SUB-QUANTIZATION/ORTHOGONALIZATION AND OPTIMIZATION OF ALGORITHM-ARCHITECTURE ADEQUACY FOR OPTIMAL POLYNOMIAL FILTERING

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ABSTRACT

This paper deals with the benefits of mixing an orthogonalization procedure to the sub-quantization of the input operation, in order to reduce the complexity and to enhance the robustness implementation of optimal polynomial filters. These non linear filters are now generic devices for real-time high speed multimedia applications. We propose an optimal polynomial filtering scheme based on Sub-quantization/ Orthogonalization operations, related to the current interest to complexity, power consumption and areas reduction.
The orthogonal polynomial basis is chosen to be in a simple closed-form expression and the input is sub-quantized with a desired sub-quantization degree. We explore, from stability and accuracy tests, the suitable choice of sub-quantization in order to achieve performances of the proposed optimal polynomial filtering scheme with robustness performances almost similar to the corresponding continuous input case.

1. INTRODUCTION

The complexity reduction for implementation of generic signal operations (such as FFT, convolution,...) on devices namely DSP, micro-controller and FPGA, has been extensively analyzed. With the expansion of real-time multimedia applications, especially in embedded systems, we are brought to optimize complexity of more complicated generic operations such as echo cancelation, noise reduction, predistorsion/equalization... with respect to some quality criteria [7]. For example, to overcome distortions induced by non linear characteristics of audio systems, studies are proposed [8, 9] in order to synthesize loudspeakers with different level of qualities, according to intended applications (industrial, real-time public applications, ...). RF Power Amplifier modeling and predistorter design has been proposed in [1, 2, 5] to linearize such systems in order to improve spectrum bandwidth utilization in multi-users applications.

NL system features is done commonly by a memoryless polynomial, thanks to its simplicity and ease of implementation. However, the real-time optimization is computationally expensive (matrix inversion, multiplication) and presents numerical instability problem.

In this paper, we explore the benefits achieved by a sub-quantization operation, leading to power complexity reduction and implementation area optimization [11], coupled with a polynomial orthogonalization procedure improving the numerical stability, for both optimal and adaptive polynomial filtering [3, 10]. We select a robust closed-form basis expression of orthogonal polynomials proposed in [1]. This basis presents the emrit to be a non-iterative procedure, compared to Hermite polynomials which are presented in a compact form but they are really derived from the Gram-Schmidt procedure, which is well known to be an iterative one.

The paper is organized as follows: we present in section 2, the sub-quantization/orthogonal polynomial optimal filtering scheme with a suitable choice of the quantization operation and the orthogonal procedure. We establish, in section 3, through simulations results, that the sub-quantized input gives the same results as the continuous one, under certain conditions. We specify the sub-quantization degree allowing to code the signal without loss of stability properties ensured by the continuous input. The accuracy of the sub-quantization/orthogonalization scheme is illustrated and discussed in section 4.

2. SUB-QUANTIZATION/ORTHOGONALIZATION SCHEME FOR OPTIMAL POLYNOMIAL FILTERING

2.1 Sub-quantization and filtering scheme

Deserving attention to optimization of algorithm-architecture adequacy for polynomial filtering with good numerical stability, the proposed identification scheme is based on a sub-quantization operation for fixed-point signal input coding, and a non iterative polynomial orthogonal procedure to enhance the performances of a real-time optimal system. Consequently we combine a sub-quantization operation with an orthogonalization procedure according to figure (1).

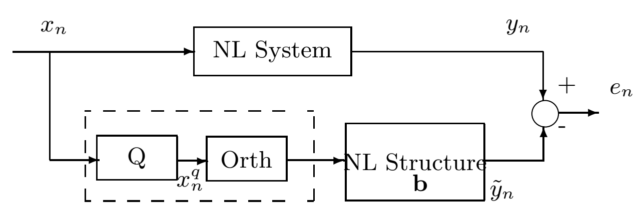

Figure 1: Sub-quantization/Orthogonalization scheme for robust optimal NL filtering implementation.

Now we propose to analyze the input/output relationship of filtering scheme with/or not the presence of the inserted box.

2.1.1 Conventional polynomial filtering scheme

Denoting by x_n the input of the NL system, and by y_n the corresponding output, at sample n. We observe N samples of x_n and y_n; and we note:

- $\mathbf{x} = [x_1, \ldots, x_N]^T$: the input data vector,
- $\mathbf{y} = [y_1, \ldots, y_N]^T$ the system output vector,
- $\tilde{\mathbf{y}} = [\tilde{y}_1, \ldots, \tilde{y}_N]^T$ the expected output vector.

The conventional input/output relationship, for a memoryless polynomial structure with order p, is given by:

$$\tilde{\mathbf{y}} = \mathbf{\Phi b}, \tag{1}$$

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where:

- $\mathbf{b} = [b_1, b_2, \ldots, b_p]^T$ the parameter vector,
- $\mathbf{\Phi}$ is the $N \times p$ matrix, characterizing the input features.

2.1.2 Orthogonal polynomial filtering scheme

The aim of orthogonalization procedures is to alleviate the numerical instability problem which rises from the inversion of the basis $\mathbf{\Phi}$ [1], and to enhance the convergence speed of adaptive algorithms [10].
Based on polynomial orthogonalization procedures, the relationship (1) becomes [1, 3]:

$$\tilde{\mathbf{y}} = \mathbf{\Psi}\beta, \tag{2}$$

The orthogonal basis $\mathbf{\Psi}$ spans the same space as the conventional basis $\mathbf{\Phi}$, and satisfies the following relationship:

$$\mathbf{\Psi} = \mathbf{\Phi}\mathbf{U}. \tag{3}$$

The orthogonal polynomial basis construction problem results on finding the upper triangular matrix $\mathbf{U}$ such that:

$$\mathbf{E}(\mathbf{\Psi^H\Psi}) = diag(d_1, \ldots, d_p),$$

where $\{d_i\}_{i=1,\ldots,p}$ ($d_1 > d_2 > \ldots d_i \ldots > d_p$) are the corresponding eigenvalues.

2.1.3 Proposed filtering scheme

The proposed scheme for NL system identification, presented in figure (1), is based on the cascade of two blocks:

- **Sub-quantization operation: Block Q**
  Achieving an uniform quantization operation into B bits signal coding. The finite alphabet set is with cardinality $N_c = 2^B$.
  The quantization procedure aims to reduce the implementation complexity of algorithms with fixed-point arithmetic.
- **Orthogonalization procedure: Block Orth**
  Instead of (2), the system's input/output relationship is given by:
  $$\tilde{\mathbf{y}} = \mathbf{\Psi_q}\beta_\mathbf{q}, \tag{4}$$
  where $\mathbf{\Psi_q}$ is defined in the same manner as $\mathbf{\Psi}$, and is applied to the sub-quantized input x_n^q.
  Since our major interest is on real-time applications, we have considered a closed-form basis expression of orthogonal polynomial proposed in [1]. The k^{th}order orthogonal polynomial basis is given by:
  $$\psi_k(x_q) = \sum_{l=1}^{k}(-1)^{l+k}\frac{(k+l)!}{(l-1!)(l+1)!(k-l)!}|x_q|^{l-1}x_q. \tag{5}$$
  Having the input sequence $\{x_q\}$, the terms $\{\psi_k\}$ are derived directly for any order k, contrary to Hermite polynomial basis which requires the knowledge of lower orders. This selected basis is generated under the hypothesis of uniformly distributed inputs in $[0, 1]$.

2.2 Optimal filtering

By minimizing the Mean Squared Error (MSE) $E[|y_n - \tilde{y}_n|^2]$, the orthogonal optimal coefficient set β_q^{opt} is then given by [12]:

$$\beta_\mathbf{q}^\mathbf{opt} = (\mathbf{\Psi_q^H\Psi_q})^{-1}\mathbf{\Psi_q^H}\mathbf{y}, \tag{6}$$

where $[.]^H$ stands for the Hermitian transpose.
Thanks to the orthogonality between elements of $\mathbf{\Psi_q}$, we expect that the matrix $(\mathbf{\Psi_q^H\Psi_q})$ has a better condition number

$$K((\mathbf{\Psi_q^H\Psi_q})) = |\frac{\lambda_{max}}{\lambda_{min}}|, \tag{7}$$

where λ_{max} and λ_{min} are, respectively, the maximum and minimum eigenvalue of the matrix $(\mathbf{\Psi_q^H\Psi_q})$.

3. LIMIT OF SUB-QUANTIZATION BENEFITS

We describe in this section the sub-quantized states generation.
Through a stability evaluation of the proposed filtering scheme, we give an idea to the system designer about the accurate sub-quantization degree, namely the value of bits number B leading to equivalent performances, ensured in the continuous case.

3.1 Sub-quantized states

To satisfy the hypothesis of x_n belonging in $[0, 1]$, considered by Raich in order to apply the orthogonal polynomials basis expression [1], we impose that the finite alphabet states vary also in $[0, 1]$ as shown in figure (2).

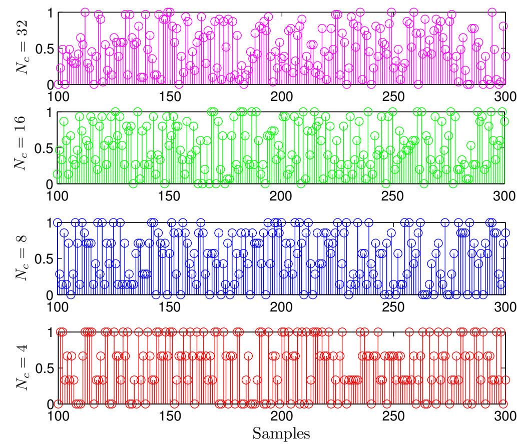


Figure 2: Sub-quantized states varying in $[0, 1]$ from 32 states to 4 states.

As expected, the quantized inputs have equivalent probability of apparition, according to the states level, as shown in figure (3).

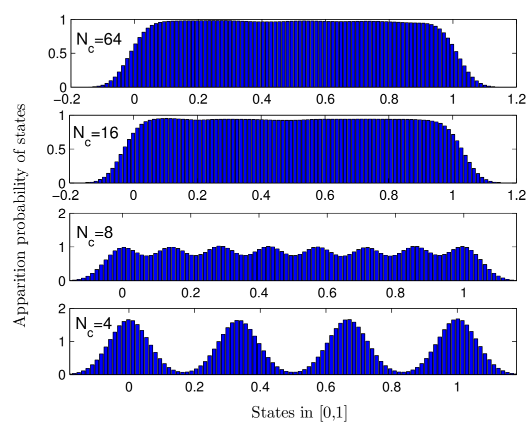


Figure 3: Effect of sub-quantization on states distribution (from 32 states to 4 states). The initial distribution were uniformly distributed in $[0, 1]$

Notations

Through all the paper, we denote by:

- QI: Sub-Quantized Input with B bits,
- QOI: Sub-Quantized and Orthogonal Input,
- CI: Continuous Input,
- COI: Continuous and Orthogonal Input.
  We note that a continuous input is equivalent to the sub-quantized input case where $B \to \infty$.

3.2 Stability of optimal polynomial filtering

The considered stability indicator is the condition number of the matrix $(\boldsymbol{\Psi}_{\mathbf{q}}^{\mathbf{H}}\boldsymbol{\Psi}_{\mathbf{q}})$ as defined in (7).

3.2.1 Real-valued uniformly distributed in $[0, 1]$ *process*

When we observe the evolution of the condition number (figure(4)-a), without orthogonal procedure, we deduce that continuous and sub-quantized inputs have high and similar values of the condition number.

For this reason, we propose to apply the orthogonal polynomial procedure.

The benefit of the introduction of the orthogonal polynomial procedure is shown on figures((4)-b,(4)-c).
Figure ((4)-c) shows that the use of the closed-form basis expression of orthogonal polynomials, under the uniformly distributed in $[0, 1]$ hypothesis, allows to reduce considerably the condition number, for both continuous and sub-quantized inputs, when considering a sub-quantization degree $B > 4$. It is interesting to note also, that coding the signal on only 6 bits is sufficient to have exactly the same robustness of the identification scheme when using continuous input (figure(4)-c).

For small states ((4)-b), the condition number is not considerably improved, only for small orders of nonlinearities, since the hypothesis of being uniformly distributed in $[0, 1]$ is not properly guaranteed (figure (3)).

3.2.2 WCDMA process

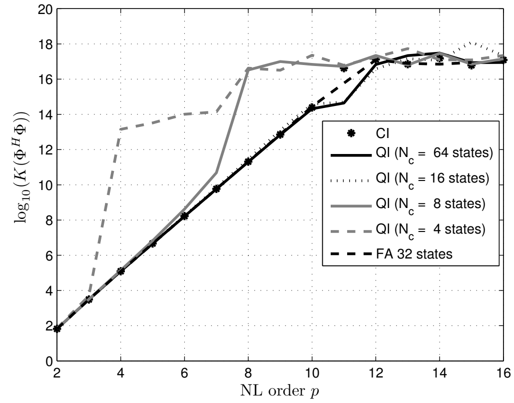
Since closed-form basis expression of orthogonal polynomial, generated under uniformly distributed in $[0, 1]$, has shown robustness for others commonly used PDF [1, 4], we propose to test the effect of combining a quantized operation on identification performances.
We focus on RF power amplifiers, which are subject to non-constant modulus signals such as WCDMA or OFDM signals where the amplitude is of Rayleigh distribution [5].
Let us consider a complex-valued signal as:

$$x_n = I_n + jL_n,$$

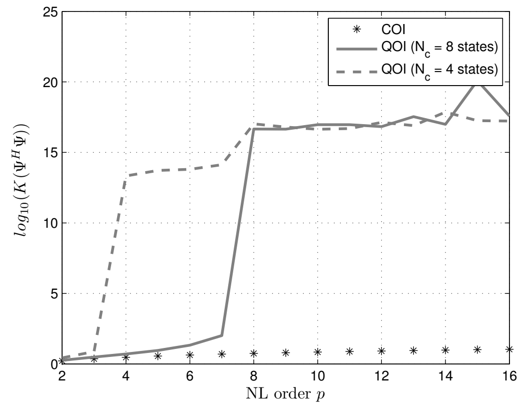
where: I_n and L_n are two real-valued gaussian processes, having the same power as the uniformly distributed in $[0, 1]$ process. $|x_n|$ is then of Rayleigh distribution.
If $I_n^q = Q(I_n)$ and $L_n^q = Q(L_n)$, then the quantized version of x_n is given by:

$$x_n^q = I_n^q + jL_n^q.$$

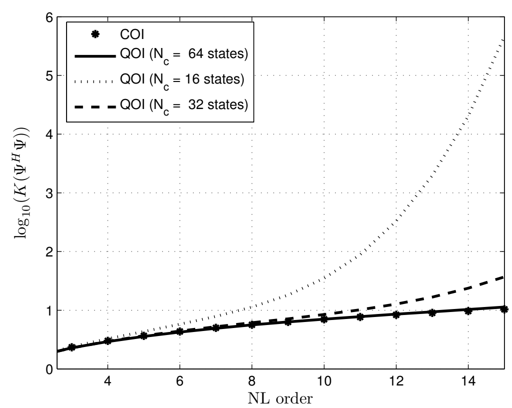
Figure (5) emphasizes the result that the sub-quantization/orthogonalization scheme for optimal polynomial filtering presents a better numerical stability than the conventional one, even with the use of gaussian input. This confirm the previous result which demonstrates that working with only 6 bits presents the same stability improvements than when coding the signal with 64 bits.



(a) Condition number of the conventional polynomial basis $\boldsymbol{\Phi}^{\mathbf{H}}\boldsymbol{\Phi}$ versus NL order, for $B = 2, 3, 4, 5$ and 6.



(b) Condition number of the orthogonal basis $\boldsymbol{\Psi}_{\mathbf{q}}^{\mathbf{H}}\boldsymbol{\Psi}_{\mathbf{q}}$ versus NL order, for a limited number of sub-quantized states ($B = 2$ and 3).



(c) Condition number of the orthogonal basis $\boldsymbol{\Psi}_{\mathbf{q}}^{\mathbf{H}}\boldsymbol{\Psi}_{\mathbf{q}}$ versus NL order, for experimental values of sub-quantized degrees ($B = 4, 5$ and 6).

Figure 4: Stability evaluation of the proposed sub-quantized/ orthogonalization based filtering scheme : evolution of the condition number for conventional and orthogonal basis versus polynomial order, for sub-quantized input cases with B bits values $(2; 3; 4; 5; 6)$ and $B \to \infty$. Case of real inputs uniformly distributed in $[0, 1]$

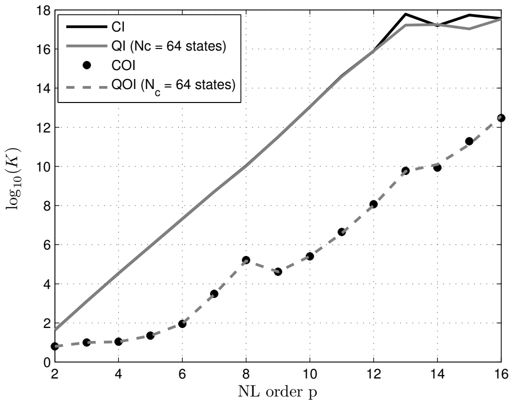


Figure 5: Stability evaluation of the proposed sub-quantized/ orthogonalization based filtering scheme: evolution of the condition number of conventional and orthogonal basis versus polynomial order, for sub-quantized input cases with B bits values $(2;3;4;5;6)$. The continuous case $(B \to \infty)$ is related to complex Gaussian input (WCDMA input).

4. ROBUSTNESS EVALUATION OF THE SUB-QUANTIZED/ORTHOGONALIZATION FILTERING

4.1 Experimental set up and simulations hypothesis

We consider a Saleh model for the NL system (figure(1)), when x_n is a real-valued process uniformly distributed in $[0, 1]$. This model is given by the following input/output relationship:

$$y_n = \frac{\alpha_a |x_n|}{1 + \beta_a |x_n|^2} exp \left[j \left(\angle x_n + \frac{\alpha_\phi |x_n|^2}{1 + \beta_\phi |x_n|^2} \right) \right]. \qquad (8)$$

The chosen parameters of Saleh model, for simulations tests are $\alpha_a = 2$, $\beta_a = 2.2$, $\alpha_\phi = 2$ and $\beta_\phi = 1$. The AM/AM conversion related to the amplitude distortion is then:

$$\frac{\alpha_a |x_n|}{1 + \beta_a |x_n|^2},$$

represented on figure (6), for $\alpha_a = 2$.

From figure (6), the tested NL system presents a high order of nonlinearity. We consider $N = 100.000$ samples, and 100 independent realizations of Monte-Carlo.

4.2 Accuracy evaluation

4.2.1 Dispersion effect

To study the accuracy of the proposed identification system, we insist on the dispersion of error measurements.
We consider the Normalized Mean Squared Error (NMSE):

$$NMSE(dB) = 10 \log_{10} \left[\frac{\sum_{n=1}^{N} |y_n - \tilde{y}_n|^2}{\sum_{n=1}^{N} |y_n|^2} \right]. \qquad (9)$$

Let us evaluate the dispersion of the NMSE, with a nonlinearity order $p = 16$ for different sub-quantization degree B, relatively for corresponding signal to quantization noise ratio defined as $RSB_Q(dB) = 10 \log_{10}(\frac{P_s}{P_Q})$ where P_s is the signal's power, and P_Q is the noise quantizer's power.

We set from figure(7) the following:

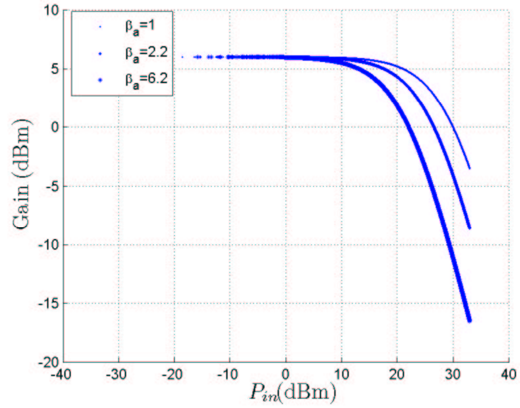


Figure 6: Non linear RF power amplifier tested : AM/AM conversion of Saleh Model when $\alpha_a = 2$ and for different nonlinearities factor β_a.

- Orthogonalization reduces the dispersion in the continuous case.
- Sub-quantization/Orthogonalization operations show accuracy improvement when coding signal with more than $B = 5$ bits.

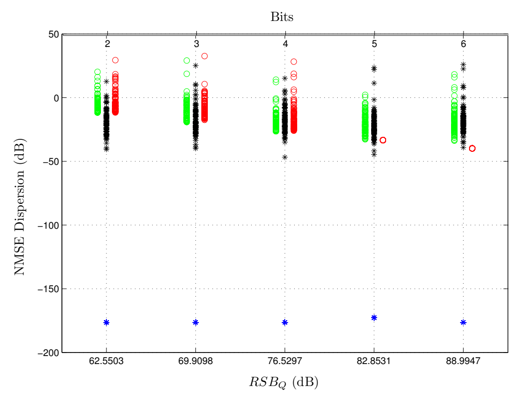


Figure 7: Accuracy evaluation of the proposed sub-quantized/ orthogonalization based filtering scheme versus sub-quantized degree B and the related signal to quantization noise ratio RSB_Q: Dispersion of the NMSE for 100 independent realizations of sub-quantized inputs and polynomial nonlinearity order $p = 16$. The continuous input $|x_n|$ is uniformly distributed in $[0, 1]$. CI: black cross, COI: blue cross, QI: green circle, QOI: red circle

For the complex-valued WCDMA process, the sub-quantized/orthogonal input shows good accuracy results, and the dispersion of the NMSE is optimized even with only $B = 3$ bits, as shown on figure (8).

4.2.2 Error evaluation

Figure (9) shows the evolution of the NMSE versus the NL order of the memoryless polynomial model, for the continuous case and the sub-quantized case for different signal bits coding B.
We can conclude, that even the best NMSE is reached when using orthogonal polynomial basis in the continuous case, the NMSE reached using the proposed sub-quantization procedure is acceptable in RF amplifier identification [6].
Furthermore, for higher orders of nonlinearities $(p > 10)$,

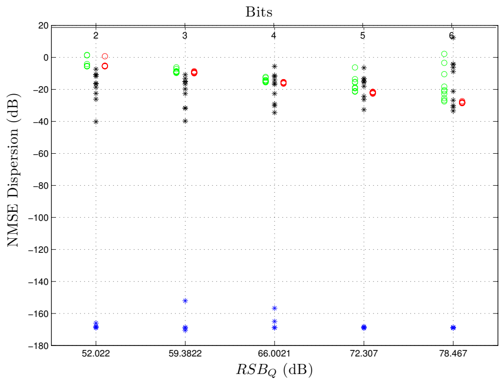


Figure 8: Accuracy evaluation of the proposed sub-quantized/ orthogonalization based filtering scheme versus sub-quantized degree B and the corresponding signal to quantization noise ratio RSB_Q: Dispersion of the NMSE for 50 independent realizations of sub-quantized inputs x_n and polynomial order $p = 16$. The continuous input $|x_n|$ is of Rayleigh distribution. CI: black cross, COI: blue cross, QI: green circle, QOI: red circle

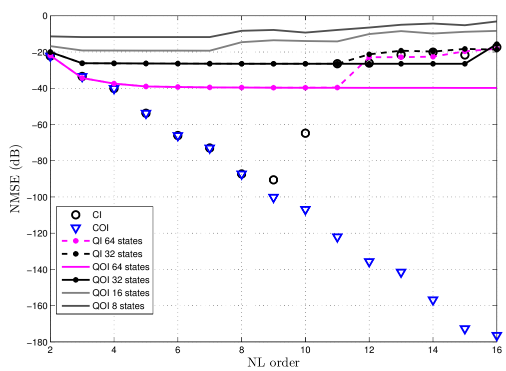


Figure 9: Error evaluation of the proposed sub-quantized/ orthogonalization based filtering scheme versus nonlinearity order, from continuous case to quantized one for different sub-quantization degree $B = log_2(N_c)$. Continuous input is real-valued uniformly distributed in $[0, 1]$.

two gains are achieved by the quantization of the input:

- Error reduction: from figure (9), we conclude that the NMSE in the continuous case, with the use of conventional polynomial $\mathbf{\Psi}$, is higher than the NMSE in the quantized case using the orthogonal polynomial basis $\mathbf{\Psi}$,
- Complexity reduction: it is sufficient to quantify the input on only 5 bits to have satisfactory results for both condition number and NMSE improvement.

According to the previous simulations results, we can deduce that a $B = 6$ bits of sub-quantization degree is sufficient to approximate the continuous case, namely:

- Ensure stability by improving the condition number when using orthogonal polynomial procedure,
- Optimize the accuracy of system identification, with dispersion errors reduction.

These results still valid when coding the signal with $B = 8$ bits. Since it is well known that implementation of operations on DSP or micro-controller needs at least $B = 8$ bits. Consequently, when reducing the complexity from $B = 64$ bits (continuous cas) to $B = 8$ bits, we expect the reduction of power and area with a minimal impact on performances.

5. CONCLUSION

In this paper we have investigated the impact of sub-quantization/orthogonalization scheme for optimal memoryless polynomial filtering, with optimization of algorithm-architecture adequacy. Orhtogonalization procedure, chosen in a closed-form expression for computational cost reduction of real-time applications, is associated to sub-quantization operations in order to ensure stability of system features design, with optimization of power consumption and areas implementation properties. Performances relying on stability and accuracy evaluation, of the proposed identification scheme have shown similarities with the conventional case, when coding signal with only $B = 8$ bits.

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