

COOPERATIVE LOCALISATION IN WIRELESS SENSOR NETWORKS USING COALITIONAL GAME THEORY

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ABSTRACT

Positioning in Wireless Sensor Networks is a key feature in many applications. Finding efficient algorithms to perform this task is of practical importance in systems where limitations on the computational power and battery life are a major issue. Forming coalitions within the set of visible nodes to a target can help reduce communication costs. We can then formulate the problem as a coalitional game where cooperation does not come for free.

1. INTRODUCTION

Wireless Sensor Networks (WSN) are receiving high interest in many communication problems and applications. The main features of WSN's are that of low-cost nodes with limited resources both in terms of computational power and battery whose purpose is sensing the environment (i.e. temperature, humidity, motion, etc). Additional requirements of accuracy or scalability may be also needed for some applications. These constraints have motivated an intensive research for efficient algorithms in WSN, offering many challenging problems in a wide range of applications. One of such problems is source localisation which has emerged as a key feature in applications like tracking, monitoring or surveillance among others. In the literature [1, 2] we can find a vast variety of applications where location plays a key role. As mentioned earlier, energy efficiency is a major issue in WSN so it will be interesting to reduce the energy consumption of the network in order to increase the network's lifetime. Using only a small subset of nodes within the set of all available nodes can suffice for getting a similar performance as considering the whole set. This is due to the fact that data may be correlated among closely located nodes. Further, in some situations it could also be beneficial to remove some of the sensing nodes due to failures and biases in the measurements that can lead to an increase in the overall error.

One way of forming small subsets within the whole set of nodes is to use game-theoretic tools. Game theory is the branch of mathematics that deals with the interaction of independent (intelligent, selfish) agents [3]. There are two different approaches in game-theory depending on whether the interest is focused on the individual agents

(non-cooperative) or in a coalition / group of agents (cooperative). We are interested in the later one (cooperative) where agents benefit from the cooperation with other agents by forming coalitions. Some of the applications where it has been applied are beamforming for interference minimization, code and power selection, rate optimization, spectrum sharing in cognitive radio, cross-layer optimization or infrastructure management among others, see [5, 4] and references therein.

In this contribution we propose a cooperative localisation and tracking scheme using game-theoretic tools. In particular, sensors are modeled as selfish agents that try to locate and track the target. For that purpose, nodes will organise into groups or coalitions that will perform the localisation task. The formation of coalitions is done following the general framework presented in [6] having into account the cost for cooperation. At the end of the coalition formation process several coalitions will result that will be tracking the target. For further energy saving only the best coalition is kept while the other coalitions will be allowed to go to sleep. This process will repeat over time in order to adaptively follow the motion of the target.

The paper is organised as follows: in Section II we present the system model. The formation of coalitions is treated in Section III and some simulations are provided in Section IV. The paper concludes in Section V.

2. SYSTEM MODEL

Through this section we describe the system model and the network behaviour. Let us assume a WSN formed by N nodes and assume the presence of a target node whose position is to be tracked. The target node broadcasts messages and a number of K nodes can hear it. Then, this subset of nodes tries to position the target. Locating nodes are not allowed to communicate for free, instead a communication cost between nodes is present. The cost is a measure of the power consumed by the communication between nodes and is proportional to the distance (at some power) between communicating entities.

2.1 Locating nodes

For getting estimates of the target position, nodes employ distance estimates. We use the standard lognormal model [1] that uses Received Signal Strength Indicator (RSSI) measurements to get distance estimates. In a real world scenario a calibration phase will be needed in order to build up a lookup table between RSSI and distance. For simplicity, we assume that the RSSI follows a linear relationship with the received power P_R . Hence, the received power follows a log-normal distribution [1] with a distance-dependent mean as

$$P_R[\text{dB}] = P_0 - 10 n_p \log_{10} \left(\frac{d}{d_0} \right) + X, \quad (1)$$

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where P_0 is the received power (in dB) at reference distance d_0 , n_p is the path-loss exponent and X is a Gaussian random variable of zero mean and variance σ_{dB} . Let us denote $P_{R,i}$ as the measured power at the i -th locating node. The maximum likelihood estimate of the distance to the target is then given by

$$d_i = d_0 10^{\left(\frac{P_0 - P_{R,i}}{10 n_p}\right)}. \quad (2)$$

Once each node has its own estimation of the distance to the target, the position estimation problem can be formulated as a linear problem [1] in the form $\mathbf{H} \mathbf{x} = \mathbf{b}$, where $\mathbf{x} = [x_0, y_0]^T$ is the target position vector and \mathbf{H} and \mathbf{b} are given by

$$\mathbf{H} = \begin{bmatrix} x_2 & y_2 \\ x_3 & y_3 \\ \vdots & \vdots \\ x_M & y_M \end{bmatrix}, \quad (3)$$

$$\mathbf{b} = \frac{1}{2} \begin{bmatrix} K_2^2 - d_2^2 + d_1^2 \\ K_3^2 - d_3^2 + d_1^2 \\ \vdots \\ K_M^2 - d_M^2 + d_1^2 \end{bmatrix}, \quad (4)$$

where $[x_i, y_i]^T$ are the node coordinates, d_i are the distance estimates at node i and $K_i^2 = x_i^2 + y_i^2$ and M is the number of nodes that are performing the localization. Without loss of generality, $d_1 > d_2 > \dots > d_M$ and node 1 is assumed to be at position $[0, 0]^T$. If the number of nodes is greater than 3, matrix \mathbf{H} is a tall matrix. The position estimate can be then calculated as

$$\hat{\mathbf{x}} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{b}. \quad (5)$$

With the distance estimator of (2), the position estimate (5) is biased. It can be easily shown that in order to get an unbiased estimate of the position the distance estimate should be modified to

$$d'_i = d_i e^{-\sigma_i^2} = d_0 10^{\left(\frac{P_0 - P_{R,i}}{10 n_p}\right)} e^{-\sigma_i^2}, \quad (6)$$

where $\sigma_i = \frac{\log(10)\sigma_{dB,i}}{10 n_p}$ and $\sigma_{dB,i}$ is the variance of the received power $P_{R,i}$ [dB]. With distance estimator (6) the estimated position of the target can be expressed as

$$\hat{\mathbf{x}} = \mathbf{x} + \mathbf{e}, \quad (7)$$

where \mathbf{e} is a zero-mean random variable that represents the position estimation error. Further, as the number of locating nodes increases the distribution of \mathbf{e} approaches a Gaussian distribution by the Central Limit Theorem. For simplicity reasons it is also assumed that communication channel between nodes is ideal and no errors occur when exchanging information among neighboring nodes.

2.2 Target movement

The target node can be placed at any arbitrary position in the network area. It is assumed that the target moves freely through the network by following a random force movement [7] given by

$$\begin{aligned} \mathbf{x}_{t+1} &= \mathbf{x}_t + \mathbf{v}_t T + \frac{1}{2} \mathbf{a}_{t+1} T^2 \\ \mathbf{v}_{t+1} &= \mathbf{v}_t + \mathbf{a}_{t+1} T, \end{aligned}$$

where \mathbf{x}_t is the target position at time instant t , \mathbf{v}_t is the target speed, \mathbf{a}_t is the acceleration and T is the elapsed

time between consecutive samples. It is assumed that the target is initially at some position $\mathbf{x}_0 = [x_0, y_0]^T$ with initial speed of $\mathbf{v}_0 = [v_0^x, v_0^y]^T$ and that the acceleration follows a Gaussian distribution, $\mathbf{a} \sim \mathcal{N}(\mathbf{0}, \sigma_a^2 \mathbf{I})$. Using the above state transition model and assuming that the position estimate (5) is a noisy measurement of the true distance as in (7), we can employ the Kalman filter for target tracking.

It is worth to mention that instead of using the Kalman filter on the joint estimate we could employ other tracking strategies based, for example, on particle filtering.

3. FORMING COALITIONS

In this section we present an algorithm for forming coalitions using game theoretical tools and having in mind communication costs. Let us first review some concepts of coalitional game theory and then analyze the properties of the game at hand.

3.1 Coalitional Game-theory

A coalitional game $\mathcal{G}(K, v)$ is defined by a set of players K and an utility function $v(\cdot)$ that assigns a value to a coalition of players in the set K . We will refer only to games in characteristic form, that is, games in which the value of a coalition depends solely on the members of that coalition and not on the members outside the coalition. Within games in characteristic form we can differentiate between Transferable Utility (TU) games and Non-Transferable Utility games (NTU). In TU games the utility function $v(\cdot)$ maps to a real number and this coalition value can be arbitrarily apportioned between the members of the coalition. In some sense it is like having a common currency for all players. In NTU games, however the payoff each player receives within one coalition depends on the joint actions that players take. In general, the utility function of NTU games does not map into a single real value but into a set of payoff vectors [3].

A common assumption made in classical coalitional game theory problems is that the game is superadditive, that is $v(A \cup B) \geq v(A) + v(B)$, where A and B are two disjoint coalitions. This means that increasing the size of the coalition is always beneficial and hence, the grand coalition will form (i.e. the coalition of all players).

In many problems however, superadditivity does not hold due to the introduction of some costs associated with the formation of coalitions. Under this setting the grand coalition will not form due to cooperation costs. The problem of how to form coalitions becomes the main challenge in such situations. In [6] a general framework for coalition formation is proposed that has been applied to wireless communication problems like beamforming and cognitive radio [8]. We follow the general framework of [6] but applied to our positioning game.

3.2 Positioning game

From the perspective of coalitional game theory (coalition formation) we can interpret our problem as a game where nodes want to locate the target node with the lowest possible error and with the lowest cost in terms of consumed power. As all nodes within the locating coalition will end up with the same position estimate and given the fact that the consumed power depends on the relative positions of the nodes, our positioning game is of clear NTU nature. We consider that the elements (players) of the game are those nodes detecting the signal of the target. Within this subset several coalitions will form based on the merge and split operations [6]. Within a coalition we assume the presence of a coalition head that acts as fusion center and performs the estimation

and tracking tasks (i.e. the one with the lowest distance to the target at the coalition formation phase).

For forming coalitions it becomes necessary to define a suitable utility function that accounts for the trade off between performance and costs. We define the value of a coalition $A \in K$ to be

$$v(A) = \begin{cases} 0 & |A| \leq 1 \\ \delta & |A| = 2 \\ (1 - \eta)Q(A) - \eta C(A) & \text{otherwise} \end{cases}$$

where the function $Q(A)$ represents a quality indicator of the coalition A and the function $C(A)$ is the associated cost function. The parameter $0 \leq \eta \leq 1$ controls the compromise between cost and performance. In our case we select the quality function to be based on the discrepancies that every node experiences between its measured distance and the final joint estimated distance within the coalition. The cost function is selected to be related to the energy consumption necessary for communication. The number $|A|$ represents the cardinality of the coalition A . As localization can only be done if three nodes are hearing the target, the value of a singleton coalition is 0. The value of a two-element coalition will be a small positive number $0 < \delta \ll 1$. This will allow the formation of bigger coalitions.

For $|A| \geq 3$ the coalition value (utility) is given by

$$v(A) = (1 - \eta) \underbrace{\left[|A| - \sum_{i \in A} \frac{(d'_i - \hat{d}_i)^2}{R^2} \right]}_{Q(A)} - \eta \underbrace{\sum_{i \in A} \frac{d_{ih}^2}{R^2}}_{C(A)}, \quad (8)$$

where d'_i is the estimated distance of the i -th node (6), \hat{d}_i is the distance of the i -th node to the joint estimated position of the target, R is the coverage range of the target (i.e. maximum distance covered by the target) and h is the index of the coalition head (fusion center). Then, the payoff that each node $j \in A$ receives is

$$\phi_j(A) = (1 - \eta) \left[1 - \frac{(d_j - \hat{d}_j)^2}{R^2} \right] - \eta \frac{d_{jh}^2}{R^2}. \quad (9)$$

It can be clearly observed that the utility will increase as locating nodes are closer (less cost) and as the discrepancy between measurement and estimation becomes smaller (less error). It is worth to mention that nodes that exhibit a high discrepancy (i.e. due to a bias in the estimation) are less likely to join a coalition.

3.3 Algorithm

The formation of the locating - tracking coalitions is achieved in a distributed manner by the application of two basic rules [6]:

$$\text{MERGE } \{C_1, \dots, C_i\} \text{ if } \left\{ \bigcup C_j \right\} \triangleright \{C_1, \dots, C_i\} \quad (10)$$

$$\text{SPLIT } \left\{ \bigcup C_j \right\} \text{ if } \{C_1, \dots, C_i\} \triangleright \left\{ \bigcup C_j \right\}, \quad (11)$$

where $\{C_1, \dots, C_n\}$ is a collection of coalitions and \triangleright represents an ordering criterium. For this particular game the ordering criterium selected is the Pareto order on the individual payoffs. The Pareto order is defined as

$$(k_1, \dots, k_n) \succ_p (l_1, \dots, l_n) \text{ if } k_i \geq l_i \text{ and } \exists k_i > l_i, i \in \{1, \dots, n\},$$

where k_i, l_i are real numbers. With these considerations, coalitions will form by successive application of merge and

Algorithm 1 System operation

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while Target is in coverage area do
  Each locating node measures the distance to the target
  if elapsedTime > sleepInterval then
    repeat
      Merge coalitions
      Split coalitions
    until Merge & Split terminates
    Select the best coalition and set all remaining coalitions to sleeping state
    elapsedTime  $\leftarrow$  0
  else
    Tracking (KF) within the best coalition
    elapsedTime  $\leftarrow$  elapsedTime + 1
  end if
end while

```

split rules using the Pareto order. Iterative application of merge and split operations is guaranteed to converge [6]. Furthermore, the procedure of forming coalitions can be done in a fully distributed manner at both coalition and individual level. However, this distributed implementation comes at the cost of increasing signalisation overhead. It is worth mentioning that nodes can handle variations in the environment such as mobility of the target by continuously refreshing the coalition structure. However, this will imply a high amount of communication overhead that will reduce the efficiency of the system. Instead, nodes can dynamically adapt to environmental changes by reducing the rate at which coalitions are formed. Obviously, this will imply a reduction in terms of error performance but it will also imply a reduction in the communication overhead. A trade-off solution between cost and performance should be made. Keeping in mind these considerations the general operation of the system will be the following: Initially nodes that are visible to the target (i.e. lie within the coverage area of the target signal) perform a measure of the received power. Each node translates its local measured value into distance by (6). If the elapsed time since last coalition formation is greater than the sleeping time, then current coalitions begin with the process of coalition formation by attempting to merge with nearby coalitions following (10). Afterwards any formed coalition is subject to a split operation into smaller coalitions if (11) is satisfied. This process is repeated until convergence, that is merge and split do not produce any new partition of the visible set. Once a final partition has been reached only the coalition with the highest utility will remain while all others will go to sleeping state in order to save energy. The winner coalition will perform the positioning and tracking task during the next sleeping interval. For simplicity, we have implemented the target tracking task in a centralized manner by using a Kalman filter. A coalition head will be selected as the node with the shortest distance estimate and all the other nodes within the coalition will communicate their measured distances to the coalition head. Once all data has been collected the target location is estimated and used to feed the Kalman filter. It is worth noting that the same approach can be done in a fully distributed way by using any distributed version of the Kalman filter (i.e. consensus based). After the tracking time, the current coalition structure will be revisited and the described process will be repeated. An illustration of the network behaviour is depicted in Figure 1 where locating nodes are the blue circles and the target node is a red square. The red line represents the true trajectory of the target while the dashed green line is the estimated position. Yellow faced nodes are the current tracking coalition. The pseudo-code of the system operation is described in Algorithm 1.

3.4 Stability remarks

The application of the proposed algorithm will end up with a sequence of tracking coalitions that evolve as the target moves through the network. Let us give first some clarifying definitions about nomenclature. The set of players $K = \{1, \dots, |K|\}$ is called the grand coalition and any non-empty subset of K is a coalition. A collection is any family $\mathcal{C} = \{C_1, \dots, C_l\}$ of mutually disjoint coalitions. Further, a collection \mathcal{C} is a partition of K if $\bigcup_{j=1}^l C_j = K$.

The stability of the coalition formation process is analysed using the concept of a defection function \mathbb{D} that assigns to each partition of the grand coalition K , some partitioned subset of the grand coalition [6]. Before ongoing with the analysis let introduce the notation $\mathcal{C}[\mathcal{P}]$ (collection \mathcal{C} in the frame of partition $\mathcal{P} = \{P_1, \dots, P_m\}$) as

$$\mathcal{C}[\mathcal{P}] = \{P_1 \cap \bigcup_{j=1}^l C_j, \dots, P_m \cap \bigcup_{j=1}^l C_j\} \setminus \{\emptyset\}.$$

With the above notation and assuming an ordering criteria \triangleright , a partition \mathcal{P} is said to be \mathbb{D} -stable [6] if:

$$\mathcal{C}[\mathcal{P}] \triangleright \mathcal{C} \text{ for all } \mathcal{C} \in \mathbb{D}(\mathcal{P}).$$

Two natural defection functions are \mathbb{D}_p which allows formation of all partitions within the grand coalition and \mathbb{D}_c , which allows formation of all collections in the grand coalition. Thus a partition \mathcal{P} is \mathbb{D}_p -stable if there is no partition \mathcal{T} of K such that $\mathcal{T} \triangleright \mathcal{P}$. Analogously, a partition \mathcal{P} is a \mathbb{D}_c -stable if for all collections $\mathcal{C} \in K$ it is satisfied that $\mathcal{C}[\mathcal{P}] \triangleright \mathcal{C}$. These definitions of stability are related to the Merge and Split rules by virtue of the following theorem [6].

Theorem 3.1. *Suppose that \triangleright is a comparison relation and \mathcal{P} is a \mathbb{D}_c -stable partition. Then*

- \mathcal{P} is the unique outcome of every iteration of the merge and split rules
- \mathcal{P} is a unique \mathbb{D}_p -stable partition
- \mathcal{P} is a unique \mathbb{D}_c -stable partition

As an immediate consequence of Theorem 3.1 we have that in the coalition formation phase of the proposed algorithm we converge to the unique \mathbb{D}_c -stable partition if such a partition exists. Two necessary and sufficient conditions for the existence of a \mathbb{D}_c -stable partition are given in [6]. However, in our particular problem such existence can not be always guaranteed as it depends on the geometry of the network (positioning nodes). If the case is such that no \mathbb{D}_c -stable partition exists then the algorithm converges to a stable partition with respect of the merge and split rules (i.e. the algorithm is guaranteed to have a finite runtime).

4. SIMULATIONS

In this section we present some numerical results in order to evaluate the performance of the proposed locating algorithm. A total of 100 nodes have been randomly deployed into an area of $50 \times 50 \text{ m}^2$ as shown in Figure 1. A number of 100 random trajectories have been simulated. The target is set to a random position on the boundaries of the locating area with an initial speed of $|v_0| = 0.2 \text{ m/s}$ and acceleration standard deviation $\sigma_a = 0.01$. For the propagation model a reference distance $d_0 = 1 \text{ m}$ have been taken with reference power $P_0 = 0 \text{ dBm}$. The path-loss exponent is $n_p = 2$ and $\sigma_{\text{dB}} = 2$. It is assumed that all nodes have the same parameters for the propagation model and that the coverage range of the target is 10 m. It is also assumed that each

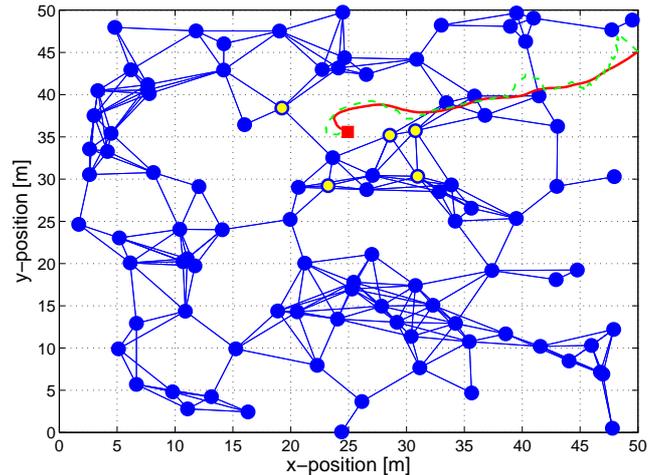


Figure 1: Illustrative example of the network operation

node performs average a number of ten measurements before locating / tracking the target. A value of $\eta = 0.65$ has been used for the simulations. For comparison purposes we have implemented a base solution where all the nodes within the range of visibility of the target (i.e. the grand coalition) cooperate and jointly perform positioning and tracking. For computation of the overhead cost due to the coalition formation process we have associated with any merging attempt between two coalitions a cost proportional to the squared distances between the centers of the coalitions. If merging is successful also the necessary broadcasting messages have been taken into account. This will provide us an approximation to the average costs involved in the merging operation. The splitting operation is assumed to be performed by the coalition head (in a centralised way) and hence, only successful splitting operations will result in a overhead cost proportional to the squared distance between splitted coalitions.

In Figure 2 it is displayed the average estimation error versus the time rate at which coalitions are rebuilt. As it can be observed the coalition formation approach exhibits (as expected) a higher mean error as the solution where all nodes cooperate. This is obviously due to the fact that not all nodes that hear the target cooperate but only a subset of them. However, if we have a look at the average cooperation costs depicted in Figure 3 it can be realised that the proposed scheme provides a better performance than the base solution despite the generated overhead even for a small duration of the sleep interval. In Figure 4 it is depicted an example of the cost evolution over time for both the grand coalition and for the tracking coalition. Peaky regions correspond to the process of coalition formation that are repeated every sleep interval.

5. CONCLUSIONS AND FUTURE DIRECTIONS

We have presented a coalition formation algorithm for localization and tracking purposes. Estimates of the target location have been made by forming coalitions over time as the target moved. The tracking part has been implemented by relying on a fusion center but a fully distributed can also be implemented. The stability of the algorithm has been analysed and the benefit in terms of communication cost at the expense of some performance degradation has been demonstrated via simulation. A characterisation of the effects of

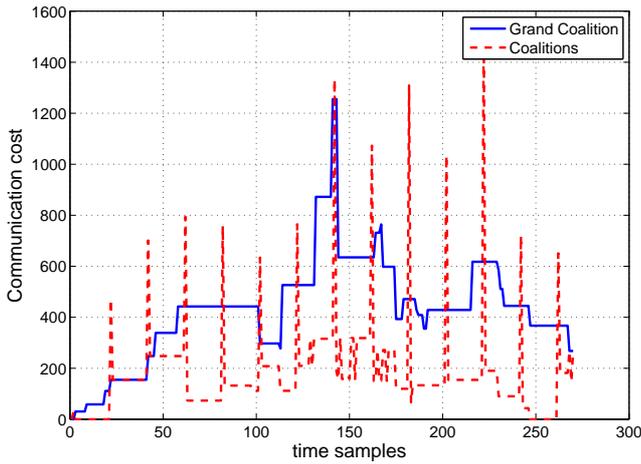


Figure 4: Cost evolution for a sample trajectory of the target

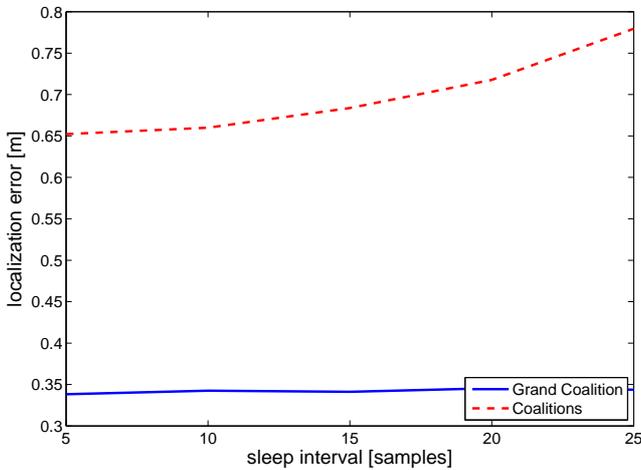


Figure 2: Average localization error

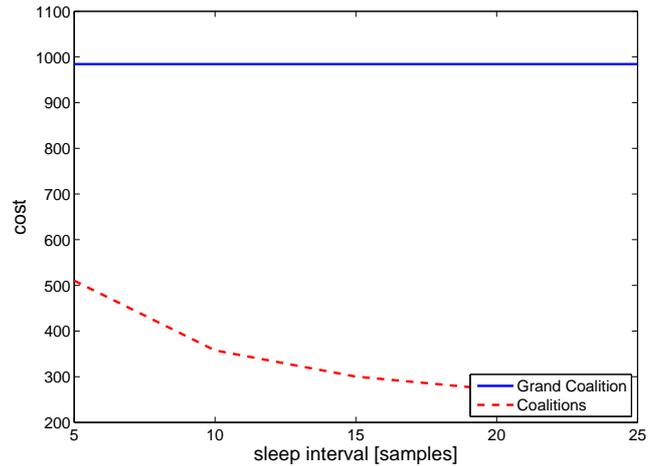


Figure 3: Average cost of communication

the different parameters at hand into the locating algorithm and a fully distributed implementation are our next steps as well as the extension to more general scenarios.

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