IMPLEMENTATION OF A WIDEBAND BEAMFORMER BASED ON A RECTANGULAR ARRAY WITH SPATIAL-ONLY INFORMATION

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Abstract. The response of a wideband beamformer based on a rectangular array with spatial-only information is studied. In this beamformer, each of the received array signals is only processed by one single coefficient and its output is an instantaneous weighted combination of the received signals. One interesting property of it is that it has a totally different beam response depending on whether we are considering the negative frequency component or the positive frequency component of the impinging signals. As a result, the implementation of this beamformer is not as straightforward as the traditional ones and a special arrangement is required for its effective operation. Two implementation schemes are proposed for complex-valued impinging signals and real-valued impinging signals, respectively, supported by design and simulation results.

Keywords. Wideband beamforming, rectangular arrays, sensor delay-lines, implementation.

1. INTRODUCTION

Wideband beamforming has found many applications in various areas ranging from sonar and radar to wireless communications [1, 2, 3, 4, 5], and it is usually achieved by the use of tapped delay-lines (TDLs) or FIR/IIR filters in its discrete form, which can form a frequency dependent response for each of the received wideband sensor signals to compensate the phase difference for different frequency components.

Recently, a class of wideband beamformers employing the so-called sensor delay-lines (SDLs) has been proposed, where there is not any form of temporal processing involved and each of the received array signals is only processed by one single coefficient [6, 7, 8]. One of the beamformers based on such a structure is the rectangular array without TDLs [9, 10, 11, 12], which basically simulates the response of a wideband linear array with TDLs. However, one key difference is that such a structure has a beamforming capability over the full azimuth range, while the traditional linear array with TDLs can only form a beam effectively over half of the azimuth range and its response over the other half is a simple repetition of the first half.

Unlike the traditional wideband beamformer with TDLs, where its coefficients are normally real-valued, the coefficients of this rectangular array are normally complex-valued in order to form a beam over the full azimuth range [12]. An interesting property of the resultant wideband beamformer is that it has a different beam response depending on whether we are considering the negative frequency component or the positive frequency component of the impinging signals. More specifically, if the beamformer has a beam in the azimuth direction ϕ for $\omega > 0$, then another beam in the direction $\phi - \pi$ for $\omega < 0$ will be formed simultaneously. As a result, the implementation of this beamformer is not as straightforward as the traditional ones and a special arrangement is required for its effective operation.

This paper is organised as follows. We will first show this specific property of its beam response in Section 2 and then provide two implementation schemes in Section 3 for complex-valued impinging signals and real-valued impinging signals, respectively. The effectiveness of the proposed implementation will be verified by simulations in Secction 4 and conclusions drawn in Section 5.

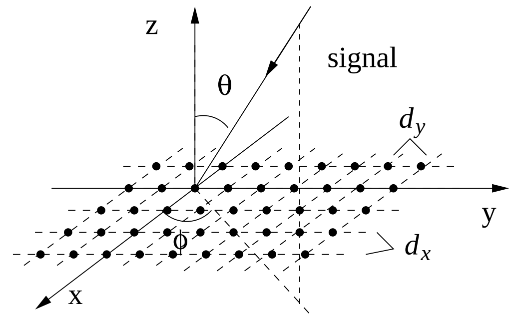

Figure 1: A uniformly spaced rectangular array, where a signal arrives from the direction (θ, ϕ).

2. BEAM RESPONSE FOR RECTANGULAR ARRAYS

Fig. 1 shows an equally spaced rectangular array with a signal arriving from the direction (θ, ϕ). The spacing of the array elements in the x and y directions is d_x and d_y, respectively.

The response of the array with respect to temporal frequency $\omega\, rad/s$ and angle of arrival (θ, ϕ) of the impinging signal is given by

$$P(\omega,\theta,\phi) = \sum_{k,l=-\infty}^{\infty} D(kd_x, ld_y) e^{-j\frac{k\omega\sin\theta\cos\phi d_x}{c}} e^{-j\frac{l\omega\sin\theta\sin\phi d_y}{c}} , \quad (1)$$

where $D(kd_x, ld_y)$ is the response of the sensor at the position (kd_x, ld_y), $k, l = \ldots, -1, 0, 1, \ldots$, and c is the wave propagation speed. Note that $D(kd_x, ld_y)$ is a constant and independent of frequency, since there are no TDLs or other frequency dependent processing for each received sensor signal.

With the following substitutions

$$\begin{aligned} \omega_1 &= \frac{\omega\sin\theta\cos\phi d_x}{c} \\ \omega_2 &= \frac{\omega\sin\theta\sin\phi d_y}{c} , \end{aligned} \quad (2)$$

we have

$$P(\omega_1,\omega_2) = \sum_{k,l=-\infty}^{\infty} D(kd_x, ld_y)\, e^{-jk\omega_1} e^{-jl\omega_2} . \quad (3)$$

We can see that the beam pattern of such a rectangular array can be obtained by first applying a 2-D (two-dimensional) Fourier transform to the array's coefficients $D(kd_x, ld_y)$ according to (3) and then using the above substitutions in (2).

From (2), we have

$$\begin{aligned} \frac{\omega_2 d_x}{\omega_1 d_y} &= \tan\phi \\ \omega &= \frac{c\omega_1}{\sin\theta\sin\phi d_y} . \end{aligned} \quad (4)$$

Thus, for a fixed θ, given any desired wideband response $P(\omega,\phi)$, we can use the relationships in (4) to express it in the form of $P(\omega_1,\omega_2)$. Then the desired coefficients $w_{m,n}$ can be obtained by applying the inverse Fourier transform to $P(\omega_1,\omega_2)$. For a desired frequency invariant response $P(\phi)$, i.e. a response independent of frequency, it can be considered as a special case and the corresponding coefficients $w_{m,n}$ can be obtained in the same way. In general $P(\omega_1,\omega_2)$ will not be symmetric and the resultant $w_{m,n}$ will be complex-valued.

An interesting property of the rectangular array response $P(\omega,\theta,\phi)$ in Equation (1) is that when $\omega = -\omega$ and $\phi = \phi - \pi$, $P(\omega,\theta,\phi)$ will have the same response, i.e.

$$\begin{aligned} & P(-\omega,\theta,\phi-\pi) \\ = & \sum_{k,l=-\infty}^{\infty} D(kd_x, ld_y) e^{-j\frac{k(-\omega)\sin\theta\cos(\phi-\pi)d_x}{c}} e^{-j\frac{l(-\omega)\sin\theta\sin(\phi-\pi)d_y}{c}} \\ = & \sum_{k,l=-\infty}^{\infty} D(kd_x, ld_y) e^{-j\frac{k\omega\sin\theta\cos\phi d_x}{c}} e^{-j\frac{l\omega\sin\theta\sin\phi d_y}{c}} \\ = & P(\omega,\theta,\phi) . \end{aligned} \quad (5)$$

3. IMPLEMENTATION

As result of the property in Equation (5), if the beamformer has a beam in the direction ϕ for $\omega > 0$, then another beam in the direction $\phi - \pi$ for $\omega < 0$ will be formed simultaneously and it will not be able to suppress the negative frequency components of the interfering signals coming from the direction $\phi - \pi$.

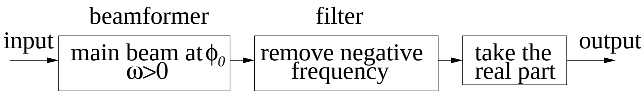

Figure 2: A block diagram for implementing the complex-valued wideband beamformer to receive a real-valued desired signal.

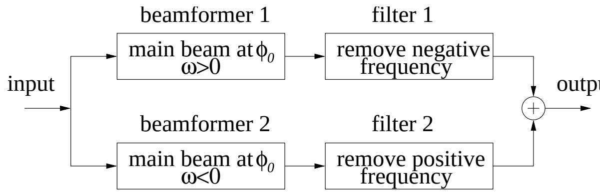

Figure 3: A block diagram for implementing the complex-valued wideband beamformer in a general case to receive a complex-valued desired signal.

For such a beamformer to work properly in a real-valued signal environment, we need to apply the complex-valued beamformer to the received array signals first, then use a complex-valued filter to remove the negative frequency component in the beamformer output, and taking the real part of this complex-valued output will give the final desired real-valued output. This process is shown in the block diagram in Fig. 2.

For the general case with complex-valued signals, we need to make sure that the full (negative and positive) spectrum of the desired signal is preserved at the beamformer output. To achieve this, we need to design two wideband beamformers. Suppose the desired direction is ϕ_0. Then one of them is designed to receive the positive part of the full spectrum ($\omega > 0$) of the desired signal from the direction ϕ_0 and its output will be processed by a filter to remove the negative part of the output spectrum; the other one to receive the negative part of the full spectrum ($\omega < 0$) of the desired signal and its output will be processed by a filter to remove the positive part of the output spectrum. The processed outputs of the two beamformers will be added together to form the final output. A block diagram for the whole process is shown in Fig. 3

4. SIMULATION RESULTS

In this section we will first give a design example to show the interesting property of this beamformer and then provide some beamspace adaptive implementation results.

4.1. Beam response example

We have designed a frequency invariant beamformer based on a 19×19 uniformly spaced rectangular array using the approach proposed in [12]. The frequency range of interest is between 400 Hz and 1600 Hz with a signal propagation speed $c = 340m/s$ and an array spacing $d_x = d_y = \lambda_{min}/2 = 34000/(2 \times 1600)cm \approx$

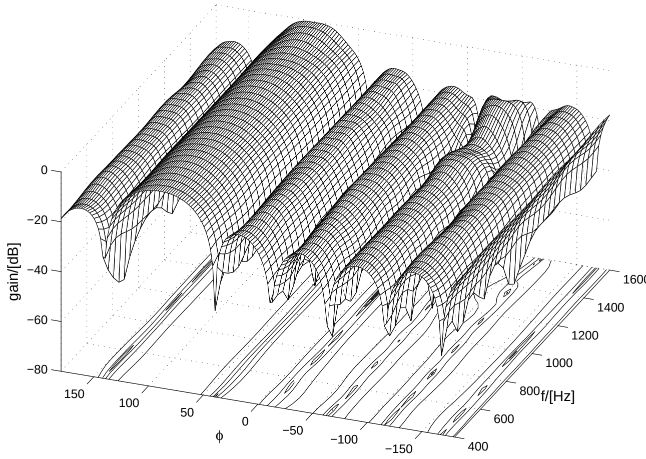

Figure 4: A design example with an off-broadside main beam ($\theta = 90^\circ$, $\phi_0 = 90^\circ$).

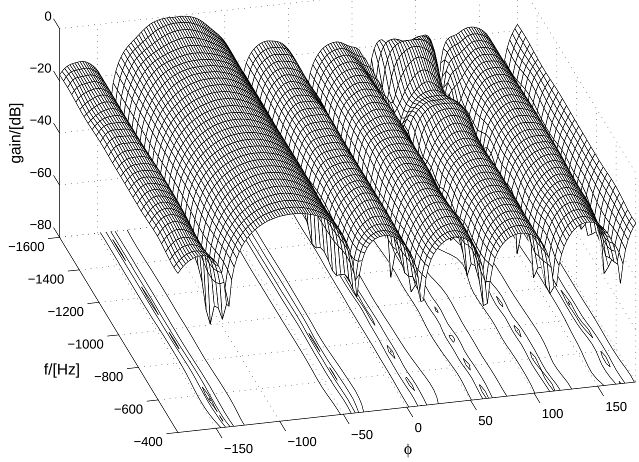

Figure 5: The response of the design in Fig. 4 over the frequency range [-1600 Hz -400 Hz] ($\theta = 90^\circ$, $\phi_0 = 90^\circ$).

$10cm$. The resultant beam pattern is shown in Fig. 4 with a main beam direction $\phi_0 = \frac{\pi}{2}$ (90°).

According to the discussion in Section 3, the design in Fig. 4 with a main beam in the direction $\phi = 90^\circ$ over the frequency range [400Hz 1600Hz] will also have a main beam in the direction $\phi = 90^\circ - 180^\circ = -90^\circ$ over the frequency range [-1600Hz $-$ 400Hz], which can be verified by checking Fig. 5. Then if we use the beamformer in Fig. 4 to receive a signal arriving from the direction 90° with a frequency range [400Hz 1600Hz], the output will also include the contribution from the signal arriving from the direction -90° with a frequency range [-1600Hz $-$ 400Hz].

4.2. Beamspace adaptive implementation results

In this part, we consider the adaptive beamforming case and use the beamspace adaptive beamformer as an example[13, 14, 15, 16], as shown in Fig. 6, where L frequency invariant beams (FIBs) are designed pointing to the directions ϕ_l, $l = 0, 1, \ldots, L-1$. One of the beams is the main beam pointing to the direction of the signal of interest and the remaining $L-1$ beams are auxiliary beams pointing to the other directions which cover all of the possible directions of the interfering signals. $\mathbf{x}[n]$ is the received array signal vector at time n, FIB_0 is the main beam with an output $d[n]$, and $y[n]$ is the final array output by combining the FIB outputs with one adaptive coefficient w_l, $l = 1, \ldots, L-1$ for each of the them. If we assume the signal of interest comes from the broadside, then we have $\phi_0 = 0$. For the auxiliary beams, they have a zero response to the signal of interest and their outputs only contain noise and interfering signals. The coefficients $\mathbf{w}$ can be adjusted by minimizing the mean square error $E\{|y[n]|^2\}$, realised by all kinds of adaptive algorithms [17]

Our simulation is based on a 19×19 uniformly spaced rectangular array with an inter-element spacing of $d_x = d_y = 10$cm. 5 FIBs are employed, which are derived from the corresponding desired responses given in Fig. 7. The signal of interest comes from the direction ($\theta = 90^\circ, \phi = 0^\circ$) and four interfering signals from the directions ($\theta = 90^\circ, \phi = 90^\circ$), ($\theta = 90^\circ, \phi = 180^\circ$),

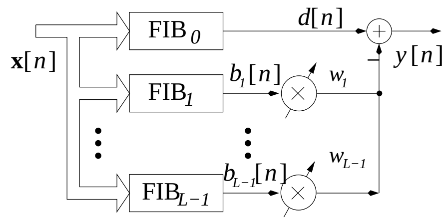

Figure 6: A general structure for beamspace adaptive beamforming.

($\theta = 90^\circ, \phi = -60^\circ$) and ($\theta = 90^\circ, \phi = -120^\circ$), respectively. All of the signals are real-valued and have a bandwidth between 560 Hz and 1440 kHz. The signal to interference ratio (SIR) is about -20 dB and the signal to noise ratio (SNR) is about 20 dB. We use a normalised LMS (least mean square) algorithm for adaptation with a stepsize of 0.05. Two cases are considered: one is to implement the FIBs as suggested in Fig. 2, and the other one is to implement it directly without considering their effects on negative frequencies.

The resultant two learning curves are shown in Fig. 8, where the dashed line is for the first case and the dotted line for the second case. The ensemble mean square residue error for the first case has reached -5 dB, whereas it stays at about 21 dB for the second case. Obviously ignoring their response to the negative frequencies has failed to separate the desired signal from its interferences.

5. CONCLUSIONS

The beam response of a wideband beamformer based on a rectangular array with spatial-only information has been studied. One interesting property of it is that it has a different response depending on whether we are considering the negative frequency component or the positive frequency component of the impinging signals.

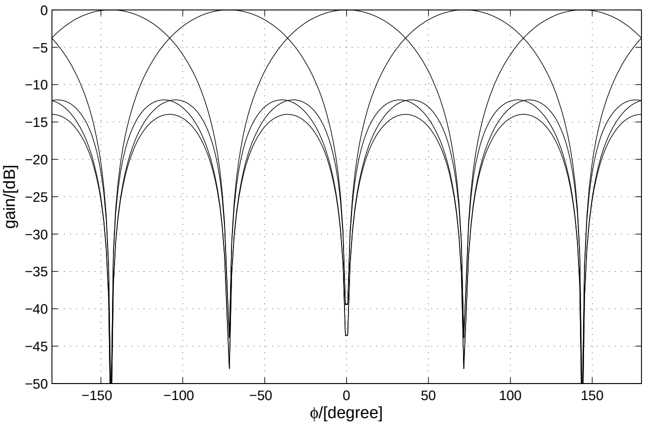

Figure 7: The desired responses for designing the five frequency invariant beams.

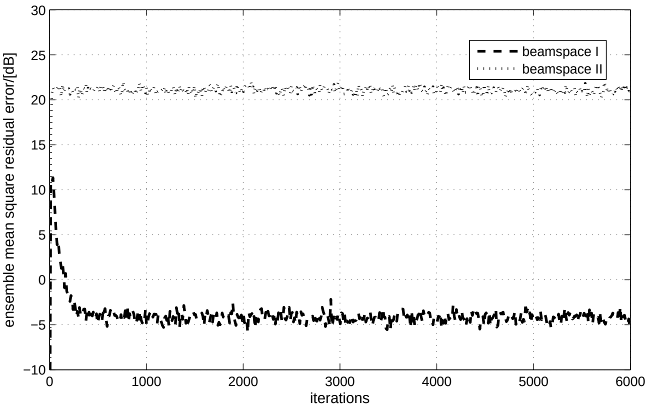

Figure 8: The two learning curves for the beamspace adaptive beamformer.

More specifically, if the beamformer has a beam in the azimuth direction ϕ for $\omega > 0$, then another beam in the direction $\phi - \pi$ for $\omega < 0$ will be formed simultaneously. As a result, the implementation of this beamformer is not as straightforward as the traditional ones and two implementation schemes have been proposed for complex-valued impinging signals and real-valued impinging signals, respectively. Beamformer design examples have shown this property clearly and the effectiveness of the proposed implementation has been verified by simulations based on the beamspace adaptive wideband beamforming structure.

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