature of $\underline{\mathbf{Q}}(z)$. In particular, the q^{th} element of $\underline{\mathbf{y}}'(z)$ can now be expressed, due to the position of the zero elements in $\underline{\mathbf{R}}(z)$, as

$$\underline{y}'_q(z) = \underline{r}_{qq}(z)\underline{x}_q(z) + \underline{n}_q{}'(z), \tag{15}$$

which is a single channel equalisation problem. This can then be solved using a standard method for SISO channel equalisation to obtain an estimate of the signal transmitted from the q^{th} transmit antenna, i.e., $\underline{x}_q(z)$. Note that in this equivalent system the polynomial element $\underline{r}_{qq}(z)$ is the spatial-temporal mode over which the q^{th} substream of data is transmitted.

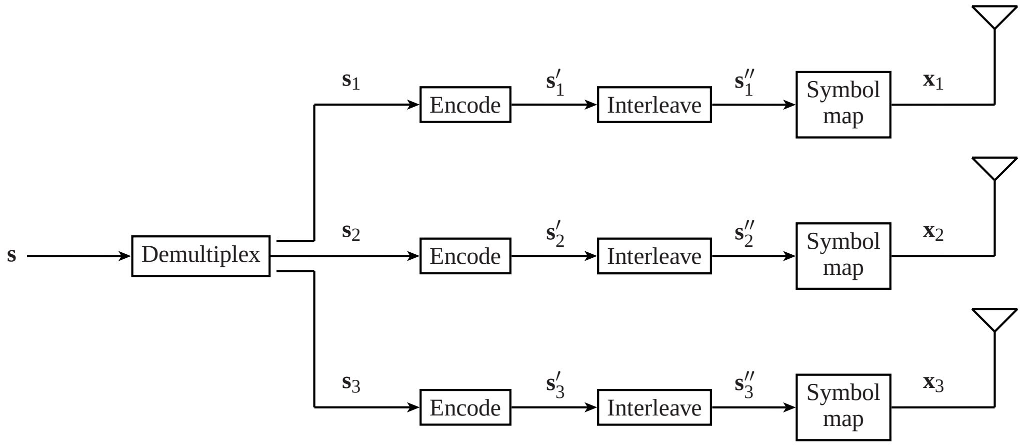

Fig. 1: Transmitter design for the H-BLAST PQRD system.

Now, the expression for the $(q-1)^{th}$ element of $\underline{\mathbf{y}}'(z)$ can be rewritten, using the estimate of the q^{th} transmitted signal, referred to as $\hat{\underline{x}}_q(z)$, as

$$\underline{y}'_{q-1}(z) - \underline{r}_{(q-1)q}(z)\hat{\underline{x}}_q(z) = \underline{r}_{(q-1)(q-1)}(z)\underline{x}_{q-1}(z) + \underline{n}'_{q-1}(z), \quad (16)$$

where all terms on the left-hand side of this expression are known and so this expression is again a single channel equalisation problem. A SISO equalisation method can again be applied to obtain an estimate of the signal $\underline{x}_{q-1}(z)$. This process is now repeated working upwards through the elements of $\underline{\mathbf{y}}'(z)$. In particular, the i^{th} element of this vector can be expressed as

$$\underline{y}_i'(z) - \sum_{j=i+1}^{q} \underline{r}_{ij}(z)\underline{x}_j(z) = \underline{r}_{ii}(z)\underline{x}_i(z) + \underline{n}_i'(z), \quad (17)$$

which, provided the set of signals is estimated according to the ordering $i = q, q-1, \ldots, 1$, will be a SISO channel equalisation problem. Each equation can then be solved to obtain an estimate of the i^{th} transmitted signal $\hat{\underline{x}}_i(z)$ using the previously estimated signals $\hat{\underline{x}}_j(z)$ for $j = i+1, \ldots, q$. As with OFDM systems, preprocessing techniques such as encoding and interleaving can be applied to improve the performance of the system.

4. TRANSMITTER AND RECEIVER DESIGN FOR THE SPATIAL-TEMPORAL MODE CHANNEL

To enable a fair comparison with previous work in the area, we have adopted the same PQRD system as implemented in [5], i.e. using a Bell Laboratories Layered Space Time encoding architecture at the transmitter and Turbo equalisation at the receiver. The overall polynomial matrix decomposition system that has been used for the simulations in Section 5 is now described.

4.1. Transmitter Design

When communicating over a MIMO system, it is advantageous to use all available antennas as this will enable the system to achieve full diversity order [7]. Horizontal Bell Laboratories Layered Space Time (H-BLAST)[1] architecture is a method for doing this [8], which operates by demultiplexing the data stream into q independent streams that can then be individually encoded, interleaved and symbol mapped in parallel prior to transmission from each of the q antennas. At the receiver each of the p streams are individually recovered before multiplexing to obtain an estimate of the initial data stream. This system design avoids the impractical high complexity observed with serial encoding [9], where the initial single data stream will be encoded and then interleaved prior to demultiplexing into a set of q substreams.

Assume that the data stream to be transmitted is $\mathbf{s} = [s(0), \ldots, s(N_2-1)]^T$. This data stream is firstly demultiplexed into q independent substreams, where the k^{th} substream is

$$\mathbf{s}_k = [s((k-1)N_2/q), \ldots, s(kN_2/q-1)]^T. \quad (18)$$

Each stream is then independently convolutionally encoded using the following code formatting polynomials

$$G0 = 1 + D3 + D4 \qquad \text{and} \qquad G1 = 1 + D1 + D3 + D4, \quad (19)$$

which are taken from the standards for the global system for mobile (GSM) communications [10]. Note that code rate is $1/2$ and so each encoded data stream will be of length $2(N_2/q)$. The encoded signal is then interleaved to randomise the encoded bits prior to transmission. This ensures that any errors appear random and, as a result, avoids long error bursts in estimates of the transmitted data. For the results presented in Section 5, an S-random interleaver with a depth of 28 bits has been applied [5]. The k^{th} independent substream is then symbol mapped to a constellation point to obtain the sequence $\mathbf{x}_k$ and then transmitted from the k^{th} transmit antenna. Note that the transmitted signal in equation (8) at time τ, where $\tau = 0, \ldots, 2(N_2/q-1)$, now relates to the independent substreams as follows

$$\mathbf{x}(\tau) = [x_1(\tau), x_2(\tau), \ldots, x_q(\tau)]^T \quad (20)$$

where $x_k(\tau)$ is the τ^{th} element of the vector $\mathbf{x}_k$. A block diagram of the transmitter design, using the H-BLAST encoding structure, can

[1] Other BLAST architectures can be used within this scheme, H-BLAST is demonstrated throughout this paper as an example.

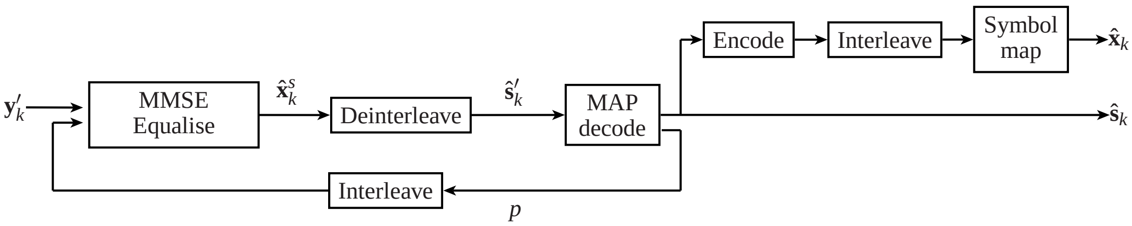

Fig. 2: Block diagram for the iterative Turbo equalisation (joint equalisation and decoding) scheme when applied to the k^{th} substream $\mathbf{y}'_k$. This process is applied to each substream of data from each receive antenna as demonstrated in Figure 3.

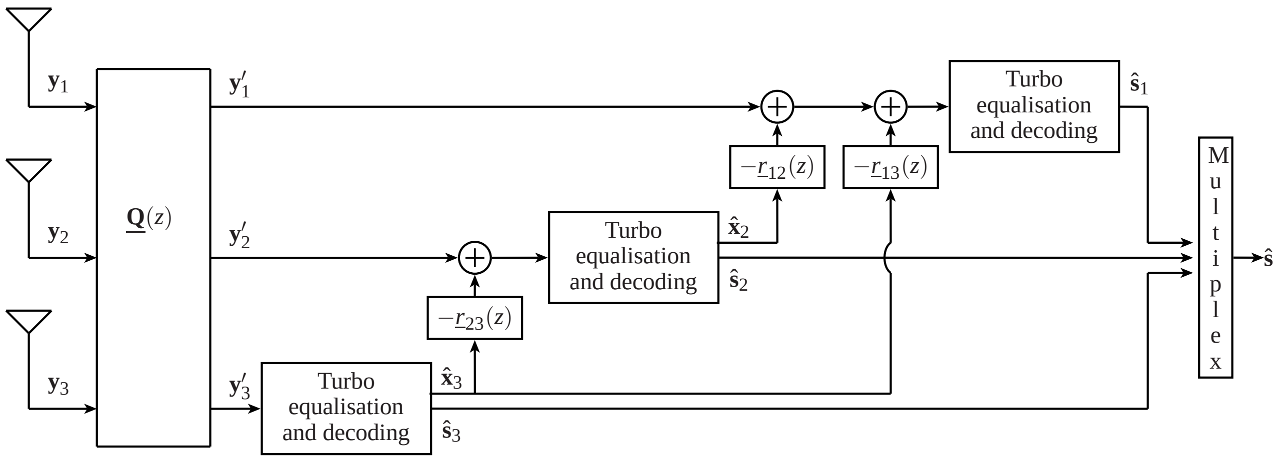

Fig. 3: Receiver design for the H-BLAST PQRD system for a three transmitter and three receiver system.

be seen in Figure 1, where it is assumed that $p = q = 3$ as these are the dimensions of our channel model in Section 5.

4.2. Receiver Design

Assume the signal received at the k^{th} antenna at time τ is $y_k(\tau)$. Then the set of received signals at time τ can be expressed in vector form as

$$\mathbf{y}(\tau) = [y_1(\tau), \ldots, y_p(\tau)]^T. \quad (21)$$

Assuming the channel matrix to the system is known at the receiver and does not vary in time with each data block, the PQRD of this matrix can be calculated according to equation (3). The received signals are firstly filtered by the paraunitary matrix to obtain $\mathbf{y}'(\tau) \in \mathbb{C}^{p\times 1}$. The q^{th} element of this vector can then be expressed as the single channel equalisation problem in equation (15) and can now be solved to obtain an estimate of the q^{th} transmitted signal, i.e., $\mathbf{x}_q$, using a standard method for SISO channel equalisation. For the results presented in this paper, an iterative process of equalisation and decoding has been implemented, which exchanges extrinsic information between the two components to improve the overall bit error rate performance. The iterative process is known as Turbo equalisation and is now explained.

At each iteration of this process the following three step routine is implemented,

1. Firstly, the equalisation of the SISO polynomial problem is performed using a minimum mean squared error (MMSE) equaliser to obtain a soft output estimate of $\mathbf{x}_q$, this is referred to as $\hat{\mathbf{x}}^s_q$. The series of coefficients of $\underline{r}_{qq}(z)$ that are used within the equalisation process are also truncated to ensure that the process is not unnecessarily computationally slow to implement. A detailed description of this equalisation function, including the truncation process, can be found in [5, 6].
2. The soft estimate of $\mathbf{x}_q$ is then deinterleaved.
3. A maximum a posteriori (MAP) decoder is then used to obtain an estimate of $\mathbf{s}_q$ from the deinterleaved $\hat{\mathbf{x}}^s_q$. For each transmitted symbol, the MAP decoder generates a hard estimate of $\mathbf{s}_q$ and also a soft estimate in the form of the *a posteriori* probability of the received sequence, referred to as p, which is then interleaved and used within the MMSE equaliser in the subsequent iteration. This exchange of information between the different stages of the joint equalisation and decoding process will enable improved performance of the system.

This routine is now repeated incorporating the soft feedback from the MAP decoder into the MMSE equaliser as this will generally demonstrate significant improvements in the BER. For a detailed description of this technique see [5, 6, 11] and references therein. For the results presented in this paper three iterations of the joint equalisation and decoding process are required. Further iterations offered no further improvement in terms of average BER performance. A block diagram of the joint Turbo equalisation scheme can be seen in Figure 2. Once an estimate of $\mathbf{s}_q$ has been obtained, this can be used to obtain an estimate of $\mathbf{s}_{q-1}$ using back substitution as described in Section 3. This process is repeated until all substreams of transmitted data have been recovered. The overall receiver design is detailed in Figure 3, where it has again been assumed that $p = q = 3$.

5. SIMULATION RESULTS

The PQRD algorithm was applied as part of a broadband MIMO communication system as described in Sections 3 and 4, where average BER results will be used to assess the performance of the system over a range of signal-to-noise ratios (SNR). The polynomial channel matrix for the system $\underline{\mathbf{A}}(z) \in \underline{\mathbb{C}}^{3\times3}$ is chosen to be of order four, where the series of coefficients of each of the polynomial elements are drawn from a circular complex Gaussian distribution with mean zero and variance $1/5$. The BER results, averaged over 1000 Monte-Carlo simulations, found when using this PQRD system are presented in Figure 4. Within the PQRD-BC algorithm, the polynomial matrices are continually truncated to ensure that their orders are not unnecessarily large and the algorithm computationally slow to implement. The truncation is performed based on proportion of the F-norm of the matrix permitted to be lost, this parameter, referred to as μ was set equal to 10^{-6} for these results. Furthermore, the algorithm was set to run until all coefficients of elements beneath the diagonal of the matrix $\underline{\mathbf{R}}(z)$ are less than 10^{-2}. Note that this stopping parameter is referred to as ε.

This process was now repeated, allowing for estimation error on the channel matrix according to equation (5), where $\underline{\mathbf{E}}(z) \in \underline{\mathbb{C}}^{3\times3}$ is a matrix of complex Gaussian noise with mean zero and variance σ^2. The average BER simulations were then repeated with varying levels for channel estimation. The observed average BER simulations found when $\sigma^2 = 0.005$ and $\sigma^2 = 0.01$ can be seen in Figure 4. Clearly the more inaccurate the channel, then the worse the performance of the system.

Note that other errors are present in the system, other than the channel estimation error as explained in Section 2.1. These errors arise from only calculating an approximate QR decomposition of the polynomial channel matrix, but can be kept small by choosing suitable values for the stopping criterion ε and truncation parameter μ when formulating the decomposition. The relative error was then calculated to check the overall accuracy of the decomposition in the presence of all errors. This measure was on average found to equal 0.0177 when $\sigma^2 = 0$, 0.2054 when $\sigma^2 = 0.005$ and 0.2257 when $\sigma^2 = 0.01$. Further analysis on the effects of the different errors upon the BER performance of the system is on going. In particular, to determine the relationship between the three distinct errors discussed and the error performance of the systems.

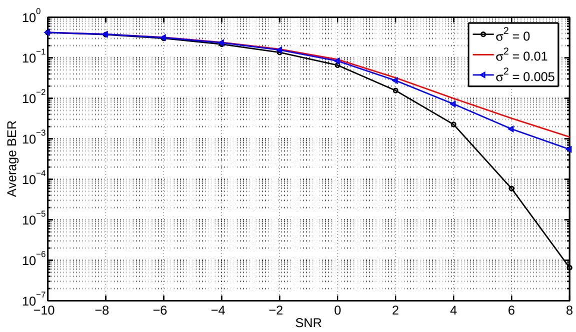

Fig. 4: Average BER performance observed when using the PQRD system with varying levels of channel state information.

6. CONCLUSIONS

This paper has demonstrated how a polynomial matrix QRD algorithm can be used as part of a broadband MIMO communications system. This paper has also discussed the possible errors that are encountered when formulating the QR decomposition of a polynomial matrix. In particular, average BER simulations have been used to illustrate the effect of various levels of channel state information upon the performance of the system. Future work aims to compare this method with other approaches of communicating over broadband MIMO channels, in particular to compare this method with a MIMO orthogonal frequency division multiplexing QRD approach, also when there is error present in the estimate of the channel matrix.

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