A CONSTRAINED TENSOR-BASED APPROACH FOR MIMO NL-CDMA SYSTEMS

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ABSTRACT

In this paper, we propose a deterministic tensor-based approach for joint channel and symbol estimation in the context of multiuser multiantenna (MIMO) CDMA communication systems. We use a new nonlinear (NL) coding allowing to obtain a third-order block-Tucker2 model for the signals received by multiple receive antennas, with a constrained structure for the core tensors that ensures the uniqueness of the tensor model. Two types of receiver are developed. First, assuming that the users' code matrices are mutually orthogonal and known at the receiver, we derive a blind algorithm composed of two steps: a separation of users' contributions in the received signals, with decoding, followed by a blind channel and symbol estimation for each user separately. Then, when the code matrices are unknown, a semi-blind receiver is proposed for jointly estimating the channels, codes and symbols of all the users. Some simulation results are provided to illustrate the performance of the proposed receivers.

1. INTRODUCTION

Since the pioneering work of Sidiropoulos, Giannakis and Bro in the context of direct sequence code-division multiple access (DS-CDMA) communication systems [11] and sensor array processing [10], tensor models have found numerous applications in image and signal processing (SP). The parallel factor (PARAFAC) model, also called canonical decomposition (CANDECOMP), independently introduced by Harshman [8] and Carroll and Chang [1] for data analysis in phonetics and psychometrics, respectively, and the Tucker model [12] are the most used tensor models for applications.

In this paper, we consider a Tucker model as a mode-n product-based transformation of an input tensor corresponding to the core tensor. Such a transformation is applied for modeling a multiuser multiantenna (MIMO) CDMA communication system, with a nonlinear (NL) coding that allows us to build a constrained structure third-order input tensor associated with each user. These input tensors are characterized by two types of matrix slices having a Vandermonde form whereas the third one has an Hankel structure. Due to this constrained structure of the core tensors, the block-Tucker2 model of the tensor of signals received by multiple receive antennas, is unique. Considering the uplink of a cooperative multiuser CDMA communication system, with users' code matrices assumed to be mutually orthogonal and known at the base station, we derive a blind tensor-based solution for channel and symbol estimation. This solution is composed of two steps: a separation of users' contributions in the received signals, with decoding, followed by a blind channel and symbol estimation for each user separately. Then, in the case of unknown codes, we propose a semi-blind receiver for jointly estimating the channels, codes and symbols of all the users.

The rest of this paper is organized as follows. Section 2 provides some tensor prerequisites. In section 3, we introduce the constrained block-Tucker2 model of the signals received by a MIMO NL-CDMA system. In section 4, assuming that the NL spreading codes are known at the receiver, we derive a blind algorithm for channel and symbol estimation. The case of unknown codes is considered in section 5 where a semi-blind receiver is proposed for jointly estimating the channels, codes and symbols of all the users. Some simulation results are presented in section 6, to illustrate the performance of the proposed tensor-based receivers, before concluding the paper in section 7.

Notations: $\mathscr{C}$ denotes the field of complex numbers.Vectors are written as bold-face lower-case letters $(\mathbf{u},\mathbf{v},\ldots)$, matrices as bold-face capital letters $(\mathbf{U},\mathbf{V},\ldots)$, and higher-order tensors as blackboard letters $(\mathbb{U},\mathbb{V}\cdots)$. $\mathbf{U}^T$, $\mathbf{U}^*$, $\mathbf{U}^H$ and $\mathbf{U}^\dagger$ stand for transpose, conjugate, transconjugate and Moore-Penrose pseudoinverse of $\mathbf{U}$, respectively. We denote by $\mathbf{U}_{i.}$ and $\mathbf{U}_{.j}$ the i^{th} row and the j^{th} column of the $(I\times J)$ matrix $\mathbf{U}$, respectively. The scalars u_i, u_{ij} and $u_{i_1\cdots i_N}$ denote the i^{th} element of $\mathbf{u}$, the $(i,j)^{th}$ element of $\mathbf{U}$ and the $(i_1,\cdots,i_N)^{th}$ element of $\mathbb{U}$, respectively. $\mathbf{I}_n$ is the identity matrix of order n and $\|.\|_F$ is the Frobenius norm. The outer product and the Kronecker product are denoted by $\circ$ and $\otimes$, respectively. The operator vec(.) forms a vector by stacking the columns of its matrix argument.

For $\mathbf{A}\in\mathscr{C}^{I\times P}$, $\mathbf{B}\in\mathscr{C}^{J\times Q}$ and $\mathbf{C}\in\mathscr{C}^{P\times Q}$, we have:

$$\mathrm{vec}\left(\mathbf{A}\mathbf{C}\mathbf{B}^T\right)=(\mathbf{B}\otimes\mathbf{A})\,\mathrm{vec}(\mathbf{C}) \tag{1}$$

2. TENSOR PREREQUISITES

For an N^{th}-order tensor $\mathbb{U}\in\mathscr{C}^{I_1\times I_2\times\cdots\times I_N}$, also called N-way array, of dimensions $I_1\times I_2\times\cdots\times I_N$, with entries $u_{i_1\cdots i_N}\in\mathscr{C}$ ($i_n=1,2,\cdots,I_n$, for $n=1,2,\cdots,N$), each index i_n is associated with a way, also called a mode, and I_n is the mode-n dimension.
In the case of a third-order tensor $\mathbb{U}\in\mathscr{C}^{I\times J\times K}$, we have three types of matrix slices, respectively called horizontal, lateral and frontal slices, and denoted by $\mathbf{U}_{i..}$, $\mathbf{U}_{.j.}$ and $\mathbf{U}_{..k}$, of respective dimensions $K\times J$, $I\times K$ and $J\times I$.

By column-wise stacking the matrix slices of a same type, we get the three following horizontal matrix unfoldings:

$$\mathbf{U}_1=[\mathbf{U}_{.1.}\cdots\mathbf{U}_{.J.}]\in\mathscr{C}^{I\times JK} \tag{2}$$

$$\mathbf{U}_2=[\mathbf{U}_{..1}\cdots\mathbf{U}_{..K}]\in\mathscr{C}^{J\times KI} \tag{3}$$

$$\mathbf{U}_3=[\mathbf{U}_{1..}\cdots\mathbf{U}_{I..}]\in\mathscr{C}^{K\times IJ} \tag{4}$$

The mode-n product of a tensor $\mathbb{U}\in\mathscr{C}^{I_1\times\cdots\times I_N}$ of order N with a matrix $\mathbf{A}\in\mathscr{C}^{J_n\times I_n}$, denoted by $\mathbb{V}=\mathbb{U}\times_n\mathbf{A}$, gives a tensor of order N and dimensions $I_1\times\cdots\times I_{n-1}\times J_n\times I_{n+1}\times\cdots\times I_N$ such as [2]:

$$v_{i_1\cdots i_{n-1}j_ni_{n+1}\cdots i_N}=\sum_{i_n=1}^{I_n}u_{i_1\cdots i_{n-1}i_ni_{n+1}\cdots i_N}a_{j_ni_n} \tag{5}$$

This mode-n product can be expressed in terms of horizontal mode-n matrix unfoldings of tensors $\mathbb{U}$ and $\mathbb{V}$ as:

$$\mathbf{V}_n=\mathbf{A}\mathbf{U}_n \tag{6}$$

For a third-order tensor $\mathbb{U} \in \mathscr{C}^{I\times J\times K}$, the Tucker model is given by:

$$u_{ijk} = \sum_{p=1}^{P}\sum_{q=1}^{Q}\sum_{r=1}^{R} g_{pqr}a_{ip}b_{jq}c_{kr} \tag{7}$$

where g_{pqr} is an element of the core tensor $\mathbb{G} \in \mathscr{C}^{P\times Q\times R}$, and a_{ip}, b_{jq} and c_{kr} are entries of the matrix factors $\mathbf{A} \in \mathscr{C}^{I\times P}$, $\mathbf{B} \in \mathscr{C}^{J\times Q}$ and $\mathbf{C} \in \mathscr{C}^{K\times R}$, respectively.

It can also be written in terms of mode-n products as:

$$\mathbb{U} = \mathbb{G} \times_1 \mathbf{A} \times_2 \mathbf{B} \times_3 \mathbf{C} \tag{8}$$

The tensor model (7)-(8) is also called a Tucker3 model. When one (two) of the matrix factors $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ is (are) an identity matrix, it is called a Tucker2 (Tucker1) model.

The Tucker model (8) can be interpreted as mode-n product-based transformations $(n = 1, 2, 3)$ of the core tensor, i.e. linear transformations defined by the matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$, applied to each mode-n vector space of the core tensor. In this case, the core tensor and the transformed tensor will be called input tensor and output tensor, respectively.

3. SIGNAL TENSOR MODELS

We now illustrate the mode-n product-based transformation introduced in the previous section, for modeling the signals received by a MIMO NL-CDMA communication system, composed of a linear and uniformly spaced array of M antennas $(m = 1, \cdots, M)$ receiving signals from Q users $(q = 1, \cdots, Q)$.

Let us define the third-order NL input signal tensor $\mathbb{U}^{(q)}(n) \in \mathscr{C}^{I\times J\times K}$ for each user q, with the following entry:

$$u_{ijk}^{(q)}(n) = \left[u^{(q)}(nK - i - k + 2)\right]^j \tag{9}$$

where $u^{(q)}(nK)$ is the symbol transmitted by the q^{th} user, at the symbol period nK, with $i = 1, \cdots, I$, $j = 1, \cdots, J$, $k = 1, \cdots, K$, and n represents the output data block number. The dimensions I, J and K represent, respectively, the channel memory expressed in symbol periods, the code nonlinearity degree and the output data block length. The three corresponding modes will be called recurrence mode (i), nonlinearity mode (j) and time mode (k). The different user sequences are assumed to be synchronized at the symbol level.

From definition (9), we can deduce that the horizontal and frontal slices have a Vandermonde structure, whereas the lateral slices have a Hankel form.

Let us assume that, at the p^{th} chip period of the n^{th} symbol period, user q transmits the following nonlinearly coded signal:

$$v_p^{(q)}(n) = \sum_{j=1}^{J} b_{pj}^{(q)} \left[u^{(q)}(n)\right]^j \tag{10}$$

where $b_{pj}^{(q)}$ is an entry of the code matrix $\mathbf{B}^{(q)} \in \mathscr{C}^{P\times J}$.

The transformed input tensor $\mathbb{V}^{(q)}(n) \in \mathscr{C}^{I\times P\times K}$, called coded input tensor, can therefore be written as:

$$\mathbb{V}^{(q)}(n) = \mathbb{U}^{(q)}(n) \times_2 \mathbf{B}^{(q)} \tag{11}$$

or equivalently

$$v_{ipk}^{(q)}(n) = v_p^{(q)}(nK - i - k + 2) = \sum_{j=1}^{J} u_{ijk}^{(q)}(n) b_{pj}^{(q)}$$

In our work, we make the following assumptions :

A1 In section 4, the code matrices $\mathbf{B}^{(q)} \in \mathscr{C}^{P\times J}$ are column-orthonormal and mutually orthogonal, which implies $P \geq JQ$.

A2 The channel between user q and antenna m is modeled as a FIR filter, time invariant over NK symbol periods, with impulse response $a_{mi}^{(q)}$ and memory $I^{(q)}$ at the symbol rate.

A3 The number Q of users, the spreading gain P (identical for all users), and an upper bound $I = \max_{q=1,\cdots,Q} \left\{I^{(q)}\right\}$ on the memory of all the channels are known by the receiver.

A4 The baseband signals received by each antenna are sampled at the chip rate.

The p^{th} signal received by antenna m from user q, associated with the $(nK - k + 1)^{th}$ symbol period, is given, in the noiseless case, by:

$$\begin{aligned} y_{mpk}^{(q)}(n) &= y_m^{(q)}(nK - k + 1 + (p-1)/P) \\ &= \sum_{i=1}^{I} a_{mi}^{(q)} v_p^{(q)}(nK - i - k + 2) \\ &= \sum_{i=1}^{I} v_{ipk}^{(q)}(n) a_{mi}^{(q)} \end{aligned}$$

The transformed tensor $\mathbb{Y}^{(q)}(n) \in \mathscr{C}^{M\times P\times K}$ contains the signals received by the M antennas, during K symbol periods of the n^{th} block, with P oversamples/symbol, corresponding to the contribution of the q^{th} user. It can be written as:

$$\mathbb{Y}^{(q)}(n) = \mathbb{V}^{(q)}(n) \times_1 \mathbf{A}^{(q)} \tag{12}$$

Replacing $\mathbb{V}^{(q)}(n)$ by its expression (11) into (12) gives:

$$\mathbb{Y}^{(q)}(n) = \mathbb{U}^{(q)}(n) \times_1 \mathbf{A}^{(q)} \times_2 \mathbf{B}^{(q)} \tag{13}$$

or equivalently, in scalar form:

$$y_{mpk}^{(q)}(n) = \sum_{i=1}^{I}\sum_{j=1}^{J} u_{ijk}^{(q)}(n) a_{mi}^{(q)} b_{pj}^{(q)} \tag{14}$$

By comparing (13) with (8), we deduce that the received signal tensor $\mathbb{Y}^{(q)}(n)$ satisfies a Tucker2 model with matrix factors $\left(\mathbf{A}^{(q)}, \mathbf{B}^{(q)}, \mathbf{I}_K\right)$ and core tensor $\mathbb{U}^{(q)}(n)$.

The overall received signal tensor is the sum of the received signal tensors $\mathbb{Y}^{(q)}(n)$, i.e.:

$$\mathbb{Y}(n) = \sum_{q=1}^{Q} \mathbb{U}^{(q)}(n) \times_1 \mathbf{A}^{(q)} \times_2 \mathbf{B}^{(q)} + \mathbb{N}(n) \tag{15}$$

where $\mathbb{N}(n) \in \mathscr{C}^{M\times P\times K}$ represents the additive noise tensor, including both measurement noise and modeling error.

We have to notice that the input signal tensors $\mathbb{U}^{(q)}(n)$ and the received signal tensor $\mathbb{Y}(n)$ are both characterized by three diversities corresponding to the modes of each tensor : recurrence (i), input nonlinearity (j), and time (k) for $\mathbb{U}^{(q)}(n)$, and space (m), code (p), time (k) for $\mathbb{Y}(n)$.

Remarks:

1. Due to the constrained structure of the core tensor $\mathbb{U}^{(q)}(n)$ and of two matrix factors $\left(\mathbf{B}^{(q)} \text{ and } \mathbf{I}_K\right)$, it is easy to deduce the uniqueness of the Tucker2 model (13) and consequently of the block-Tucker2 model (15). This model can also be viewed as a constrained version of the decomposition in rank-(I, J, *) terms, introduced in [7].

2. For $I=J=K=1$,which corresponds to the case of a memoryless channel and a linear coding, (14) simplifies as

$$y_{mnp}^{(q)} = a_{mq}u_{nq}b_{pq}$$

and the received signal tensor $\mathbb{Y} \in \mathscr{C}^{M\times N\times P}$, in the noiseless case, becomes:

$$y_{mnp} = \sum_{q=1}^{Q} a_{mq}u_{nq}b_{pq}$$

This is the PARAFAC model for DS-CDMA sytems, proposed in [11]. With this model, each user's contribution to the received signal is a rank-one tensor component.

4. BLIND CHANNEL AND SYMBOL ESTIMATION

In this section, we assume that the code matrices are mutually orthogonal and known at the receiver which gives, in the noiseless case, for $q \in \{1,\cdots,Q\}$:

$$\mathbb{S}^{(q)}(n) = \mathbb{Y}(n) \times_2 \left[\mathbf{B}^{(q)}\right]^H \tag{16}$$

$$= \mathbb{U}^{(q)}(n) \times_1 \mathbf{A}^{(q)} \in \mathscr{C}^{M\times J\times K} \tag{17}$$

This transformation (16) allows to simultaneously separate and decode the information transmitted by the q^{th} user. The resulting decoded received signal tensor satisfies a Tucker1 model, two of its factor matrices being equal to the identity matrices of order J and K.

The horizontal mode-1 unfolded matrix form of the tensor $\mathbb{S}^{(q)}(n)$ can be deduced from (6) :

$$\mathbf{S}_1^{(q)}(n) = \mathbf{A}^{(q)}\mathbf{U}_1^{(q)}(n) \in \mathscr{C}^{M\times JK} \tag{18}$$

In the sequel, we assume that $\mathbf{U}_1^{(q)}(n) \in \mathscr{C}^{I\times JK}$ is right-invertible, i.e. full row-rank, which implies $I \leq JK$.

Due to the column-block structure (2) of the horizontal matrix unfoldings $\mathbf{U}_1^{(q)}(n)$ and $\mathbf{S}_1^{(q)}(n)$, (18) can be rewritten as:

$$\mathbf{S}_{.j.}^{(q)}(n) = \mathbf{A}^{(q)}\mathbf{U}_{.j.}^{(q)}(n), \quad j=1,\cdots,J \tag{19}$$

To improve the symbol estimation, we take the redundancy and constrained structure of input tensors into account, i.e. the Hankel and Vandermonde structures of their matrix slices. Instead of estimating the Hankel matrix $\mathbf{U}_{.j.}^{(q)}(n) \in \mathscr{C}^{I\times K}$, we estimate its generator vector $\mathbf{u}^{(q,j)}(n) \in \mathscr{C}^{I+K-1}$ defined as:

$$\mathbf{u}^{(q,j)}(n) = \Big[\left(u^{(q)}(nK)\right)^j \; \left(u^{(q)}(nK-1)\right)^j \; \cdots \; \left(u^{(q)}((n-1)K-I+2)\right)^j\Big]^T \tag{21}$$

Applying the vec operator to (19) and using the identity (1) with $\mathbf{B} = \mathbf{I}_K$, we get:

$$\mathrm{vec}\left(\mathbf{S}_{.j.}^{(q)}(n)\right) = \left(\mathbf{I}_K \otimes \mathbf{A}^{(q)}\right)\mathrm{vec}\left(\mathbf{U}_{.j.}^{(q)}(n)\right) \quad j=1,\cdots,J \tag{22}$$

with

$$\mathrm{vec}\left(\mathbf{U}_{.j.}^{(q)}(n)\right) = \mathbf{M}\mathbf{u}^{(q,j)}(n) \tag{23}$$

1. Randomly initialize $\hat{\mathbf{A}}_0^{(q)}(1)$, $q=1,\cdots,Q$ and form $\mathbf{M}$ as defined in (24).
2. For $n=1,\cdots,N$, $q=1,\cdots,Q$, $it=0$:
   (a) Compute the decoded signal tensor $\mathbb{S}^{(q)}(n)$ using (16).
   (b) Iterate until convergence ($it = it+1$)
      i. Symbol estimation:
      $$\hat{\mathbf{u}}_{it}^{(q,1)}(n) = \left(\left(\mathbf{I}_K \otimes \hat{\mathbf{A}}_{it-1}^{(q)}(n)\right)\mathbf{M}\right)^{\dagger} \mathrm{vec}\left(\mathbf{S}_{.1.}^{(q)}(n)\right)$$
      Improvement of the symbol estimation using the Vandermonde structure.
      ii. Channel estimation:
      $$\hat{\mathbf{A}}_{it}^{(q)}(n) = \mathbf{S}_1^{(q)}(n)\left[\hat{\mathbf{U}}_{1,it}^{(q)}(n)\right]^{\dagger} \tag{20}$$
      iii. Return to step 2b until convergence.
   (c) Projection of the estimated symbols onto the alphabet used by user q.
3. Return to step 2 until $n=N$.

Table 1: Blind channel and symbol estimation algorithm.

and

$$\mathbf{M} = \begin{bmatrix} \mathbf{I}_I & \vdots & \mathbf{0}_{I\times(K-1)} & & \\ \mathbf{0}_{I\times 1} & \vdots & \mathbf{I}_I & \vdots & \mathbf{0}_{I\times(K-2)} \\ \vdots & & & & \vdots \\ \mathbf{0}_{I\times(K-2)} & \vdots & \mathbf{I}_I & \vdots & \mathbf{0}_{I\times 1} \\ & \mathbf{0}_{I\times(K-1)} & & \vdots & \mathbf{I}_I \end{bmatrix} \in \mathscr{R}^{IK\times(I+K-1)} \tag{24}$$

where $\mathbf{0}_{I\times K}$ denotes the null matrix of dimensions $I\times K$. Substituting (23) into (22) for $j=1$ gives the following LS solution for the generator vector:

$$\hat{\mathbf{u}}^{(q,1)}(n) = \left(\left(\mathbf{I}_K \otimes \mathbf{A}^{(q)}\right)\mathbf{M}\right)^{\dagger}\mathrm{vec}\left(\mathbf{S}_{.1.}^{(q)}(n)\right). \tag{25}$$

It is also possible to take the Vandermonde structure into account for improving the symbol estimation. Due to a lack of space, this step is not detailed in the paper. An alternating least squares (ALS) based solution for blind channel and symbol estimation is presented in Table 1. This algorithm includes a projection of the estimated symbols onto the alphabet used by each user.

5. SEMI-BLIND CHANNEL/CODE/SYMBOL ESTIMATION

In this section, we present a semi-blind channel/code/symbol estimation algorithm that can be applied when the code matrices are unknown at the receiver. This algorithm is inspired from the results in [9].

Let us define the tensors:

$$\mathbb{T}^{(q)}(n) = \mathbb{U}^{(q)}(n) \times_2 \mathbf{B}^{(q)} \in \mathscr{C}^{I\times P\times K} \tag{26}$$

Noting that $\mathbb{Y}^{(q)}(n) = \mathbb{T}^{(q)}(n) \times_1 \mathbf{A}^{(q)} = \mathbb{S}^{(q)}(n) \times_2 \mathbf{B}^{(q)}$, with $\mathbb{S}^{(q)}(n)$ defined in (17), and using (6), the horizontal mode-1 and

-2 matrix unfoldings of $\mathbb{Y}(n)$, in the noiseless case, can be written as:

$$\mathbf{Y}_1(n) = \sum_{q=1}^{Q} \mathbf{Y}_1^{(q)}(n) = \mathbf{A}\mathbf{T}(n) \in \mathscr{C}^{M\times PK} \quad (27)$$

$$\mathbf{Y}_2(n) = \sum_{q=1}^{Q} \mathbf{Y}_2^{(q)}(n) = \mathbf{B}\mathbf{S}(n) \in \mathscr{C}^{P\times KM} \quad (28)$$

with the following matrix partitionings:

$$\mathbf{A} = \left[\mathbf{A}^{(1)} \cdots \mathbf{A}^{(Q)}\right] \qquad \mathbf{B} = \left[\mathbf{B}^{(1)} \cdots \mathbf{B}^{(Q)}\right] \quad (29)$$

$$\mathbf{T}(n) = \left[\mathbf{T}_1^{(1)^T}(n) \cdots \mathbf{T}_1^{(Q)^T}(n)\right]^T \quad (30)$$

$$\mathbf{S}(n) = \left[\mathbf{S}_2^{(1)^T}(n) \cdots \mathbf{S}_2^{(Q)^T}(n)\right]^T \quad (31)$$

We also consider the vertical mode-3 matrix unfolding of $\mathbb{Y}(n)$ given by:

$$\mathbf{Y}_3(n) = \begin{bmatrix} \mathbf{Y}_{.1.}(n) \\ \vdots \\ \mathbf{Y}_{.P.}(n) \end{bmatrix} = (\mathbf{B}\odot\mathbf{A})\,\mathbf{U}(n) \in \mathscr{C}^{PM\times K} \quad (32)$$

where $\odot$ denotes the block column-wise Kronecker product:

$$\mathbf{B}\odot\mathbf{A} = \left[\mathbf{B}^{(1)}\otimes\mathbf{A}^{(1)} \cdots \mathbf{B}^{(Q)}\otimes\mathbf{A}^{(Q)}\right] \in \mathscr{C}^{PM\times QJI} \quad (33)$$

and

$$\mathbf{U}(n) = \begin{bmatrix} \mathbf{U}_3^{(1)}(n) \\ \vdots \\ \mathbf{U}_3^{(Q)}(n) \end{bmatrix} \in \mathscr{C}^{QJI\times K} \quad (34)$$

with

$$\mathbf{U}_3^{(q)}(n) = \begin{bmatrix} \mathbf{U}_{.1.}^{(q)}(n) \\ \vdots \\ \mathbf{U}_{.J.}^{(q)}(n) \end{bmatrix} \in \mathscr{C}^{JI\times K} \quad (35)$$

In order to take the Hankel structure of each matrix slice $\mathbf{U}_{.j.}^{(q)}(n)$ into account, we define the matrix $\mathbf{G}(n) \in \mathscr{C}^{Q(I+K-1)\times J}$ that contains all the generator vectors $\mathbf{u}^{(q,j)}(n)$ defined in (21):

$$\mathbf{G}(n) = [\mathbf{G}_{.1}(n) \cdots \mathbf{G}_{.J}(n)] = \begin{bmatrix} \mathbf{G}^{(1)}(n) \\ \vdots \\ \mathbf{G}^{(Q)}(n) \end{bmatrix} \quad (36)$$

with

$$\mathbf{G}_{.j}(n) = \begin{bmatrix} \mathbf{u}^{(1,j)}(n) \\ \vdots \\ \mathbf{u}^{(Q,j)}(n) \end{bmatrix} \in \mathscr{C}^{Q(I+K-1)}$$

$$\mathbf{G}^{(q)}(n) = \left[\mathbf{u}^{(q,1)}(n) \cdots \mathbf{u}^{(q,J)}(n)\right] \in \mathscr{C}^{(I+K-1)\times J}$$

Decomposing M defined in (24) into K blocks as:

$$\mathbf{M} = \left[\mathbf{M}^{(1)^T} \cdots \mathbf{M}^{(K)^T}\right]^T \in \mathscr{C}^{IK\times(I+K-1)} \quad (37)$$

we have:

$$\mathrm{vec}\,(\mathbf{Y}_3(n)) = \mathbf{D}\mathbf{u}(n) \quad (38)$$

where $\mathbf{u}(n) = \begin{bmatrix} \mathrm{vec}\left(\mathbf{G}^{(1)}(n)\right) \\ \vdots \\ \mathrm{vec}\left(\mathbf{G}^{(Q)}(n)\right) \end{bmatrix} \in \mathscr{C}^{QJ(I+K-1)}$ contains all the generator vectors $\mathbf{u}^{(q,j)}(n)$, for $q = 1,\cdots,Q$, $j = 1,\cdots,J$, and

$$\mathbf{D} = \left[\mathbf{D}^{(1)} \cdots \mathbf{D}^{(Q)}\right] \in \mathscr{C}^{PMK\times QJ(I+K-1)} \quad (39)$$

with:

$$\mathbf{D}^{(q)} = \left(\mathbf{I}_K \otimes \left(\mathbf{B}^{q)} \otimes \mathbf{A}^{(q)}\right)\right) \begin{bmatrix} \mathbf{I}_J \otimes \mathbf{M}^{(1)} \\ \vdots \\ \mathbf{I}_J \otimes \mathbf{M}^{(K)} \end{bmatrix} \quad (40)$$

The proposed semi-blind joint channel/code/symbol estimation algorithm, summarized in Table 2, is derived by applying the ALS technique to Eq (27), (28) and (38). We have the following necessary conditions for identifiability: $PK \geq QI$, $KM \geq QJ$, and $PMK \geq QJ(I+K-1)$.

1. Initialization ($it = 0$, $n = 1$): Randomly initialize $\hat{\mathbf{A}}_0^{(q)}(1)$ and $\hat{\mathbf{B}}_0^{(q)}(1)$, form $\hat{\mathbb{U}}_0^{(q)}(1) = \mathbb{U}^{(q)}(1)$, for $q = 1,\cdots,Q$ and $\mathbf{M}$ defined in (24).
2. Supervised phase ($n = 1$): joint channel and code estimation using the first data tensor $\mathbb{Y}(1)$, associated with the known input tensors $\mathbb{U}^{(q)}(1)$, $q = 1,\cdots,Q$, and a 2 step-ALS algorithm.
   (a) Compute: $\hat{\mathbb{T}}_{it}^{(q)}(n) = \hat{\mathbb{U}}_{it}^{(q)}(n) \times_2 \hat{\mathbf{B}}_{it}^{(q)}(n)$, $\hat{\mathbb{S}}_{it}^{(q)}(n) = \hat{\mathbb{U}}_{it}^{(q)}(n) \times_1 \hat{\mathbf{A}}_{it}^{(q)}(n)$, for $q = 1,\cdots,Q$ and form $\hat{\mathbf{T}}_{it}(n)$ and $\hat{\mathbf{S}}_{it}(n)$ using (30) and (31).
   $$\hat{\mathbf{A}}_{it}(n) = \mathbf{Y}_1(n)\hat{\mathbf{T}}_{it}^{\dagger}(n), \quad \hat{\mathbf{B}}_{it}(n) = \mathbf{Y}_2(n)\hat{\mathbf{S}}_{it}^{\dagger}(n). \quad (41)$$
   (b) Return to step 2a until convergence, with $it = it + 1$.
3. Blind phase ($n = n+1$):
   (a) Initialization ($it = 0$): $\hat{\mathbf{A}}_0^{(q)}(n) = \hat{\mathbf{A}}_\infty^{(q)}(n-1)$ and $\hat{\mathbf{B}}_0^{(q)}(n) = \hat{\mathbf{B}}_\infty^{(q)}(n-1)$ where $\hat{\mathbf{A}}_\infty^{(q)}(n-1)$ and $\hat{\mathbf{B}}_\infty^{(q)}(n-1)$ are the estimated matrices obtained at convergence, for the data block $n-1$.
   (b) Symbol estimation: Compute $\mathbf{D}_{it}(n)$ using (39)-(40) with $\mathbf{A}^{(q)}$ and $\mathbf{B}^{(q)}$ replaced by $\hat{\mathbf{A}}_{it}^{(q)}(n)$ and $\hat{\mathbf{B}}_{it}^{(q)}(n)$.
   $$\hat{\mathbf{u}}_{it}(n) = \mathbf{D}_{it}(n)^{\dagger}\mathrm{vec}\,(\mathbf{Y}_3(n)) \quad (42)$$
   Improvement of the symbol estimation using the Vandermonde structure.
   (c) Channel and code estimation using Eq.(41)
   (d) Return to step 3b until convergence, with $it = it + 1$.
   (e) Projection of the estimated symbols onto the alphabet used by each user.
   (f) Return to step 3a with $n = n+1$ if $n \leq N$, otherwise stop.

Table 2: Semi-blind channel/code/symbol/estimation algorithm.

6. SIMULATION RESULTS

We now present some Monte Carlo simulation results to illustrate the performance of the proposed receivers. The transmitted symbols are 4-PSK modulated. The number of users and the channel memory are $Q = 2$ and $I = 3$, respectively. The spreading codes are first assumed to be known at the receiver, the code matrices $\mathbf{B}^{(q)} \in \mathscr{C}^{P\times J}$, $q = 1,\cdots,Q$, being Fourier matrices, with $P = JQ$, in the blind case. In the semi-blind case, the components of the

M \ SNR (dB)	0	10	20	30	40
2	16.23	11.20	11.92	11.49	15.58
3	4.38	1.68	3.17	0	0
4	1.25	0	0	0	0
5	0.82	0	0	0	0

Table 3: Blind receiver with known codes. SER versus SNR for $M \in \{2,3,4,5\}$ and $J = 2$.

J \ SNR (dB)	0	10	20	30	40
1	64.25	65.10	65.82	67.21	69.23
2	4.38	1.68	3.17	0	0
3	3.46	5.50	5.19	5.77	5.77
4	4.59	0.05	8.70	10.65	11.88

Table 4: Blind receiver with known codes. SER versus SNR for $J \in \{1,2,3,4\}$ and $M = 3$.

code matrices are randomly generated from a 16-QAM alphabet $\{1+2n+j\cdot(1+2k)|n,k=-2,-1,0,1\}$, with $P = 4$. The Monte Carlo simulations were carried out with 10 different randomly generated channel models, and 10 additive complex white Gaussian noises for each model. The performance is evaluated in terms of symbol error rate (SER). From these simulation results, we can conclude that the nonlinear coding provides a very important performance improvement.

Tables 3 and 4 show the SER obtained with the blind receiver for $\{J = 2; M = 2,3,4,5\}$ and $\{M = 3; J = 1,2,3,4\}$, after processing of $N = 20$ blocks, each one corresponding to $K = 10$ transmitted symbols per user. As expected, the SER decreases when the antenna number increases, and it is quasi constant for a SNR greater than 10 dB. The best compromise is obtained for $J = 2$ and $M = 3$. Tables 5 and 6 show the SER obtained with the semi-blind receiver for $\{J = 2; M = 2,3,4,5\}$ and $\{M = 3; J = 1,2,3,4\}$, respectively. From these simulation results, we can conclude that the proposed semi-blind receiver provides very good performance with J = 2 and $M = 3$ for any SNR equal to or greater than 10 dB.

It is to be noticed that the SER was averaged over 65 % of the experiments corresponding to the best SERs in the case of the blind receiver, while it was averaged on all the Monte Carlo simulations in the semi-blind case.

7. CONCLUSION

In this paper, a nonlinear coding has been proposed for MIMO CDMA communication systems. Such a coding allows to define, for each user, a third-order input tensor with a constrained structure such as two types of matrix slices have a Vandermonde form whereas the third one has an Hankel structure. The received signals tensor satisfies a constrained block-Tucker2 model that is unique. Assuming that the users' code matrices are mutually orthogonal and known at the receiver, an ALS-based blind channel and symbol estimation method has been derived. Then, in the case of unknown codes, a semi-blind receiver has been proposed for jointly estimating the channels, codes and symbols of all the users. Both the blind and semi-blind solutions take the constrained structure of the core tensors into account, which allows to get very good performances in terms of symbol recovery, as illustrated by simulations.

Several perspectives of this work can be drawn, as for instance the development of adaptive methods that take all the constrained structure of the input tensors into account, i.e. both the Hankel and Vandermonde structures. An optimization of the code matrices is also a topic for future work. More general multiantenna/multicode transmission and multipath propagation scenarii, recently introduced for MIMO CDMA systems ([3], [4], [5], [6]) will be also considered for MIMO NL-CDMA systems.

M \ SNR (dB)	0	10	20	30	40
2	40.91	1.34	0.03	0	0
3	16.14	0.20	0	0	0
4	9.20	0.09	0	0	0
5	4.36	0	0	0	0

Table 5: Semi-blind Receiver with code estimation. SER versus SNR for $M \in \{2,3,4,5\}$ and $J = 2$.

J \ SNR (dB)	0	10	20	30	40
1	18.67	0.34	0	0	0
2	16.14	0.20	0	0	0
3	70.78	27.53	0.67	0.03	0
4	73.72	73.55	51.28	21.78	29.40

Table 6: Semi-blind receiver with code estimation. SER versus SNR for $J \in \{1,2,3,4\}$ and $M = 3$.

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