THEORETICAL EXPRESSION OF ERROR EVENT PROBABILITY FOR A TRELLIS CHAOS CODED MODULATION CONCATENATED WITH SPACE-TIME BLOK CODE

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ABSTRACT

In this paper, we propose a new insights for the chaos coded modulation (CCM) schemes originally proposed by Kozic & al. using the approximation of the distance spectrum distribution with some usual laws, a complete study of the performances of these CCM schemes when they are concatenated with a Space Time Block Code (STBC) is proposed. Accurate bounds are obtained even in the case of time selective channels.

1. INTRODUCTION

Chaotic signals, i.e. signals which can be described as outputs of nonlinear dynamical systems exhibiting chaotic behavior. The possibility of using these signals to carry information was proposed in 1993 [1] and, since then, chaotic communications has been an important topic in digital communications. The seemingly unpredictable behaviour of chaotic systems renders their use in secure communication systems highly attractive. Due to their extreme sensitivity to initial conditions which, for example, facilitates theoretically the separation of merging paths in a trellis based code, these systems have also been considered as good potential candidates for channel encoding [2-3]. This explains why chaotic modulations and channel encoders derived from chaotic systems have been extensively studied in the open literature. There are several types of chaos based channel encoders, According to us, our type those which transmit a complex quasi-continuous alphabet i.e. those which are inherently chaotic in all their characteristics. These channel encoders exhibit a high spectral efficiency and can be compared to Trellis Coded Modulation (TCM) schemes. Many works deal with the optimization of such coders and, among them, perhaps the most famous ones were those named Chaos Coded Modulation (CCM) schemes. However, the weakness of such transceiver was their poor BER performance since they did not have even better performances than un-coded systems such as Binary Phase Shift Keying (BPSK) [4-5]. This was particularly the case for the systems which use CSK (Chaos Shift Keying) Modulation [6-7]. Nevertheless, some recent studies have stressed the fact that Chaos Coded Modulation (CCM) systems, working at a joint waveform and coding level, can be efficient in additive white Gaussian noise channels [8-9]. These promising works on the AWGN channel have been recently further extended by Escribano & al in the case of Rayleigh flat fading channels [10]. In this paper, we use Chaos Coded Modulation designs of S. Kozic [11-12] and we optimize them using the distance spectrum. Using this optimization step, we study the performances of the proposed Chaos Coded Modulation designs when we concatenate them with a Space Time Block Code (STBC) such as the famous Alamouti's scheme [13]. Concatenation of a Trellis Coded Modulation (TCM) with a STBC code is recognized as a performing alternative to the use of Space Time Trellis Codes (STTC) [14-15]. The strength of our study consists to study the case of block fading channels and we derive accurate BER bounds. The contributions of our paper are thus the following ones:

- Detailed study of the distance spectra of the chaos based encoders.
- Derivation of accurate BER bounds for quasi-static block fading channels.

The rest of the paper is organized as follows. In Section 2, we give new insights for the chaos coded modulation schemes proposed by S. Kozic. In section 3, we study the performances of the concatenation of the Chaos Coded Modulation (CCM) together with the Alamouti's STBC code for quasi static block fading channels. The concluding remarks are eventually given in Section 4.

2. CHAOS CODED MODULATION SCHEME, DISTANCE SPECTRUM STUDY

2.1 CHAOTIC CODER STRUCTURE

We consider the Chaos-Coded modulation scheme of Fig. 1. This scheme was originally given by S. Kozic in his PhD works [12]. The scheme of Figure.1 can be represented by means of a convolutional coder of rate $\eta=1/(n.(Q+1))$, where at each time step k, one bit b_k enters the coder and a vector of $(Q+1)$ bits $\mathrm{v} = [\mathrm{v}_Q,\mathrm{v}_{Q-1},\ldots,\mathrm{v}_0]^T$ is produced. The signal constellation is realized by a weighted sum of vectors

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$2^{-i}.\boldsymbol{A}^{(Q-i+1)}$ *mod* (1) where $\boldsymbol{A}$ is some matrix which optimizes the distance spectrum of the code. This mapping, due to the modulus operation, is a highly non-linear operation and serves as a chaos generator. Henceforth, we have a system which combines a convolutional coder with a high dimensional mapping in the same way as Multi-level Trellis Coded Modulation (M-TCM). The corresponding convolutional coder is classically described by:

$$h_i(D) = \frac{\mathrm{v}_i(D)}{b(D)} = t_{i,Q} + t_{i,Q-1}.D + ... + t_{i,0}.D^Q \qquad (1)$$

S. Kozic defines several possible matrices $\boldsymbol{T}=\{t_{i,j}\}$ in his work which give good performances:

$$\boldsymbol{T}_{shift} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \cdots \end{bmatrix} \quad \boldsymbol{T}_{e-shift} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & \cdots \end{bmatrix} \quad \boldsymbol{T}_{tent} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & \cdots \end{bmatrix}$$

Concerning, the choice of the matrix $\boldsymbol{A}$, we can write the transmitted vector at the output of the modulator:

$$\boldsymbol{x}_k = \sum_{i=0}^{Q_a} 2^{-(i+1)}.\boldsymbol{A}^{Q_a - i}.\mathbf{v}_i(D) + \sum_{i=Q_a+1}^{Q} 2^{-(i+1)}.\mathbf{v}_i(D) \bmod(1) \qquad (2)$$

Before transmitting $\boldsymbol{x}_k$ on the channel propagation medium, we modulate each of its components in NRZ-BPSK in order to obtain a average zero mean value to better interface with a zero mean additive noise such as for AWGN channel i.e: $\boldsymbol{x}_k \rightarrow 2\,\boldsymbol{x}_k - 1$.

Rather than a global optimization algorithm which should look for the convolutional coder together with the mapping process, we choose to fix a convolutional coder structure and then we work on the mapping process by using a particular form of matrix $\boldsymbol{A}$. We found that the choice $\boldsymbol{T}_{i,j}=\boldsymbol{T}_{shift}$ for $i = j$ and $\boldsymbol{T}_{i,j}=\boldsymbol{T}_{tent}$ for $i \neq j$ enables to obtain a large set of performing non-linear mapping with $\boldsymbol{A}$. For example, in the case $n = 2$, using this choice for matrices $\boldsymbol{T}$, we are looking for matrices $\boldsymbol{A}$ with the following structure:

$$\boldsymbol{A} = \begin{pmatrix} 1 & -1 \\ a_{21} & 1 \end{pmatrix}$$

and we optimize the choice of a_{21} using the distance spectrum.

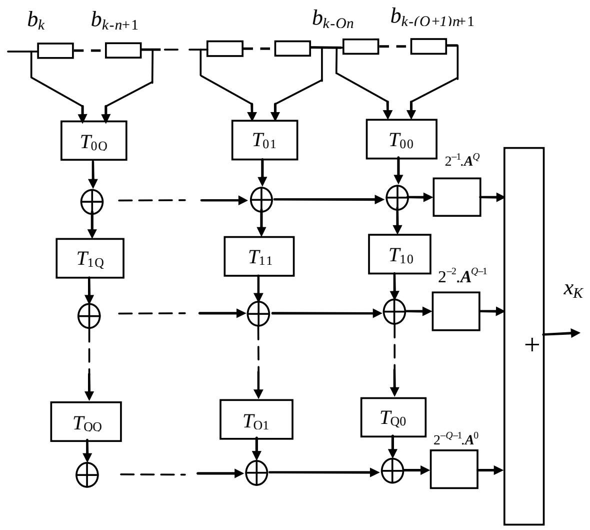

Figure 1: Trellis chaos coded modulation encoder

The choice of the remaining parameters $a_{i,j}$ is done using the distance spectrum of the code. The state of the coder is defined by vector $\boldsymbol{S}_k$: $\boldsymbol{S}_k=[\mathrm{b}_k,\ldots,b_{k-n},\ldots \mathrm{b}_{k-Qn},\ldots,\mathrm{b}_{k-(Q+1).n+1}]^T$

Concerning the choice of Q, it's clear that the Viterbi decoding algorithm is rapidly limited by the complexity in the number of states which is equal to $2^{\,n.(Q+1)}$. Practically, the number $n(Q+1)$ should not exceed 12 which correspond to 4096 states. For $n = 2$, this gives a maximum value of Q equal to 5. The choice of Q_a, is related to the chaotic behaviour of the coder.

2.2 SPECTRUM DISTANCE ANALYSIS

In order to optimize the coders, we study their distance spectrum. To do this, we have to determine the trajectories in the trellis which start with a common state $\boldsymbol{S}_i = \boldsymbol{S}_i^*$ and evolve in disjoint paths for (L-1) time steps and then merge again into the same state $\boldsymbol{S}_i = \boldsymbol{S}_k^*$ not necessarily equal to $\boldsymbol{S}_i$. This kind of trajectory in the trellis defines a loop and the loop is characterized by its initial state $\boldsymbol{S}_i$, its final state $\boldsymbol{S}_k$ and its length L. The distance of corresponding codewords belonging to the two competing paths in the loop is:

$$d^2_{L,S_i,S_k} = \sum_{m=1}^{L-1} \left\| \boldsymbol{x}_m - \boldsymbol{x}_m^* \right\|^2 \qquad (3)$$

The problem of the computation of (3) is that, unlike linear codes when we can choose a reference path equal to a all zero sequence, due to the non-linear mapping, we have to test all the possible transmitted sequence for a given loop length together with all the possible starting states. Hence, the distance spectrum computation problem is of non polynomial complexity and in straightforward manner requires the inspection of all possible initial conditions and all possible controlled trajectories. For example, there are $2^{\,n.(Q+1)}.2^{nL}$ different controlled trajectories of length L. In order to compute the distance spectrum with a reasonable complexity while keeping a sufficient accuracy, we form all the possible pair of sequences starting from a given state and both converging towards an other state after L steps with L belonging to the interval $[Qn+1,\, n.(Q+m)]$, i.e. the length of the loop varies from $Qn+1$ (the constraint length of the code plus one) to to $n.(Q+m)$ (we limit practically the search to m = 2 or 3 in our case due to the computation burden). We have partitioned the distance spectrum into subsets by distinguishing error events which entail one error bit, error events which entail two error bits, error events which entail three error bits and so on... In practice, we limit our search to error events which entail five maximum error bits since simulation results evidenced that it was sufficient to obtain accurate upper bounds for the BER.

We obtain for example with matrices: $\boldsymbol{T}_{i,j}=\boldsymbol{T}_{shift}$ for $i = j$ and $\boldsymbol{T}_{i,j}=\boldsymbol{T}_{tent}$ (i.e. $n = 2$) for $i \neq j$ and $a_{21} = 8$, $Q = Q_a = 3$, the distance spectrum illustrated on figure 2.

In fact, we found that, in a majority of cases, the shape of the distance spectrum is close to a Rayleigh distribution with the following probability density function:

$$f_C(x) = \frac{(x-m_j)}{\sigma_j^2}.e^{-(x-m_j)^2/2.\sigma_j^2}, x \geq m_j \tag{4}$$
$$f_C(x) = 0, x < m_j$$

For example, with the distance spectrum plotted on figure.2, we calculate parameters μ_j and σ_j^2 to obtain the best fitting between the pdf of the distance spectrum and $f_c(x,m_j,\sigma_j^2)$ we obtain with classical MMSE technique: $\mu_j \cong \sigma_j^2 \cong 6.7$. This corresponds to a minimum free distance of the coder equal to $d_{free} \cong 6.7$. We have developed an original EM (Expectation-Maximization) algorithm to obtain the approximated Rayleigh distribution of the distance spectrum in the general case where the distance spectrum looks like a mixture of Rayleigh laws [16].

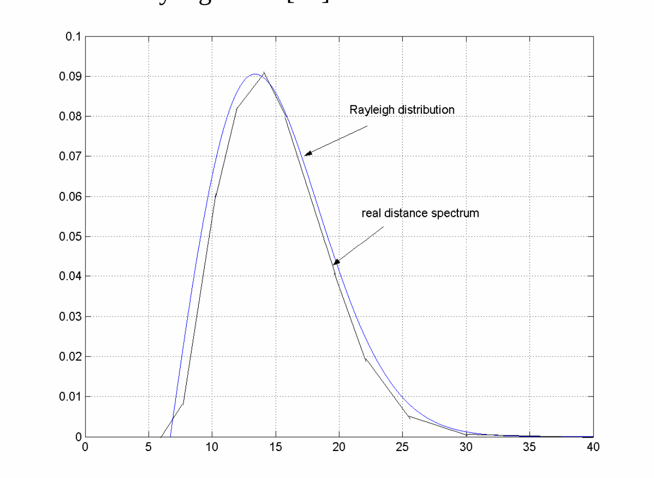

Figure 2: Distance spectrum of the chaos coded modulation

To end this part, we give some BER results on AWGN channels, using the optimization obtained by the distance spectrum computation to find good modulation parameters. Due to a lack of place we only give simulation results. For $n = 2$, we obtain the following result on figure 3

The chaotic coder outperforms uncoded BPSK at high SNR's due to good asymptotic properties with a moderate high free distance. The weakness of this kind of code is their poor coding rate.

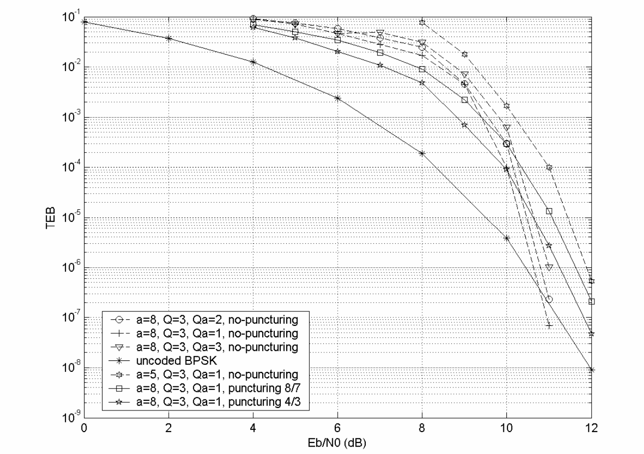

Figure 3: Performances of Trellis Chaos-Coded Modulation over AWGN channels for $n = 2$, $Q = 3$

There are several solutions to improve this. The first is to make input bits enter the coder by groups of k bits. In this case, the coding rate becomes equal to: $k/(n.(Q+1))$. However, this considerably reduces the correlation degree between consecutive states and renders the trellis non-binary. We found that the penalty encountered by this method too much important (using $k = 2$ results in 4 dB losses compared to $k = 1$) so we prefer using puncturing to increase the coding rate of our proposed coders.

3. CHAOS CODED MODULATION SCHEME CONCATENATED WITH STBC

3.1 Computation of the Pairwise Error Probability

the concatenation of the chaotic encoder with a (Space Time Block Code) STBC, as the well known Alamouti's scheme, is illustrated on figure 4:

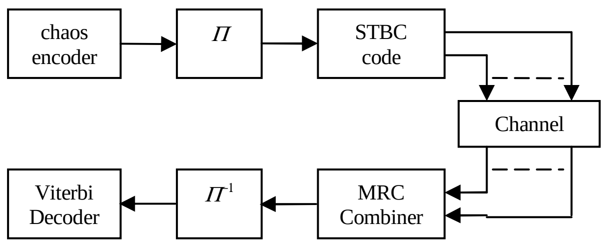

Figure 4: Concatenation of the chaos coded modulation encoder and a STBC code

We consider $\boldsymbol{x}_k$ is a vector of two transmitted analog symbols: $\boldsymbol{x}_k = \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix}$.

These two symbols are transmitted using the well known Alamouti's scheme $\boldsymbol{S} = \begin{pmatrix} x_1(k) & -x_2^*(k) \\ x_2(k) & x_1^*(k) \end{pmatrix}$ or, in the case of real transmitted symbols, $\boldsymbol{S} = \begin{pmatrix} x_1(k) & -x_2(k) \\ x_2(k) & x_1(k) \end{pmatrix}$.

In the case of quasi-static block fading channels, channel remains constant over the duration of a transmitted packet but changes from packet to packet and there is no need to use an interleaver between the chaos-based channel encoder and the STBC. The received signal within two consecutive time slots can be written in the following way:

$$\begin{aligned} y_1(n) &= h_{11}x_1(n) + h_{21}x_2(n) + n_1(n) \\ y_1(n+1) = y_2(n) &= -h_{11}x_2^*(n) + h_{21}x_1^*(n) + n_2(n) \end{aligned} \tag{5}$$

using the Maximum Ratio Combiner (MRC) we obtain the two decision variables $Z_1(n)$ and $Z_2(n)$:

$$\begin{aligned} Z_1(n) &= (|h_{11}|^2 + |h_{21}|^2).x_1(n) + h_{11}^*.n_1(n) + h_{21}.n_2^*(n) \\ Z_2(n) &= (|h_{11}|^2 + |h_{21}|^2).x_2(n) + h_{21}^*.n_1(n) - h_{11}.n_2^*(n) \end{aligned} \tag{6}$$

there will be an error event when having sent the sequence $\boldsymbol{x}$ each time the decoder chooses $\boldsymbol{x}' \neq \boldsymbol{x}$, both sequences starting in the same state and merging again in possibly other state after L steps, when the decision metric implying $\boldsymbol{x}'$ will be inferior to those with $\boldsymbol{x}$. Assuming ML decoding, this is equivalent to:

$$P_e\left(x \to x' \middle| h_{11}, h_{21}, x\right) =$$
$$\Pr oba\Big(\sum_{n=m}^{M+m-1} \left| Z_1(n) - \left(|h_{11}|^2 + |h_{21}|^2\right) x_1(n) \right|^2 + \left| Z_2(n) - \left(|h_{11}|^2 + |h_{21}|^2\right) x_2(n) \right|^2 \Big)$$
$$< \sum_{n=m}^{L+m-1} \left| Z_1(n) - \left(|h_{11}|^2 + |h_{21}|^2\right) x_1'(n) \right|^2 + \left| Z_2(n) - \left(|h_{11}|^2 + |h_{21}|^2\right) x_2'(n) \right| \quad (7)$$

After some manipulation, we obtain the conditional probability of error:

$$P_e\left(x \to x' \middle| h_{11}, h_{21}, x\right) =$$
$$\frac{1}{2} erfc \left(\frac{ \left(|h_{11}|^2 + |h_{21}|^2\right)^{1/2} \left(\sum_{n=m}^{L+m-1} [x_1(n) - x_1'(n)]^2 + [x_2(n) - x_2'(n)]^2 \right)^{1/2} }{2\sqrt{2}\sigma} \right) \quad (8)$$

In fact, since h_{11} and h_{21} are complex Gaussian random variables, $B=(|h_{11}|^2+|h_{21}|^2)$ is a central chi-squared distributed random variable, whose probability density function (PDF) is given by:

$$p_B(x) = \frac{1}{\sigma_h^d . 2^{d/2} . \Gamma(d/2)} . x^{d/2-1} . \exp(-\frac{x}{2.\sigma_h^2}) \quad (9)$$

To derive P_e, we need to Averag (8) over the distribution of B (9) [17]:

$$P_e(\boldsymbol{x} \to \boldsymbol{x}' | \boldsymbol{x}) = \frac{1}{2} - \frac{C.\sigma^2.\sigma_h}{(C^2.\sigma_h^2 + 4.\sigma^2)^{3/2}} - \frac{C.\sigma_h}{2.(C^2.\sigma_h^2 + 4.\sigma^2)^{1/2}} \quad (10)$$

with: $C^2 = \sum_{n=m}^{L+m-1} \left[x_1(n) - x_1'(n)\right]^2 + \left[x_2(n) - x_2'(n)\right]^2$

and: $\sigma_h^2 = E[|h_{11}|^2] = E[|h_{21}|^2]$

To compute the probability $P_e(\boldsymbol{x} \to \boldsymbol{x}')$, it is then necessary to study the distance spectrum of the channel coder, i.e, we need to average (10) over the distribution of the quantity C. Using Gaussian distributions [16], we obtain:

$$P_e\left(x \to x'\right) = \sum_{j=1}^{L} \pi_j \sum_{n=1}^{+\infty} \frac{(-1)^{n+1} . 2^n . n . (n+1) . (2n+1)!}{\left((n+1)!\right)^2} . \frac{\sigma^{2(n+1)}}{\sqrt{2\pi} . \sigma_j . \sigma_h^{2(n+1)}} .$$
$$\left[\sum_{p=n+1}^{+\infty} \frac{(-1)^p . p!}{(p-n)!} . \left(\frac{I_{p-n}}{m_j^{p+1}} + \left(m_j\right)^{p-n} . J_p \right) \right] \quad (11)$$

with:
$$I_p = \int_{d_{\min}-m_j}^{m_j} u^p . e^{-u^2/2.\sigma_j^2} . du$$
$$J_p = \int_{m_j}^{+\infty} \frac{e^{-u^2/2.\sigma_j^2}}{u^{p-1}} . du$$

I_n and J_n can be computed recursively. We have:

$$J_{p+2} = \frac{e^{-m_j^2/2.\sigma_j^2}}{(p+1).(m_j)^{p+1}} - \frac{1}{\sigma_j^2.(p+1)} . J_p$$

Depending on the parity of p, the former formula enables to calculate J_p given the first values J_1 or J_0.

For J_1, we have:

$$J_1 = \int_{m_j}^{+\infty} \frac{e^{-u^2/2\sigma_j^2}}{u} du = -\sigma_j^2 . \int_{m_j}^{+\infty} \frac{\left(-u/\sigma_j^2\right) . e^{-u^2/2.\sigma_j^2}}{u^2} du =$$
$$\int_{m_j^2/2\sigma_j^2}^{+\infty} \frac{e^{-x}}{x} . dx = EI_1\left(m_j^2 / 2.\sigma_j^2\right)$$

With: $EI_1(x) = \int_x^{+\infty} \frac{e^{-t}}{t} . dt$

For J_0, we have:

$$J_0 = \int_{m_j}^{+\infty} e^{-u^2/2.\sigma_j^2} . du = \sqrt{2} . \sigma_j \int_{m_j/\sqrt{2}.\sigma_j}^{+\infty} e^{-x^2} . dx = \sqrt{\frac{\pi}{2}} . \sigma_j . \text{erfc}(m_j / \sqrt{2}.\sigma_j)$$

For the computation of I_p:

$$e^{-u^2/2.\sigma_j^2} = \sum_{n=0}^{+\infty} (-1)^n \frac{u^{2n}}{n\ !} . \frac{1}{(2.\sigma_j^2)^n} = \sum_{n=0}^{+\infty} (-1)^n \frac{u^{2n}}{2^n . n\ !} . \frac{1}{\sigma_j^{2n}}$$

$$I_p = \int_{d_{\min}-m_j}^{m_j} u^p . e^{-u^2/2\sigma_j^2} . du = \sum_{n=0}^{+\infty} \frac{(-1)^n}{2^n . n! . \sigma_j^{2n}} . \int_{d_{\min}-m_j}^{m_j} u^{2n+p} . du =$$
$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{2^n . n! . (2n+p+1) . \sigma_j^{2n}} . \left[m_j^{2n+p+1} - \left(d_{\min} - m_j\right)^{2n+p+1} \right]$$

The expressions of the Pairwise Error Probability (11) are serial expansions in terms of $\boldsymbol{\sigma^2}$, the variance of the additive white Gaussian noise on the link. It can also be expressed in terms of E_b/N_0 taking into account that:

$$E_b / N_0 = E_s / (n.(Q+1).N_0) = \frac{1}{3.(Q+1).n.N_0} = \frac{1}{6.(Q+1).n.\sigma^2}$$
$$\sigma^2 = \frac{1}{6.(Q+1).n.(E_b / N_0)} \quad (12)$$

E_s=1/3 corresponds to the average energy per transmitted symbol.

To obtain P_b we have to average some approximations [18], finally we obtain this expression:

$$P_b \le \sum_{\text{w}=1}^{+\infty} \text{w}.P_{e,\text{w}}(\boldsymbol{x} \to \boldsymbol{x}') \quad (13)$$

With $P_e(x \rightarrow x')$ denoting the Pairwise Error Probability (PEP) for pair of sequences which having w error length weight. $P_{e,w}(x \rightarrow x')$, has been given in (11).

3.2 Simulation results

We checked formula (13) using the chaotic coder of Fig. 1 (a = 8, n =2, Q = 3) and the coder optimized for n = 3, Q = 2. We compare the upper bound of (13) with the simulation results on Figure 5. One can see that the two curves are very close to each other in the two cases (n = 2 and n = 3) showing the accuracy of the formulas (11) and (13). The system for n = 3, Q = 2 has a high diversity gain at high SNR's since its slope is much more important that those for the case n = 2, Q = 3. This is due to a better free distance.

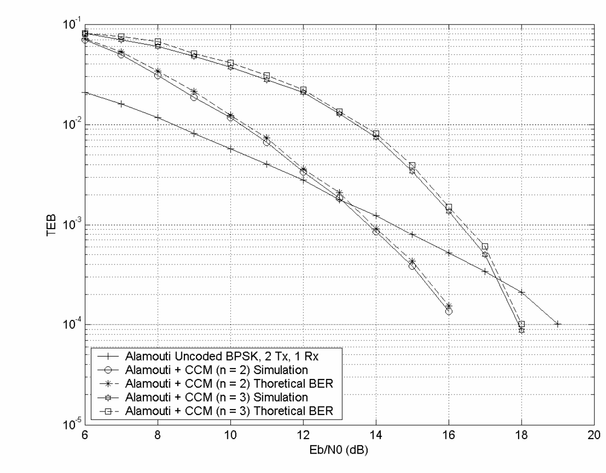

Figure 5: Performance comparison between simulation results and the Upper-Bound results

4. CONCLUSION

Based on the former work of Kozic & al, we have proposed new insights for the stud of performing Chaos Coded Modulation (CCM) schemes. We have studied in detail the distance spectrum of some CCM schemes. Using the approximations of their pdf's with some well known laws, we have been able to study the performances of these CCM schemes when they are concatenated with a Space Time Block Code (STBC). We obtained accurate BER bounds for quasi-static block fading channels. The provided simulation results, together with the computation of the BER bounds, illustrate the accuracy of our closed form expressions except.

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