Optimum Waveform Design and Clutter Rejection Processing for MIMO Radar

Hai Deng* and Braham Himed†
*Florida International University, Miami, FL 33174, USA
†Air Force Research Lab, Dayton, Ohio, 45433, USA

Introduction

Multiple-Input Multiple-Output (MIMO) radar has the capability to simultaneously transmit and receive multiple orthogonal waveforms to improve radar performances [1, 2]. The unique structure of MIMO radar, as shown in Fig.1, allows transmitting waveform and signal processing to be optimized to achieve a better radar performance in various areas. Multiple matched filters are used at the receiver for each antenna element to exclusively receive the waveforms transmitted by each element. It has been demonstrated that virtual transmitting and receiving beams can be formed to achieve an optimum signal-to-noise ratio (SNR) in target detection, through digital signal processing at the receiver using arbitrary transmitting waveforms. The actual transmitting antenna beam can be optimized to minimize the radiation power in any direction (or all directions) to achieve a low probability of intercept (LPI) property, without degrading target detection performance. In previous work involving MIMO radar virtual beamforming systems, only SNR and white noise were considered for target detection performance evaluation. However, for some applications such as ground moving target detection, using airborne assets, ground clutter interference is a much more serious threat for reliable target detection. In this work, the MIMO system is further investigated for clutter rejection as well as waveform design for optimizing the actual radar radiation pattern.

Clutter and Target Models for MIMO Radar

Consider a MIMO radar with L transmitting antenna elements and the transmitting waveform is assumed to be

$$\mathbf{s} = [e^{j\varphi_1} e^{j\varphi_2} \cdots e^{j\varphi_L}]^T \quad , \tag{1}$$

where φ_l is the initial phase of the waveform transmitted at element $l, (1 \le l \le L)$. It is assumed that the MIMO radar system operates with a low pulse repetition frequency (PRF) such that there is no range ambiguity for the echoes. For clutter patch i in the clutter ring shown in Fig. 2, the relative time delay vector from the L antenna elements to the clutter patch is

$$\mathbf{d}_i = [0\ \tau_{i1}\ \tau_{i2} \cdots \tau_{iL-1}]^T \tag{2}$$

and the transmit and receive steering vectors for the clutter patch in the conventional sense are:

$$\mathbf{v}_{it} = [1\ e^{-j\omega\tau_{i1}}\ e^{-j\omega\tau_{i2}} \cdots e^{-j\omega\tau_{iL-1}}]^T \tag{3}$$

and

$$\mathbf{v}_{ir} = [1\ e^{-j\omega\tau_{i1}}\ e^{-j\omega\tau_{i2}} \cdots e^{-j\omega\tau_{iL-1}}]^T . \tag{4}$$

The MIMO radar steering vector for the i-th clutter patch is:

$$\mathbf{v}_{is} = \mathbf{v}_{it} \otimes \mathbf{v}_{ir} , \tag{5}$$

where $\otimes$ denotes the Kronecker product.

Considering the initial phases of the transmitted waveforms , one may define the MIMO radar echo of the i-th clutter patch in Fig. 2 as:

$$\boldsymbol{\chi}_{ci} = \alpha_{ci}\mathbf{v}_i , \tag{6}$$

where

$$\mathbf{v}_i = [diag(\mathbf{s})\mathbf{v}_{it}] \otimes \mathbf{v}_{ir} \tag{7}$$

and α_{ci} is a random complex echo magnitude from the i-th clutter patch. If the number of clutter patches in a clutter ring is N, the received clutter for the clutter ring is given by:

$$\boldsymbol{\chi}_c = \sum_{i=1}^{N} \boldsymbol{\chi}_{ci} = \sum_{i=1}^{N} \alpha_{ci}\mathbf{v}_i . \tag{8}$$

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The random complex magnitudes of clutter patches are assumed to be independent and identically distributed (iid) and therefore, it follows that:

$$E\{\alpha_i \alpha_j^*\} = \xi_i \delta_{i-j} \tag{9}$$

Accordingly, the clutter covariance matrix of the clutter ring is:

$$\mathbf{R}_c = \sum_{i=1}^{N} \xi_i \mathbf{v}_i \mathbf{v}_i^H \tag{10}$$

The relative time delay vector from L antenna elements to the target is assumed to be:

$$\mathbf{d}_t = [0\ \tau_{t1}\ \tau_{t2} \cdots \tau_{tL-1}]^T \tag{11}$$

The target transmit and receive steering vectors are respectively:

$$\mathbf{v}_{tt} = [1\ e^{-j\omega\tau_{t1}}\ e^{-j\omega\tau_{t2}} \cdots e^{-j\omega\tau_{tL-1}}]^T \tag{12}$$

and

$$\mathbf{v}_{tr} = [1\ e^{-j\omega\tau_{t1}}\ e^{-j\omega\tau_{t2}} \cdots e^{-j\omega\tau_{tL-1}}]^T \tag{13}$$

The expected target signal vector received from the matched filter outputs in Fig. 1 is determined by:

$$\mathbf{t} = \alpha_t [diag(\mathbf{s})\mathbf{v}_{tt}] \otimes \mathbf{v}_{tr} \tag{14}$$

where α_t is the random complex target signal magnitude with its power is given by:

$$E\{|\alpha_t|^2\} = \xi_t \tag{15}$$

However, in most cases for MIMIO radar with closely-located antenna elements, the target signal may be considered to be deterministic with an unknown amplitude and phase.

Signal Processing and Waveform Optimization

The optimal signal processing is to design the best digital filter coefficient vector **w** in Fig. 1 to maximize the output target signal-to-interference plus noise ratio (SINR). Because the clutter and receiver noise are uncorrelated and considered to be zero-mean Gaussian distributed, the covariance matrix of the clutter and noise at the outputs of the matched filters in Fig.1 is given by:

$$\mathbf{R} = \mathbf{R}_c + \mathbf{R}_n = \mathbf{R}_c + \sigma_n^2 \mathbf{I} \tag{16}$$

where σ_n^2 is the receiver white noise variance and **I** is the unit matrix of size $L^2 \times L^2$.

With both clutter and noise assumed to be Gaussian-distributed, the maximum likelihood ratio test can be applied to determine the best target detector. Hence, the optimal filter coefficient vector is obtained as:

$$\mathbf{w} = k\mathbf{t}^H \mathbf{R}^{-1}, \tag{17}$$

where k is a constant, normally defined as $\mathbf{w} = 1/(\mathbf{t}^H \mathbf{R}^{-1} \mathbf{t})$. Accordingly, the final output for the MIMO radar shown in Fig. 1 is given by:

$$\Lambda = \mathbf{w}^T \mathbf{x} = \frac{\mathbf{t}^H \mathbf{R}^{-1} \mathbf{x}}{\mathbf{t}^H \mathbf{R}^{-1} \mathbf{t}} \tag{18}$$

An important metric for radar clutter rejection performance evaluation is Signal to Interference plus Noise ratio (SINR) improvement factor (IF), defined as:

$$IF_{SINR} = \frac{SINR_o}{SINR_i}, \tag{19}$$

where $SINR_o$ is the SINR at the final output and $SINR_i$ is the SINR for each channel at the output of the matched filters in Fig. 1. The SINR improvement factor can further be expressed as:

$$IF_{SINR} = \frac{SINR_o}{SINR_i} = \frac{\xi_t |\mathbf{w}^T \mathbf{t}|^2}{\mathbf{w}^H \mathbf{R}^{-1} \mathbf{w}} \Big/ \left(\frac{\xi_t}{\sigma_n^2 + \xi_c} \right) \tag{20}$$

The initial phases of the transmitted waveforms do not directly affect the target detection results because the phases can be compensated for through the digital filtering process. However, the phases of the transmitted waveforms affect the actual transmitted beam patterns. In order to achieve a low probability of intercept (LPI) property, we can choose the phases of the transmitted waveforms to make the radiation power of the physical antenna beams to be roughly uniform in all directions. Previous work has demonstrated that by making the waveform phases uniformly distributed in $(0, 2\pi)$, the formed antenna beam radiation power is roughly uniform in all directions.

Simulation Results

The proposed approach is simulated based upon the MIMO radar configuration shown in Fig. 1, with a linear antenna array consisting of 16 elements, uniformly spaced at half-wavelength $(\lambda/2)$. The carrier frequency is 450 MHz. The distance from the radar to the range ring is 100km. The clutter-to-noise ratio (CNR) prior to signal processing is 40dB and the signal-to-noise ratio (SNR) is 0dB. The target is fixed in the direction of 0° on the range ring. The clutter patch is located on the same range ring with an extension of 1° in azimuth. When the location of the clutter patch is changed from 1° to 360°, the performance of the SINR improvement factor, after the MIMO clutter rejection processing, is shown in Fig. 3. The results shown in Fig. 3 indicate that the clutter can be almost completely eliminated through the MIMO clutter rejection processing proposed in this work, except when the clutter patch is very close to the target. Because the processing is only performed in the space domain, when the clutter patches exist simultaneously in all directions, the total removal of all clutter is not feasible.

The initial phases of the orthogonal waveforms used to generate our simulation results in Fig. 3 are randomly and uniformly chosen from a value between 0 and 2π. Subsequently, the actual radiation beam pattern of the MIMO radar is shown in Fig. 4. Even the maximum radiation power is limited under a certain value in any direction to minimize the probability of radar signal interception, the best target detection and clutter rejection performance is still achieved.

References

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[2] F. Robey and et al, "MIMO radar theory and imaging: degrees of freedom and resolution," *2003 Conf. Record of the 37th Asilomar Conf. on Signals, Systems and Computers*, vol. 1, Nov. 2003.

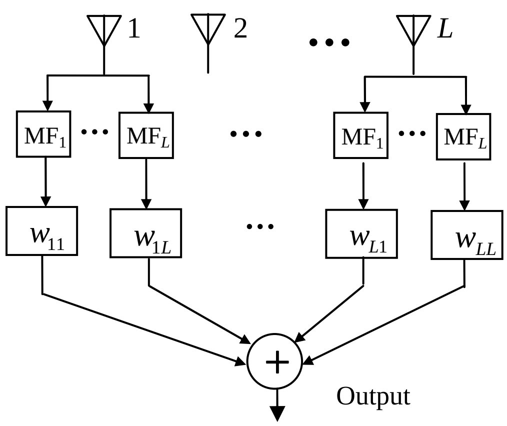

Fig. 1 Basic structure of MIMO radar processor

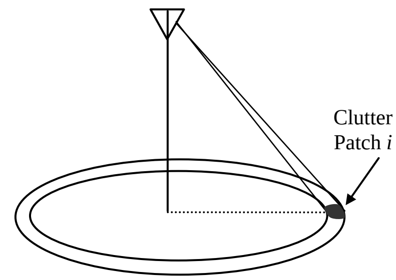

Fig. 2 Clutter patch *i* in a clutter ring for MIMO radar operation

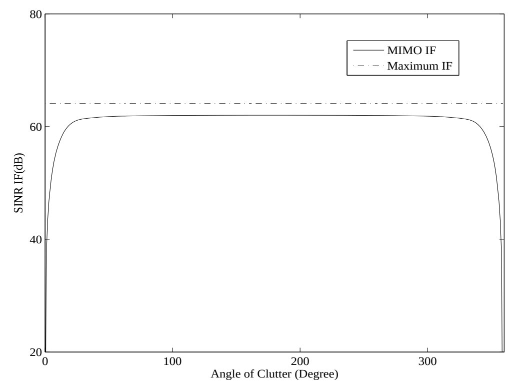

Fig. 3 Signal-to-interference plus noise ratio (SINR) improvement factor for clutter at different angles

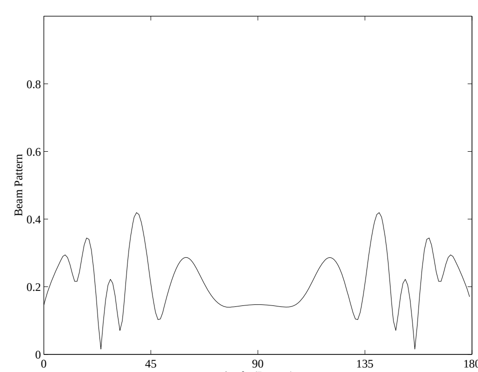

Fig. 4 Actual radiation beam pattern generated by the transmitted waveforms to achieve LPI property