EEG ANALYSIS USING BI-FREQUENCY COHERENCE

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ABSTRACT

Bi-frequency Coherence is a normalized spectrum function based on Loève Spectrum. In this work, we investigated the use of Bi-frequency Coherence function for correlation of two non-stationary processes, particularly for EEG signals. We define a procedure to achieve Bi-frequency Coherence starting from Loève Spectrum. Then we present a numerical estimation of this approach. We compare our method with traditional coherence and TF-coherence functions, by means of some examples to show its advantages. It is shown that for some non-stationary processes, the proposed Bi-frequency Coherence function may extract underlying coupling better than other approaches.

1. INTRODUCTION

Inspection of the relations between two or more processes occurred simultaneously, has a significant role in signal processing. Analysis of stochastic processes, from this point of view, is a challenging issue. Various methods have been proposed to show coupling between stochastic processes such as cross correlation, cross spectra, coherence function, etc. Coherence function is one of the most widely used techniques to analyze such processes and find applications in a wide range of disciplines from optics to neuroscience[1, 2, 3, 4, 5, 6]. Coherence function is simply obtained by normalizing cross-spectrum of two processes with their auto-spectra [1] as shown in eq. (1):

$$|C_{XY}(\omega)|^2 = \frac{|S_{XY}(\omega)|^2}{S_X(\omega)S_Y(\omega)} \tag{1}$$

where the cross-spectrum of $X(t)$ and $Y(t)$ is obtained by taking the Fourier transform of the cross-correlation function of two processes:

$$S_{XY}(\omega) = \mathscr{F}\{R_{XY}(\tau)\} \tag{2}$$

and the cross-correlation is calculated as,

$$R_{XY}(\tau) = \mathbf{E}[X(t)Y(t+\tau)] \tag{3}$$

Here, $\mathscr{F}$ and $\mathbf{E}$ denote the Fourier transform and expectation operators respectively.

Coherence function, which is indeed a frequency domain representation, is very special since, most of the time, processes under consideration (thought to be coupled) are triggered by oscillatory events. By definition, coherence function is effective only on wide-sense-stationary, stochastic processes. However, almost all signals encountered in nature, like physiological signals, are said to be non-stationary. Thus some further analysis methods are needed to overcome this theoretical shortcoming of the coherence function. A method is suggested to observe evolution of coherency by time, called "Time-Frequency (TF) Coherency" [7, 8, 9]. Basically, TF Coherence is a reflection of TF "auto" spectra on "cross" case:

$$|C_{XY}(\omega,t)|^2 = \frac{|S_{XY}(\omega,t)|^2}{S_X(\omega,t)S_Y(\omega,t)} \tag{4}$$

TF Coherence function is based upon the fact that second moment of the processes is dependent on time

$$R_{XY}(\tau,t) = \mathbf{E}[X(t)Y(t+\tau)] \tag{5}$$

so is the cross spectrum,

$$S_{XY}(\omega,t) = \mathscr{F}\{R_{XY}(\tau,t)\} \tag{6}$$

It is obvious that TF-Coherence is very useful examining time evolution of coherence function. However, as in the previous case, the assumption of being wide sense stationary still has to be ensured even though in a manner of changing by time. Loosely speaking, coherence function is useful to show coupling in single frequency, correlation of two processes in ω_i frequency for instance. On the other hand, TF-coherence yields information about evolution of correlation in ω_i frequency by time. The question is what if the coupling exists between two different frequencies ω_i and ω_j. Such processes exist and referred as "Harmonizable Processes" [10, 11]. In the rest of this paper, we will revisit the definition of non-stationarity and try to extend it to investigate bi-frequency spectral concept.

On the other hand, the concept of being coupled involves stationarity in some manner. To be more precise, we intend to extract "stationarity" between non-stationary processes. From this point of view, one may define coupling as being stationary of two or more processes according to each other with respect to some manner.

2. BI-FREQUENCY COHERENCE FUNCTION

2.1 Bi-frequency Spectrum

The question is if there is a way to show coupling between two different processes in different frequencies. The answer should hold the properties of coherence function besides it should be applicable to non-stationary processes. As a starting point, we must go back to the auto-correlation

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function, which can be seen as the ground of coherence function, and drop the wide sense stationarity (WSS) assumption.

In a more comprehensive way, auto-correlation function of a stochastic process $X(t)$ is given by,

$$R_X(t_1,t_2) = \mathbf{E}[X(t_1)X^*(t_2)]. \tag{7}$$

Since the auto-correlation function $R_X(t_1,t_2)$ involves two time parameters, its frequency domain representation or "power spectrum" should have two dimensions as well, and both these dimensions should refer to frequencies. The Bi-frequency or Loève Spectrum of this process is defined as 2-dimensional (2D) Fourier transform of the above auto-correlation function[10]:

$$S_X(\omega_1,\omega_2) = \mathscr{F}_{t_1}\{\mathscr{F}_{t_2}\{R_X(t_1,t_2)\}\} \tag{8}$$

without any assumptions on time parameters. Let the Fourier transform of any realization of the process $X(t)$ is defined by equation (9), for any observation $x(t)$.

$$X(\omega) = \mathscr{F}\{x(t)\} \tag{9}$$

It can easily be shown that the Bi-frequency Spectrum $S_X(\omega_1,\omega_2)$ can be calculated as the expected value of the outer product of Fourier transform $X(\omega)$ with its complex conjugate as,

$$S_X(\omega_1,\omega_2) = \mathbf{E}[X(\omega_1)X^*(\omega_2)] \tag{10}$$

In contrast to the WSS case, $S_X(\omega_1,\omega_2)$ has imaginary components. Moreover, it is symmetric with respect to the $\omega_1 = \omega_2$ axis whereas on the $\omega_1 = \omega_2$ line it is exactly equal to its 1D counterpart "Power Spectrum". We can then say that, while $\omega_1 \neq \omega_2$, $S_X(\omega_1,\omega_2)$ reveals information about relations between different frequency components of the process.

2.2 Bi-frequency Coherence

A cross spectrum function (11) can be defined in bi-frequency plane as in auto-spectrum case (9).

$$S_{XY}(\omega_1,\omega_2) = \mathbf{E}[X(\omega_1)Y^*(\omega_2)] \tag{11}$$

Normalizing procedure can then be defined as (12), since (11) is an inner product in Hilbert Space.

$$|C_{XY}(\omega_1,\omega_2)|^2 = \frac{|S_{XY}(\omega_1,\omega_2)|^2}{S_X(\omega_1)S_Y(\omega_2)} \tag{12}$$

For the case, when $\omega_1 = \omega_2$, $C_{XY}(\omega_1,\omega_2)$ function is equal to its 1D counterpart, i.e., the coherence function given in (1). We can state that, bi-frequency coherence function is a combination of weighting coefficients which are produced by linear relations between two non-stationary processes in variant oscillations.

3. NUMERICAL CALCULATIONS

3.1 Spectral Estimation

The calculation of a spectra of a stochastic process requires estimation. Any spectral estimation technique found in the literature may be used for calculations [12]. For simplicity, we use Welch's modified periodograms [13] in our experiments.

$$\hat{S}_{XY}(\omega_1,\omega_2) = \frac{1}{N}\sum_i^N P^i_{XY}(\omega_1,\omega_2) \tag{13}$$

where

$$P^i_{XY}(\omega_1,\omega_2) = X_i(\omega_1)Y_i^*(\omega_2)$$

and

$$X_i(\omega) = \int x(\tau)h(t-i,\tau)e^{-j\omega\tau}d\tau \tag{14}$$

3.2 Trust Level

Since estimation of the spectrum relies on some approximation process (i.e. expected value of a quantity) we have to determine in what condition one can trust these results. In literature a line called confidence limits is added to the coherence graphs as a representation of the trust level and values below this level is assumed to be zero as the evidence of lack of coupling [1]. The confidence limit is a measure of the expectation variance and obtained as;

$$1-(1-\alpha)^{1/(L-1)} \tag{15}$$

Here α and L represents the interval of the confidence limits and the number of observation instances respectively. %95 confidence interval is used for this work.

4. EXPERIMENTAL RESULTS

We used both synthetic signals and real EEG signals in order to show that there may be some situations where the bi-frequency method may be more useful then the conventional coherence and the joint TF methods. Synthetic signals were generated according to two different cases each of which took under consideration different scenarios. 10-second long, 25 observations were generated as synthetic signals.

100 msec. non-overlapping Hamming windows were used for the analysis. Sampling frequency was chosen as 500 Hz. Upper confidence limit within 95% confidence interval was 0.0012 for bi-frequency-coherence and traditional coherence. Confidence limit, for TF-coherence, was 0.1178,

4.1 Synthetic Signals

Case 1: Null condition. We considered two uncorrelated processes as the null situation for the first case. We chose two Normal distributed random processes with zero mean and unit variance.

$$x_1(t) = \varphi_1(t) \quad \varphi_1 \sim \mathscr{N}(0,1)$$

and

$$y_1(t) = \varphi_2(t) \quad \varphi_2 \sim \mathscr{N}(0,1)$$

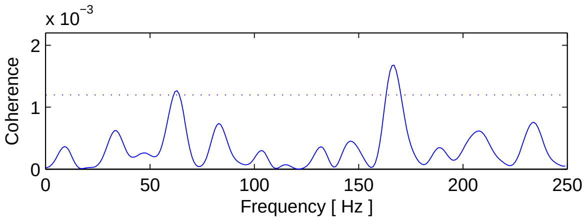

Figure 1: Traditional coherence function for Case 1. Dotted straight line shows upper confidence limit within 95% confidence interval.

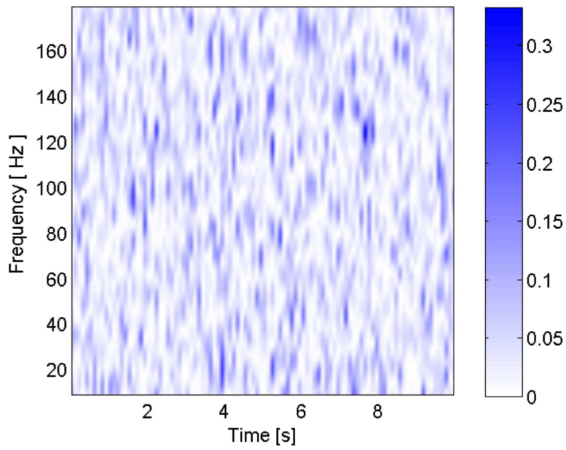

Figure 2: TF-coherence function for Case 1. Confidence limit is calculated as 0.1178. No significant pattern is observed in the absence of coupling, as expected

Case 2: Coupling in different frequencies. In this case, we have two signals correlated in different frequencies, with additive noise, i.e.,

$$x_2(t) = a sin(\theta_1(t)) + \varphi_1(t), \quad \varphi_1 \sim \mathcal{N}(0,1)$$

and

$$y_2(t) = b sin(\theta_2(t)) + \varphi_2(t), \quad \varphi_2 \sim \mathcal{N}(0,1)$$

where the frequencies are chosen as,

$$\frac{1}{2\pi}\frac{d\theta_1(t)}{dt} = \begin{cases} 25Hz, & 3.2 \text{ sec} \leq \text{t} < 4 \text{ sec}; \\ 72Hz, & 6.3 \text{ sec} \leq \text{t} < 7.1 \text{ sec}; \\ 0, & \text{otherwise.} \end{cases}$$

and

$$f_2(t) = \begin{cases} 57Hz, & 3.2 \text{ sec} \leq \text{t} < 4 \text{ sec}; \\ 37Hz, & 6.3 \text{ sec} \leq \text{t} < 7.1 \text{ sec}; \\ 0, & \text{otherwise.} \end{cases}$$

where,

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}, \quad i = 1,2$$

The bi-frequency coherence function is the only approach that extracts the underlying information in this case. For the analysis of processes having coupling in different frequencies as in this example, the traditional coherence and the TF-coherence functions give the same results as for the uncorrelated processes as shown in figures.

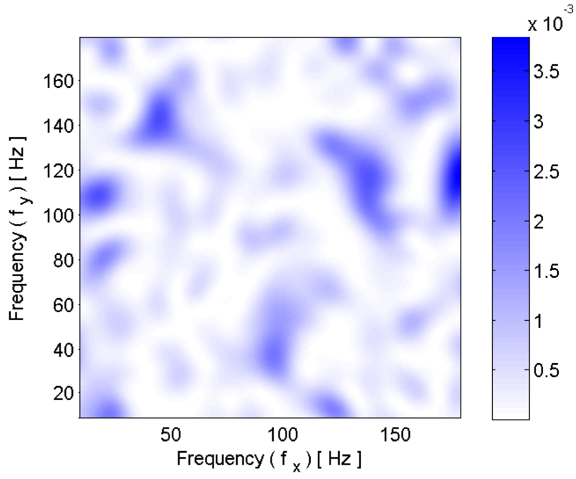

Figure 3: Bi-frequency coherence function for Case 1. No significant pattern is observed, as expected

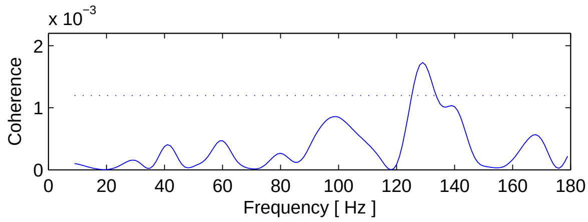

Figure 4: Coherence for Case 2.

4.2 Bi-frequency Coherence Analysis of EEG Signals

Conventionally, coherence function (1) has been widely used to determine coupling between two processes. As it is shown in the previous section, the shortcomings of the traditional coherence may be overcome using the proposed bi-frequency coherence. Hence the coupling between two EEG channels was investigated in this study. Recordings were taken from BCI III competition [14]. $4-9$ measurements were taken from a subject in three sessions. With visual feedback on a screen, left or right imaginary movement task is performed by the subject in each recording [15].

Off-diagonals indicates some patterns both in Fig.7 and Fig.8. Hence, one can deduce that linear relations exist among different frequencies. Also, these patterns may be used to distinguish different tasks from each other.

5. CONCLUSIONS

In this study, we investigate the use of Bi-frequency Coherence function for non-stationary processes, particularly for EEG signals.

We discuss some conventional methods used for analyzing cross relations of EEG signals and conceptional deficiencies of these methods. We present the Bi-frequency Coher-

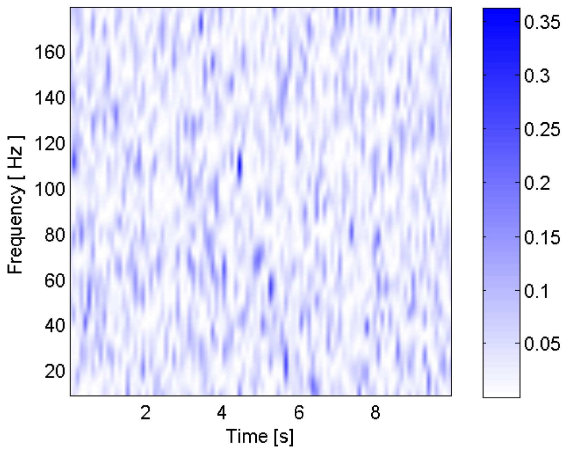

Figure 5: TF-coherence for Case 2.

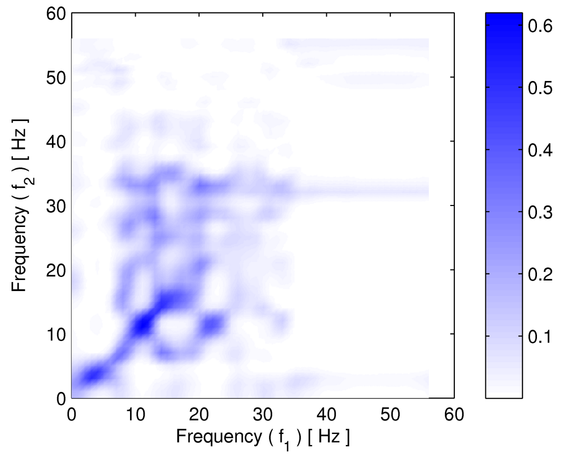

Figure 7: Imaginary right movement

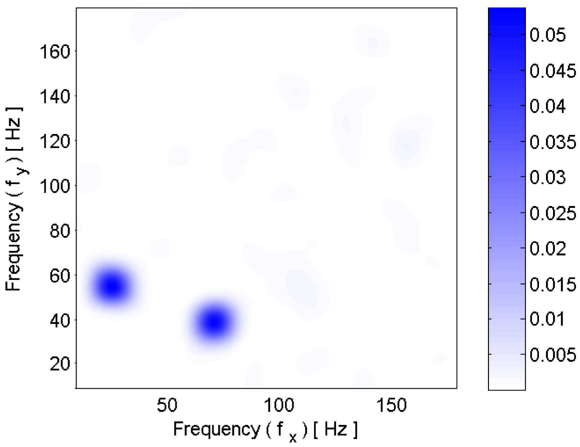

Figure 6: Bi-frequency coherence for Case 2.

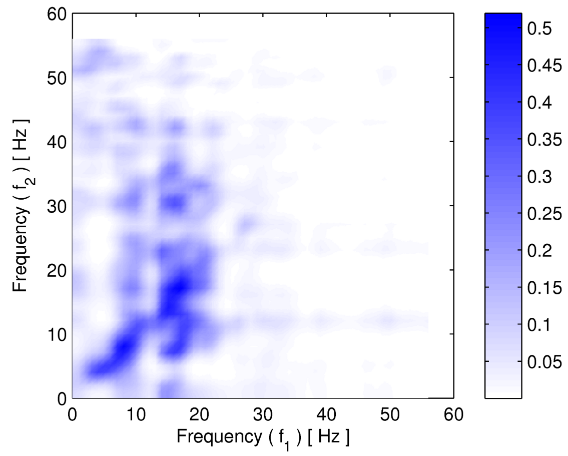

Figure 8: Imaginary left movement

ence function by means of the Loève Spectrum. It is shown that Bi-frequency Coherence can be used as an alternative way of dealing those shortcomings. Consequently, in some situations, it is shown that TF Coherence may have some problems, and Bi-frequency Coherence is more convenient in extracting the information. Furthermore a sample EEG signal couple were analyzed with very encouraging results. It is shown that EEG couples have some components that could not be determined by TF methods but extracted using Bi-frequency Coherence.

Joint TF spectral approaches assume that processes under investigation are stationary around a definite time duration, even though in a manner of evolution. From this point of view, one can say that joint TF spectral estimation methods investigate non-stationarity by tracing information about stationarity (like in conventional frequency methods) changing over time. In this work, it is shown that being non-stationary may exist on the far side of being "stationary changing by time" generally for any non-stationary process by using synthetic signals. Though bi-frequency plane is not commonly used in electro-physiological signal analysis, the need may arise similar to the EEG signal example shown here.

Fourier based methods are considered while calculating Bi-frequency Coherence in the present study. Other approaches such as wavelet, AR models etc. may be employed for calculating Bi-frequency Coherence as well. Moreover, investigating time evolution of Bi-frequency Coherence function may yield some extra information. Another interesting information is the phase that Bi-frequency function carries. This may be useful for the investigation about which process is leading (triggering) the other [1], which is frequently searched in most conventional coherence studies.

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