DATA-AIDED DOA ESTIMATION OF SINGLE SOURCE WITH TIME-VARIANT RAYLEIGH AMPLITUDES

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ABSTRACT

This paper focuses on the data-aided (DA) direction of arrival (DOA) estimation of a single narrow-band source in time-varying Rayleigh fading amplitude. The time-variant fading amplitude is modeled by considering the Jakes' and the first order autoregressive (AR1) correlation models. Closed-form expressions of the CRB for DOA alone are derived for fast and slow Rayleigh fading amplitude. As a special case, the CRB under uncorrelated fading Rayleigh channel is derived. A analytical approximate expressions of the CRB are derived for low and high SNR that enable the derivation of a number of properties that describe the bound's dependence on key parameters such as SNR, channel correlation. A high signal-to-noise-ratio maximum likelihood (ML) estimator based on the AR1 correlation model is derived. The main objective is to reduce algorithm complexity to a single-dimensional search on the DOA parameter alone as in the static-channel DOA estimator. Finally, simulation results illustrate the performance of the estimator and confirm the validity of the theoretical analysis.

Index Terms−DOA estimation, ML estimator, Cramér Rao bound, Time-varying fading channel, Jakes' channel model, AR1 channel model.

1. INTRODUCTION

Estimating the direction of arrival (DOA) of propagating plane waves incident on an array of sensors is an important problem in array signal processing due to its applications in radar, sonar, mobile communications, and so on (e.g., [1, 2, 3, 4]). Stochastic and deterministic CRBs derivation for the DOA parameter alone has been an intensive research field because the performances of several high-resolution DOA estimation methods are known to be comparable to these bounds under certain mild conditions. These bounds have been derived for circular and non-circular complex Gaussian sources under uniform white noise field in [5] and [8] respectively. In particular, the DOA estimation problem of a single source has been extensively studied for a static channel (e.g., [6, 7]). A fast and explicit approximate ML algorithm with lower computational complexity has been developed in [6]. The ML DOA estimation for a constant-modulus signal is addressed in [7] which utilizes the available knowledge of the signal waveform. In recent years, DOA estimation for non-circular complex signal with discrete distributions (e.g., binary phase shift keying (BPSK) and offset quadrature phase shift keying (OQPSK) modulated signals) which are widely used in communication systems, has attracted more attention due to the performance gain from the non-circular properties (e.g., [9, 12, 11]). In [10] a closed-form expressions of the DA CRB and stochastic CRB for DOA alone has been derived for BPSK and QPSK modulated signals in the case of one narrowband source corrupted by additive white Gaussian noise (AWGN) channel. We note that in [10], the channel amplitude is assumed constant over the observation interval. In radar applications, H. Gu [4] developed a radar tracking algorithm for multiple moving targets where the targets amplitudes are assumed deterministic and time-variant. However, in many applications requiring DOA estimation (e.g., mobile communication, radar), the assumption that the channel amplitude is constant throughout the observation period is not valid.

In this paper, basing on the formulation in [10], we consider the problem of estimating the DOA of one source by assuming that the Rayleigh fading amplitude of the associated target vary in time according to Jakes' or first order autoregressive (AR1) correlation models. We derive closed-form expressions for the DA CRB for the DOA parameter alone with correlated and uncorrelated time-varying Rayleigh fading amplitude. This bound enables to evaluate the effect of the amplitude's time variation on DOA estimation. We present a simple estimation procedure derived through an approximate, high-SNR maximum-likelihood (ML) approach based on a simplified model for the amplitude fading process. The estimation procedure requires only a single-dimensional parameter search.

The paper is organized as follows. Section 2 describes the signal model, the Jakes' and AR1 correlation models and pose the estimation problem. In Section 3, exact and approximate closed-form expressions for the CRB of the DOA parameter alone are derived for fast amplitude fading, slow amplitude fading and uncorrelated amplitude fading models. In this section, we also prove different properties of the derived bound. In Section 4, the ML estimator is derived for a high SNR approximation. Finally, simulation results are presented in Section 5.

2. SIGNAL MODEL AND PROBLEM FORMULATION

Let an arbitrary array of M sensors receive a single target with unknown DOA. Over the observation interval, the Rayleigh fading amplitude of the target is assumed to vary in time according to Jakes' or first order autoregressive (AR1) correlation models. Assuming a receiver with ideal sample timing and perfect synchronization, the $M\times 1$ array snapshot complex vectors at the output of the matched filter can be modeled as

$$\mathbf{y}_n = s_n h_n \mathbf{a}(\theta) + \mathbf{n}_n,\; n = 0,\ldots,N-1 \qquad (2.1)$$

where $\mathbf{a}(\theta)$ is the steering vector parametrized by the scalar unknown DOA parameter θ. We suppose $\|\mathbf{a}(\theta)\|^2 = M$. The transmitted signal s_n is assumed known with $|s_n|^2 = 1$. The M-variate additive noise vectors $(\mathbf{n}_k)_{k=0,\ldots,N-1}$ are assumed

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to be i.i.d. zero-mean complex circular Gaussian with covariance matrix $\mathrm{E}(\mathbf{n}_k\mathbf{n}_k^H) = \sigma_n^2\mathbf{I}$. The process h_k is the sample of the fading amplitude of the target assumed to be zero-mean circular complex Gaussian with unknown variance σ_h^2 and correlation function given by:

$$R_h^J(m) \stackrel{\text{def}}{=} \sigma_h^2 \mathrm{E}(h_n h_{n-m}^*) = \sigma_h^2 J_0(2\pi f_d T m),$$

where $J_0(.)$ is the first kind 0th-order Bessel function, T is the symbol period and f_d denotes the maximum Doppler shift. This is frequently referred to as the Jakes' model [14]. The signal-to-noise ratio (SNR) is defined as $\rho \stackrel{\text{def}}{=} \frac{\sigma_h^2}{\sigma_n^2}$.

Collecting the samples of the received signal to form a vector $\mathbf{y} \stackrel{\text{def}}{=} (\mathbf{y}_0^T, \ldots, \mathbf{y}_{N-1}^T)^T$ yields the following model

$$\mathbf{y} = \mathbf{SAh} + \mathbf{n}, \tag{2.2}$$

where $\mathbf{A} \stackrel{\text{def}}{=} \mathbf{I} \otimes \mathbf{a}(\theta)$, $\mathbf{S} \stackrel{\text{def}}{=} \mathrm{Diag}(s_0, \ldots, s_{N-1}) \otimes \mathbf{I}$, $\mathbf{h} \stackrel{\text{def}}{=} (h_0, \ldots, h_{N-1})^T$ and $\mathbf{n} \stackrel{\text{def}}{=} \left(\mathbf{n}_0^T, \ldots, \mathbf{n}_{N-1}^T\right)^T$ is a $NM \times 1$ noise vector with covariance matrix $\sigma_n^2\mathbf{I}$. Since the transmitted symbols s_n are known, $\mathbf{y}$ is a zero-mean complex Gaussian random vector, with correlation matrix given by

$$\mathbf{R}_y \stackrel{\text{def}}{=} \mathrm{E}(\mathbf{y}\mathbf{y}^H) = \mathbf{S}\mathbf{A}\mathbf{R}_h\mathbf{A}^H\mathbf{S}^H + \sigma_n^2\mathbf{I}, \tag{2.3}$$

where $\mathbf{R}_h \stackrel{\text{def}}{=} \mathrm{E}(\mathbf{h}\mathbf{h}^H)$ is the fading amplitude correlation matrix.
Since $|s_n|^2 = 1$ for all n, the matrix $\mathbf{S}$ is unitary (i.e., $\mathbf{S}\mathbf{S}^H = \mathbf{S}^H\mathbf{S} = \mathbf{I}$). Subsequently, the probability density function (PDF) of $\mathbf{y}$ is the same as the PDF of $\mathbf{z} \stackrel{\text{def}}{=} \mathbf{S}^H\mathbf{y}$, and which is given by:

$$p(\mathbf{y};\alpha) = p(\mathbf{z};\alpha) = \frac{1}{\pi^{NM}\det(\mathbf{R}_z)} e^{-\mathbf{z}^H\mathbf{R}_z^{-1}\mathbf{z}}, \tag{2.4}$$

where $\mathbf{R}_z \stackrel{\text{def}}{=} \mathbf{S}^H\mathbf{R}_y\mathbf{S} = \mathbf{A}\mathbf{R}_h\mathbf{A}^H + \sigma_n^2\mathbf{I}$ is the covariance matrix of the vector $\mathbf{z}$.

AR1 model of fading Among various channel models, the information theoretic results in [15] show that the first-order AR model provides a sufficiently accurate model for time fading channels $h_k = \gamma h_{k-1} + e_k$ where $e_k \sim \mathcal{N}(0, \sigma_h^2(1-\gamma^2))$ is the additive driving noise and where $\gamma \stackrel{\text{def}}{=} J_0(2\pi f_d T)$ is assumed to be unknown. The fading amplitude at time n is constrained to follow a sequence from a known initial state, say h_0:

$$h_n = \gamma^n h_0 + \sum_{k=0}^{n-1} \gamma^k e_{n-k}. \tag{2.5}$$

The correlation over m signalling intervals is given by

$$R_h^{\mathrm{AR}}(m) = \mathrm{E}(h_n h_{n+m}^*) = \sigma_h^2 \gamma^{|m|},$$

and it depends on the mobility environment (and on the symbol time T) at hand. Consequently, the covariance matrix for the AR1 channel model depends on the unknown parameter γ can be written as

$$\mathbf{R}_h^{\mathrm{AR}} = \sigma_h^2 \begin{pmatrix} 1 & \gamma & \gamma^2 & \ldots & \gamma^{N-1} \\ \gamma & 1 & \gamma & \ldots & \gamma^{N-2} \\ \gamma^2 & \gamma & 1 & \ldots & \gamma^{N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \gamma^{N-1} & \gamma^{N-2} & \gamma^{N-3} & \ddots & 1 \end{pmatrix} \tag{2.6}$$

It is clear that for $\gamma = 0$ the channel becomes an uncorrelated fading process, and for $\gamma = 1$, the channel is simply a realization of a single random variable (slow fading).

We note that the derivation of analytical expression of the CRB and the ML DOA estimator are difficult tasks under Jakes' fading amplitude. Thus, for the sake of analytical tractability, we choose to model the fading amplitude as an AR1 process.

The estimation problem can now be formulated as follows: Given the received signal $\mathbf{y}$ whose PDF is given by (2.4) and an unknown parameter vector $\alpha \stackrel{\text{def}}{=} (\theta, \sigma_n^2, \sigma_h^2, \gamma)^T$, estimate θ. In this problem, θ is the parameter of interest and the other parameters are nuisance parameters.

3. CRB EVALUATION

The CRB for zero-mean, circular complex, Gaussian measurements vector depend on unknown vector parameter α is given by the circular complex Gaussian Slepian-Bangs formula [16, rel. B.3.25].

$$\begin{aligned} \mathrm{CRB}(\alpha) &= (\mathbf{I}_\alpha)^{-1} \\ (\mathbf{I}_\alpha)_{k,l} &\stackrel{\text{def}}{=} \mathrm{Tr}\left(\mathbf{R}_z^{-1}\frac{\partial \mathbf{R}_z}{\partial \alpha_k}\mathbf{R}_z^{-1}\frac{\partial \mathbf{R}_z}{\partial \alpha_l}\right),\ k,l = 1,\ldots,4. \end{aligned}$$

The expression of the CRB for DOA alone proved in [13], is summarized by the following result.

Result 1 *The DA CRB of the parameter θ alone is decoupled from that of the other parameters under AR1 fading amplitudes, and is given by:*

$$\mathrm{CRB}(\theta) = \mathrm{CRB}_0^{\mathrm{DA}}(\theta) \frac{N\sigma_h^2}{\mathrm{Tr}\left(\mathbf{R}_h^2\left(\mathbf{R}_h + \frac{\sigma_n^2}{M}\mathbf{I}\right)^{-1}\right)} \tag{3.7}$$

where $\mathrm{CRB}_0^{\mathrm{DA}}(\theta) = \frac{1}{N\rho}\frac{1}{\alpha}$ and where α is the purely geometrical factor[1] $2\mathbf{a}'^H(\theta)\Pi_{\mathbf{a}(\theta)}^{\perp}\mathbf{a}'(\theta)$ with $\Pi_{\mathbf{a}(\theta)}^{\perp} \stackrel{\text{def}}{=} \mathbf{I} - \mathbf{a}(\theta)\mathbf{a}^H(\theta)/M$ and $\mathbf{a}'(\theta) \stackrel{\text{def}}{=} \frac{\partial \mathbf{a}(\theta)}{\partial \theta}$.

We note that the $\mathrm{CRB}_0^{\mathrm{DA}}(\theta)$ is the DA CRB derived in [10] when the amplitudes fading is assumed constant within the observation period.

Remark 1 *We note that the results obtained with the simplified AR1 correlation model are not numerically identical to the results obtained with the Jakes' model except for high SNR, the analytical insight obtained under the AR1 correlation model also applies to the Jakes' model (see Section 5).*

In the special cases of slow fading amplitude ($i.e., \gamma = 1 \,\text{and}\, \mathbf{R}_h = \sigma_h^2\mathbf{1}\mathbf{1}^T$) and uncorrelated fading amplitude ($i.e., \gamma = 0, \text{and}\, \mathbf{R}_h = \sigma_h^2\mathbf{I}$), the result 1 can be extended to the following result.

Result 2 *The CRB for DOA alone over slow and uncorrelated fading amplitudes are given by* [2]

[1]The parameter α is equal the following values $\alpha_{\mathrm{ULA}} = \pi^2\frac{M(M-1)}{6}\cos^2(\theta)$ [resp. $\alpha_{\mathrm{UCA}} = \frac{M\pi^2}{4\sin^2\pi/M}$] for uniform linear [resp. uniform circular] array.

[2]Where the superscripts Slow and Uncor of $\mathrm{CRB}^{\mathrm{Slow}}(\rho)$ and $\mathrm{CRB}^{\mathrm{Uncor}}(\rho)$ refer slow and uncorrelated channel fading respectively.

$$\mathrm{CRB}^{\mathrm{Slow}}(\theta) = \frac{1}{N}\left(\frac{1}{\alpha}\left[\frac{1}{\rho}+\frac{1}{MN\rho^2}\right]\right) \quad (3.8)$$

$$\mathrm{CRB}^{\mathrm{Uncor}}(\theta) = \frac{1}{N}\left(\frac{1}{\alpha}\left[\frac{1}{\rho}+\frac{1}{M\rho^2}\right]\right), \quad (3.9)$$

We remak that the bound (3.9) is the conventional stochastic CRB for DOA alone of one source derived under the circular complex Gaussian distribution [8]. From (3.8) and (3.9), we have

$$\begin{aligned}\mathrm{CRB}^{\mathrm{Uncor}}(\theta) &\geq \mathrm{CRB}^{\mathrm{Slow}}(\theta) \text{ for all SNR}\\ \mathrm{CRB}^{\mathrm{Uncor}}(\theta) &\approx \mathrm{CRB}^{\mathrm{Slow}}(\theta)\approx\mathrm{CRB}_0^{\mathrm{DA}}(\theta) \text{ for high SNR}\\ \mathrm{CRB}^{\mathrm{Uncor}}(\theta) &\approx N\,\mathrm{CRB}^{\mathrm{Slow}}(\theta)\approx\frac{1}{N\rho^2}\frac{1}{M\alpha} \text{ for low SNR}\end{aligned}$$

3.1 Approximate expressions for CRB

To get more insights on the CRB, we obtain in the following approximate expressions for the CRB given by (3.7) in the high and low SNR regimes that enable the derivation of the properties below.

3.1.1 High and low SNR expressions

For high and low SNR cases, we have

$$\begin{aligned}\left(\mathbf{R}_h+\frac{\sigma_n^2}{M}\mathbf{I}\right)^{-1} &\approx \mathbf{R}_h^{-1} \text{ for high SNR}\\ \left(\mathbf{R}_h+\frac{\sigma_n^2}{M}\mathbf{I}\right)^{-1} &\approx \frac{\sigma_n^2}{M}\mathbf{I} \text{ for low SNR,}\end{aligned}$$

hence, the channel-dependent term of the denominator of Eq. (3.7) can be approximated as:

$$\begin{aligned}\mathrm{Tr}\left(\mathbf{R}_h^2\left(\mathbf{R}_h+\frac{\sigma_n^2}{M}\mathbf{I}\right)^{-1}\right) &\approx N\sigma_h^2 \text{ for high SNR}\\ \mathrm{Tr}\left(\mathbf{R}_h^2\left(\mathbf{R}_h+\frac{\sigma_n^2}{M}\mathbf{I}\right)^{-1}\right) &\approx \frac{\sigma_n^2}{M}\mathrm{Tr}(\mathbf{R}_h^2) \text{ for low SNR}\end{aligned}$$

Consequently, the expressions of the CRB for DOA alone for high and low SNR cases are given by:

$$\mathrm{CRB}^{\mathrm{high}}(\theta) = \mathrm{CRB}_0^{\mathrm{DA}}(\theta) \text{ for high SNR} \quad (3.10)$$

$$\mathrm{CRB}^{\mathrm{low}}(\theta) = \frac{1}{\rho^2\beta}\frac{1}{M\alpha} \text{ for low SNR} \quad (3.11)$$

where the channel-dependent parameter β is given by $\frac{1}{\sigma_h^4}\mathrm{Tr}(\mathbf{R}_h^2)$. We remark that the CRB given by (3.10) is identical to the DA CRB derived in [10] when the amplitudes fading is assumed constant within the observation period.

3.1.2 CRB properties

The following properties follow immediately from (3.10) and (3.11).

Property 1 *For high SNR, the CRB for DOA alone is approximately inversely proportional to SNR and does not depend on the parameter of the channel.*

This property implies that the CRBs for DOA alone associated to Jakes' and AR1 correlation models are identical for high SNR. We also note that for a correlation model, the CRBs for DOA alone associated to slow, fast and uncorrelated fading channel are identical for high SNR.

Property 2 *For low SNR, the CRB for DOA alone is approximately inversely proportional to ρ^2 (decreasing rapidly with SNR).*

Note that the parameter β is a monotone decreasing function of f_dT for Jakes' and AR1 correlation models as illustrated in Fig. 1.

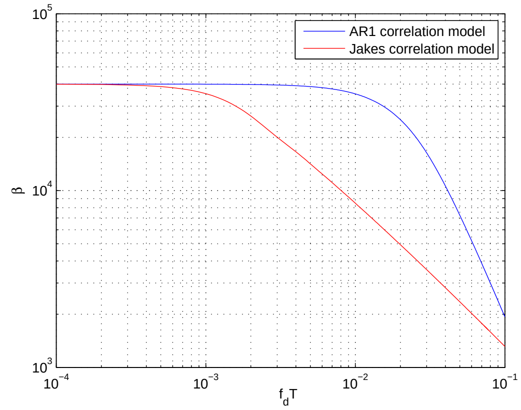

Fig.1 The channel-dependent parameter β for the Jakes and AR1 correlation model versus f_dT with $N=200$.

From this figure, we see that β decreases rapidly for Jakes' correlation model contrary to AR1 correlation model for which β remains quite constant up to $f_dT = 0.0032$. As the CRB (3.11) approximately inversely proportional to β, we have the following property

Property 3 *For low SNR, the CRB for DOA alone is a monotonically decreasing function of the channel correlation parameter γ which varies from uncorrelated fading bound ($\gamma=0$) to the slow fading bound ($\gamma=1$).*

4. ML DOA ESTIMATOR

The direct maximization of the likelihood function (2.4) with respect to the unknown parameter α is a difficult task. To facility the derivation of the ML estimates of α, we choose to model the variation of the amplitude fading as AR1 process with covariance matrix is given by (2.6). Using the Markovianity property of the AR1 process the log-likelihood function proved in [13] is given by (after drooping the constant term)

$$\begin{aligned}L(\alpha) = &-\Big(\ln(\det(\mathbf{R}))-(N-1)\ln(\det(\mathbf{C}))+\mathbf{z}_0^H\mathbf{R}^{-1}\mathbf{z}_0\\ &+\sum_{n=1}^{N-1}\overline{\mathbf{z}}_n^H\mathbf{C}^{-1}\overline{\mathbf{z}}_n\Big)\end{aligned} \quad (4.12)$$

where $\overline{\mathbf{z}}_n \overset{\text{def}}{=} (\mathbf{z}_n - \frac{\gamma\sigma_h^2}{M\sigma_h^2+\sigma_n^2}\mathbf{a}(\theta)\mathbf{a}^H(\theta)\mathbf{z}_{n-1})$, $\mathbf{R} \overset{\text{def}}{=} \sigma_h^2\mathbf{a}(\theta)\mathbf{a}^H(\theta) + \sigma_n^2\mathbf{I}$ and $\mathbf{C} \overset{\text{def}}{=} \sigma_h^2\left(1-\frac{\gamma^2M\sigma_h^2}{M\sigma_h^2+\sigma_n^2}\right)\mathbf{a}(\theta)\mathbf{a}^H(\theta)+\sigma_n^2\mathbf{I}$

The following result proved in [13], shows that it is possible to reduce the optimization problem, under a high SNR approximation, to a single-parameter search with respect to the DOA parameter θ.

Result 3 *For high SNR environment, the joint ML estimates that maximize the log-likelihood function* (4.12) *are given by the following:*

$\hat{\theta}_{ML}$ *is obtained by the maximizing with respect to* θ

$$F(\theta) = -\Big(N\ln(\hat{\sigma}_{n,ML}^{2(M-1)}\hat{\sigma}_{h,ML}^{2}) + (N-1)\ln(1-\hat{\gamma}_{ML}^{2}) + \mathbf{z}_0^H\left(\frac{1}{\hat{\sigma}_{n,ML}^2}\Pi_{\mathbf{a}(\theta)}^{\perp} + \frac{1}{M^2\hat{\sigma}_{h,ML}^2}\mathbf{a}(\theta)\mathbf{a}^H(\theta)\right)\mathbf{z}_0 + \sum_{n=1}^{N-1}\tilde{\mathbf{z}}_n^H\tilde{\mathbf{C}}\tilde{\mathbf{z}}_n\Big) \quad (4.13)$$

where $\tilde{\mathbf{z}}_n \stackrel{\text{def}}{=} \mathbf{z}_n - \frac{\hat{\gamma}_{ML}}{M}\mathbf{a}(\theta)\mathbf{a}^H(\theta)\mathbf{z}_{n-1}$ *and* $\tilde{\mathbf{C}} \stackrel{\text{def}}{=} \frac{1}{\hat{\sigma}_{n,ML}^2}\Pi_{\mathbf{a}(\theta)}^{\perp} + \frac{1}{M^2\hat{\sigma}_{h,ML}^2(1-\hat{\gamma}_{ML}^2)}\mathbf{a}(\theta)\mathbf{a}^H(\theta)$ *and where* $\hat{\sigma}_{h,ML}^2$, $\hat{\sigma}_{n,ML}^2$ *and* $\hat{\gamma}_{ML}$ *are the estimates of the nuisance parameters given by*

$$\hat{\gamma}_{ML} = -\frac{k_{2,z}(\theta)}{2k_{4,z}(\theta)} \quad (4.14)$$

$$\hat{\sigma}_{h,ML}^2 = \frac{1}{N}\left(k_{3,z}(\theta) + \frac{1}{1-\hat{\gamma}_{ML}^2}(-\hat{\gamma}_{ML}k_{2,z}(\theta) + \hat{\gamma}_{ML}^2 k_{1,z}(\theta))\right) \quad (4.15)$$

$$\hat{\sigma}_{n,ML}^2 = \frac{1}{N(M-1)}\sum_{n=0}^{N-1}\mathbf{z}_n^H\Pi_{\mathbf{a}(\theta)}^{\perp}\mathbf{z}_n, \quad (4.16)$$

where the the DOA-dependent coefficients $k_{l,z}(\theta)$, $l=1,\dots,4$, *are given by*

$$k_{1,z}(\theta) \stackrel{\text{def}}{=} \frac{1}{M^2}\left(\sum_{n=1}^{N-1}(\mathbf{z}_n^H\mathbf{a}(\theta)\mathbf{a}^H(\theta)\mathbf{z}_n + \mathbf{z}_{n-1}^H\mathbf{a}(\theta)\mathbf{a}^H(\theta)\mathbf{z}_{n-1})\right)$$

$$k_{2,z}(\theta) \stackrel{\text{def}}{=} \frac{1}{M^2}\left(\sum_{n=1}^{N-1}(\mathbf{z}_n^H\mathbf{a}(\theta)\mathbf{a}^H(\theta)\mathbf{z}_{n-1} + \mathbf{z}_{n-1}^H\mathbf{a}(\theta)\mathbf{a}^H(\theta)\mathbf{z}_n)\right)$$

$$k_{3,z}(\theta) \stackrel{\text{def}}{=} \frac{1}{M^2}\sum_{n=0}^{N-1}\mathbf{z}_n^H\mathbf{a}(\theta)\mathbf{a}^H(\theta)\mathbf{z}_n$$

$$k_{4,z}(\theta) \stackrel{\text{def}}{=} k_{3,z}(\theta) - k_{1,z}(\theta)$$

From (4.14) and (4.16), using the high SNR condition, we get for the true values of θ that

$$\hat{\gamma}_{ML} = \frac{\sum_{n=1}^{N-1}(\mathbf{z}_n^H\mathbf{a}(\theta)\mathbf{a}^H(\theta)\mathbf{z}_{n-1} + \mathbf{z}_{n-1}^H\mathbf{a}(\theta)\mathbf{a}^H(\theta)\mathbf{z}_n)}{\sum_{n=2}^{N-1}\mathbf{z}_{n-1}^H\mathbf{a}(\theta)\mathbf{a}^H(\theta)\mathbf{z}_{n-1}} \stackrel{N\to\infty}{\longrightarrow} \frac{(N-1)M^2\sigma_h^2 J_0(2\pi f_d T)}{(N-2)(M^2\sigma_h^2 + M\sigma_n^2)} \approx J_0(2\pi f_d T) \quad (4.17)$$

$$\hat{\sigma}_{n,ML}^2 \stackrel{N\to\infty}{\longrightarrow} \sigma_n^2. \quad (4.18)$$

Similarly, from (4.15) using (4.17) and the high SNR condition, we get after some manipulation that

$$\hat{\sigma}_{h,ML}^2 \stackrel{N\to\infty}{\longrightarrow} \sigma_h^2.$$

Consequently, $\hat{\gamma}_{ML}$, $\hat{\sigma}_{h,ML}^2$ and $\hat{\sigma}_{n,ML}^2$ are the consistent estimators of γ, σ_h^2 and σ_n^2, respectively at high SNR.

5. SIMULATION RESULTS

The purpose of this section is to illustrate the behavior of the derived CRB for DOA alone and the performance of the derived estimator.

Assume that a single narrowband source impinges on a uniform linear array (ULA) of sensors $M=6$ separated by a half-wavelength for which $\mathbf{a} = \left(1, e^{i\theta}, \dots, e^{i(M-1)\theta}\right)$, where $\theta = \pi\sin\alpha$, with α the DOA relative to the normal of array broadside. The channel is simulated according to the Jakes and AR1 correlation model [14, 15] with doppler-time product of $f_d T$. In our simulations, each value of the MSE is obtained by averaging over 1000 independent runs. The number of sample is fixed at $N=200$.

We begin with Fig.2, which compares $\text{CRB}^{\text{Slow}}(\theta)$ (3.8), $\text{CRB}^{\text{Uncor}}(\theta)$ (3.9) and the exact CRB over Jakes' and AR1 correlation model (3.7) with two values of $f_d T$ versus SNR. From this figure, we see that all these bounds are identical for high SNR except for low SNR where the fast bound decreasing when $f_d T$ is increasing as predicted by the Properties 1 and 3. On the other hand, we observe that the CRB associated with Jakes' model remains close to the CRB associated with AR1 model for low SNR.

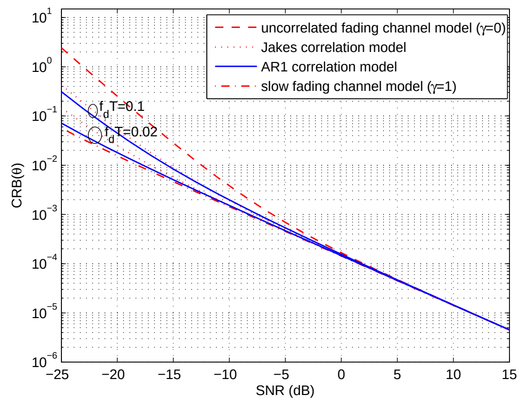

Fig.2 Exact CRB on DOA estimation with Jakes and AR1 correlation model for two values of $f_d T$, $\text{CRB}^{\text{Slow}}(\theta)$ and $\text{CRB}^{\text{Uncor}}(\theta)$ versus SNR with $N=200$.

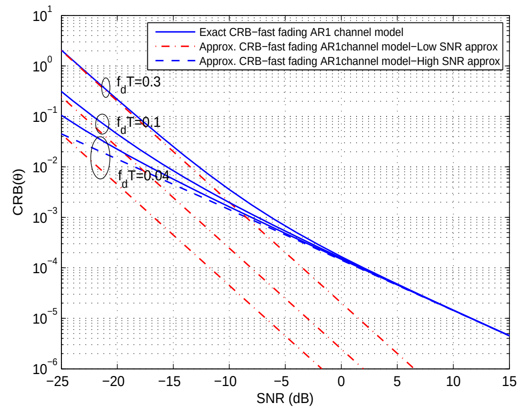

Fig.3 Exact CRB and its approximations for the fast fading AR1 correlation model for three values of $f_d T$ versus SNR with $N=200$.

Fig.3 exhibits the domain of validity of the low and high approximations of the CRB given by Eqs. (3.11) and (3.10), respectively. We can see from this figure that the domain of validity depends on the values of $f_d T$, where for low SNR the exact CRB equals to its low approximation bound for a large low SNR range except when $f_d T$ is decreasing (the amplitude fading becomes slow fading). In contrast to the low SNR case, we observe that the approximates CRB for high SNR does not depend on $f_d T$ which is identical to its exact bound for large SNR range when $f_d T$ decreasing.

Fig.4 presents the dependence of the CRB for DOA alone on the Jakes' and AR1 correlation models for low SNR throughout the Doppler-time product $f_d T$.

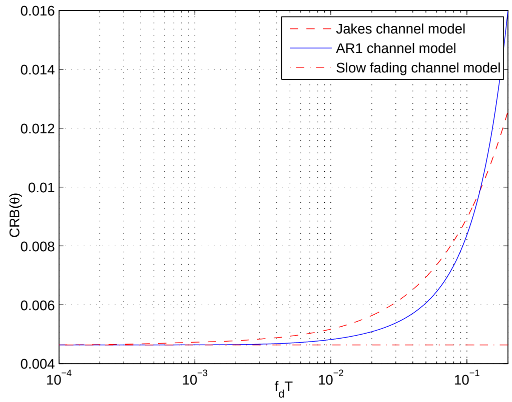

Fig.4 Exact CRB(θ) for the Jakes and AR1 correlation model, and $\mathrm{CRB}^{\mathrm{Slow}}(\theta)$ versus f_dT with SNR$=-15dB$ and $N=200$.

We observe from this figure that as the Doppler-time product f_dT increases, the CRBs associated with Jakes' [resp. AR1] correlation model remain quite constant up to Doppler-time product values of 0.0007 [resp. 0.0035], for which these bounds are identical to the CRB associated to the slow amplitude fading. We also see that the bounds increase when the time-Doppler product increases as predicted by Property 3.

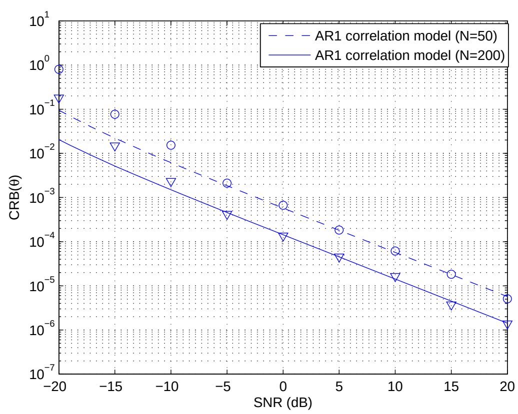

Fig.5 Exact CRB(θ) for the AR1 correlation model and estimated MSE $\mathrm{E}(\hat{\theta}_{ML}-\theta)^2$ given by the ML estimator versus SNR fir two values of N with $f_dT=0.01$.

Fig.5 illustrates the Result 3 by comparing the exact CRB (3.7) and the minimum mean square error (MSE) of DOA estimate given by the asymptotic high SNR ML estimator for the AR1 time-variant amplitude fading versus SNR. From this figure, we observe a good agreement between the derived CRB and the estimated MSE for high SNR. On the other hand, we note that the asymptotic ML estimator still gives a valid estimate of DOA parameter for small values of N and for low SNR.

6. CONCLUSION

The effect of time-variant Rayleigh amplitude fading on DOA estimation of single source was studied. A closed-form expression of the DA CRB for DOA alone is derived with AR1 fading amplitudes. As special cases, the CRBs for DOA alone over slow and uncorrelated amplitude fading are also derived from the general expression of the fast amplitude fading bound. We have also derived analytical approximate expressions for the CRB of the DOA alone for low and high SNR. Some properties that highlight how the bound depends on key parameters such as SNR and time-Doppler product were derived. These properties show that the DA CRB for DOA alone is insensitive to the channel-dependent time-Doppler product for high SNR expect for low SNR. The ML approach for estimating DOA parameter based on a mismatched AR1 channel-correlation model upon which a high SNR estimator was derived. The estimator was compressed into a single-parameter search over the DOA parameter alone.

Issues that were not addressed in this paper are the ML estimator and the CRB on DOA estimation of multiple targets over time-varying amplitudes. A paper in preparation deals with these issues.

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