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PERFORMANCE ANALYSIS OF TURBO-EQUALIZED SYSTEMS OVER FREQUENCY-SELECTIVE BLOCK-FADING CHANNELS

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ABSTRACT

In this paper [1], we investigate the finite-length system performance of turbo-equalized systems over frequency-selective block-fading channels for packet data transmissions. First, we derive a tight upper bound on the frame error rate (FER) performance of the system under maximum-likelihood sequence estimation based on an approximation of the Euclidean distance distribution at the output of the noiseless inter-symbol interfering channel. Then, we show that the intersymbol interference can be viewed as an additional fading factor depending on the channel selectivity, but also on the coding scheme and the interleaver type. By introducing the interference-to-fading ratio, we evaluate the impact of the interference in the performance degradation compared to the effect of the fading. We show that a two-stage interleaver structure is needed to achieve coding diversity and turbo-equalization gain.

1. INTRODUCTION

In data packet communication systems over frequency-selective channels, turbo-equalization [1] is an efficient technique that combines signal detection and error correction in an iterative scheme leading to substantial gains in intersymbol interference (ISI) mitigation in comparison with systems using disjoint signal detection and correction. System performance is usually addressed in terms of bit error rate (BER) in the context of static ISI channels using iterative detection and decoding approaches based on EXIT chart analysis [2] assuming infinite coded sequence [3]. However, in data packet communication systems, a finite packet length is used, and system performance is actually measured in terms of frame error rate (FER) rather than of BER. Finite length system performance can be predicted by the analytic assessment of the corresponding maximum likelihood (ML) receiver.

The performance of maximum likelihood sequence estimation was first analyzed by Forney [4, 5] for uncoded transmission over static ISI channels. Upper and lower bounds were derived based on the Euclidean distance distribution at the output of the noiseless ISI channel. This distribution is estimated using a trellis-based approach using the state diagram of the channel to calculate the transfer function of the channel. In [6], the authors proposed to upper bound the performance of a serially concatenated turbo-coded system, assuming *uniform interleaving*. This approach was applied to a turbo-equalized system by viewing the ISI channel as a rate-1 trellis encoder. This approach has been applied in [7, 8] to the case of turbo-equalized systems over partial response channels. However, for a multipath-fading channel, where the channel is time-variant, trellis-based approaches can not be applied, because the transfer function depends on the particular channel realization. For this type of channels finite length system performance was essentially studied by means of numerical simulations like in [9].

In this paper, we propose a novel approach for the evaluation of the Euclidean distance distribution at the output of uncorrelated block fading ISI channels, based on the autocorrelation functions of the error sequence and channel response. We found evidence that union bounds provide tight FER approximation (eventhough the bounds are loose for BER evaluation) of turbo-equalized systems with finite frame length. In particular, we show that a two-stage interleaver is needed for the block diversity transmission scheme to achieve simultaneously coding diversity and turbo-equalization gain.

The remaining of this paper is organized as follows. In Section 2, we introduce the turbo-equalized system model with block diversity. In Section 3, we address the problem formulation. In Section 4, we investigate the distribution of the Euclidean distance at the output of noiseless ISI fading channel and we derive an approximation of this distribution in order to evaluate a tight upper bound on the FER performance. In Section 5, we discuss the diversity issues in the turbo-equalized system and the interleaver structure. Finally, conclusions are given in Section 6.

2. SYSTEM MODEL

We consider a bit-interleaved coded communication system with B-blocks diversity as shown in Figure 1. A sequence of K information bits $\mathbf{d}$ is encoded into a sequence $\mathbf{c}'$ of N coded bits using a rate $R = K/N$ linear error correcting code (ECC). The coded sequence of bits is then interleaved using a random interleaver of length N which in turn is mapped into a sequence of symbols $\mathbf{x}$ using a binary phase shift keying (BPSK) modulation alphabet $\{-1,+1\}$. The modulated sequence $\mathbf{x}$ is then divided into B blocks $(\mathbf{x}_1,\cdots,\mathbf{x}_B)$ of equal length $M = N/B$, assumed to be an integer. Each block of symbols $\mathbf{x}_b = (x_{b,1},\cdots,x_{b,M})$ for $1 \le b \le B$ is transmitted through an ISI channel modeled by its equivalent discrete time finite impulse response of length L denoted by $\mathbf{h}_b = (h_{b,0},...,h_{b,L-1})$, which is assumed to be constant over a block period but changes from one block to another. We assume that channel tap coefficients are modeled by real-valued independent and identically distributed (i.i.d.) Gaussian random variables with zero mean and variance $1/L$. According to this model, the received sequence samples corresponding to the b-th transmitted block are given by,

$$r_{b,m} = \sum_{\ell=0}^{L-1} h_{b,\ell} x_{b,m-\ell} + w_{b,m},\ m = 1,\cdots,M,$$

where $w_{b,m}$ is an independent additive white Gaussian noise (AWGN) with variance σ_w^2. The average signal to noise ratio (SNR) is defined by $\rho = E_s/N_0 = E_s/(2\sigma_w^2)$. At the receiver side, we consider a turbo-equalizer for iterative detection and decoding with perfect channel state information. In order to characterize the FER, we extend the union bound on the FER under maximum likelihood sequence detection and decoding initially proposed by Benedetto [6] for serially concatenated codes.

3. MAXIMUM-LIKELIHOOD PERFORMANCE

In this section, we present a novel approach to extend Benedetto's upper bound to the case of ISI channels. We apply the proposed approach to uncorrelated time variant channels. Let $\mathscr{X}$ denote the

[1]This work was supported by the project "Urbanisme des Radiocommunications" of the Pôle de compétitivité SYSTEM@TIC.

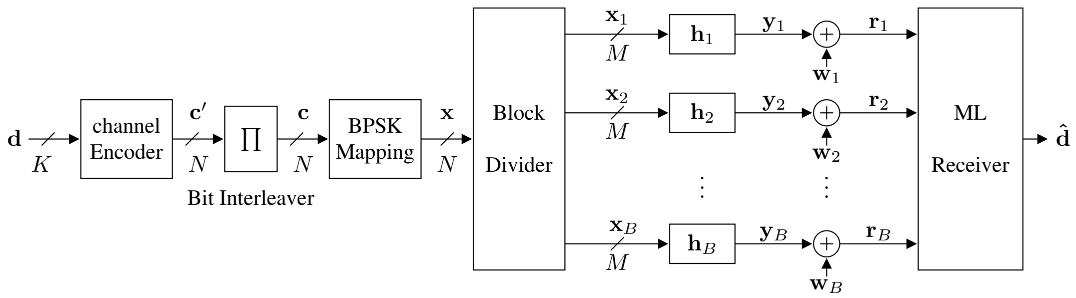

Figure 1: Model of a turbo-equalized system over frequency-selective block-fading channels.

ensemble of the 2^K possible transmitted sequences. The maximum likelihood receiver estimates the transmitted sequence $\mathbf{x}$ by the coded sequence $\hat{\mathbf{x}}$ taken from $\mathscr{X}$ given by

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in\mathscr{X}} \sum_{b=1}^{B} ||\mathbf{r}_b - \mathbf{h}_b * \mathbf{x}_b||^2.$$

For a given set of channel realizations $\mathscr{H} = \{\mathbf{h}_1, \cdots, \mathbf{h}_B\}$, the pairwise error probability (PEP) between two arbitrary coded sequences $\mathbf{x}$ and $\hat{\mathbf{x}}$ is given in [4] by

$$P_2(\mathbf{x},\hat{\mathbf{x}}|\mathscr{H}) = Q\left(\sqrt{d_E^2(\mathbf{x},\hat{\mathbf{x}}|\mathscr{H})\rho/2}\right), \tag{1}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{+\infty} e^{-t^2/2}dt$, and

$$d_E^2(\mathbf{x},\hat{\mathbf{x}}|\mathscr{H}) = \sum_{b=1}^{B} d_E^2(\mathbf{x}_b,\hat{\mathbf{x}}_b|\mathbf{h}_b), \tag{2}$$

is the squared Euclidean distance between $\mathbf{x}$ and $\hat{\mathbf{x}}$ at the output of noiseless ISI block-fading channels which depends on the error sequence between the pair of coded sequences $\mathbf{e} \triangleq \hat{\mathbf{x}} - \mathbf{x}$. Therefore, the Euclidean distance will be denoted in the rest of this paper as a function of the error sequence as $d_E^2(\mathbf{e}|\mathscr{H})$. It can be evaluated within each block by

$$d_E^2(\mathbf{e}_b|\mathbf{h}_b) \triangleq \|\mathbf{h}_b * \mathbf{e}_b\|^2 = \sum_{m=1}^{M+L-1}\left|\sum_{\ell=0}^{L-1} h_{b,\ell}e_{b,m-\ell}\right|^2, \tag{3}$$

where the first sum on the right hand side is performed over $M+L-1$ symbols to take account of the dispersive nature of the ISI channel. Moreover, we assume that symbols outside a block period are known by the receiver and consequently the corresponding error values are assumed to be identically zeros i.e. $e_{b,m} = 0$ for $m \notin [1,M]$.

Assuming a uniform interleaver, and by extending the results of [6], the union bound on the average FER performance of the maximum-likelihood receiver, denoted by P_{sub}, conditioned on a particular set of channel realizations $\mathscr{H}$ is given by

$$P_{sub}(\rho|\mathscr{H}) \triangleq \sum_{d_E^2} A(d_E^2|\mathscr{H}) \times Q\left(\sqrt{d_E^2\rho/2}\right), \tag{4}$$

where $A(d_E^2|\mathscr{H})$ is the output squared Euclidean distance enumerator of the concatenated system defined by,

$$A(d_E^2|\mathscr{H}) \triangleq \sum_{d=d_{min}}^{N} A^c(d) \times \frac{A_d^{ch}(d_E^2|\mathscr{H})}{\binom{N}{d}}, \tag{5}$$

where d_{min} is the minimum free distance of the ECC, $A^c(d)$ is the output weight enumerator of the ECC defined as the total number of binary error sequences with Hamming weight d, and $A_d^{ch}(d_E^2|\mathscr{H})$ is the input-output Euclidean distance enumerator of the ISI channel, defined as the average number of binary error sequences with Hamming weight d and output squared Euclidean distance d_E^2, where the average is taken over $\mathscr{X}$.

Since the error function $Q(.)$ is a rapidly decreasing function, the dominant terms in the union bound are those with low error weight d. The first term $A^c(d)$ depends on the used code and can be evaluated in different ways. Since we are interested in an upper bound, $A^c(d)$ can be upper bounded as in [6] by neglecting the length of the error events compared to codeword length N, as follows

$$A^c(d) \leq \sum_{k=1}^{k_{max}} \binom{RN}{k} \sum_{\substack{d_1,\cdots,d_k \\ d_1+\cdots+d_k=d}} a(d_1)\times\cdots\times a(d_k), \tag{6}$$

where $a(d_i)$ are the output weight enumerators of error events of the ECC and $k_{max} = \lfloor d/d_{min} \rfloor$.

The main problem in evaluating (5) arises from the evaluation of the term $A_d^{ch}(d_E^2|\mathscr{H})$. Most of previous works has addressed this problem in the context of static channels using enumeration techniques, hence computationally prohibitive for large channel memory, like in [10] and references therein. We present in this paper a novel approach in the evaluation of the Euclidean distance distribution which provides an analytical tool to predict channel effects on the Euclidean distance and can be applied for general static channels as well as time variant channels.

We start by first noticing that the quantity

$$P_d(d_E^2|\mathscr{H}) \triangleq \frac{A_d^{ch}(d_E^2|\mathscr{H})}{\binom{N}{d}} \tag{7}$$

can be interpreted as the conditional probability of obtaining a squared output Euclidean distance d_E^2 with a weight d error sequence given $\mathscr{H}$. For the BPSK mapping scheme, the non-zero elements e_n of the error sequence take their values from the ensemble $\{-2,+2\}$ depending on the transmitted symbols x_n in such a way that $\mathbf{x}+\mathbf{e}$ is a valid coded sequence. A given binary error sequence of weight d with fixed error positions, can generate one of 2^d signed error sequences $\mathbf{e}$ depending on the transmitted sequence $\mathbf{x}$. By averaging over $\mathscr{X}$, we assume that each signed error sequence $\mathbf{e}$ can be obtained with equal probability $P = 2^{-d}$ as long as $d << K$. This is equivalent to assuming error elements as independent. The same simplifying assumption was made by [7] justified by the use of a high rate linear code. Under this assumption the conditional probability in the equation (7) can be rewritten as

$$P_d(d_E^2|\mathscr{H}) \triangleq \frac{\tilde{A}_d^{ch}(d_E^2|\mathscr{H})}{2^d\binom{N}{d}} \tag{8}$$

where $\tilde{A}_d^{ch}(\cdot)$ is the *total* number of signed error sequences $\mathbf{e} \in \mathscr{E}_d$ leading to a squared Euclidean distance d_E^2, where $\mathscr{E}_d$ denotes the ensemble of all signed error sequences with Hamming weight d with cardinality $|\mathscr{E}_d| = 2^d \binom{N}{d}$. The independence assumption is traduced in (8) by a probability space extension.

Now, instead of enumerating the quantity $\tilde{A}_d^{ch}(d_E^2|\mathscr{H})$ as in [10] for example, we evaluate directly the probability $P_d(d_E^2|\mathscr{H})$ over the probability space $\mathscr{E}_d$. This is a true probability mass function (pmf) of the discrete random variable d_E^2. By averaging (4) over all possible channel realizations, d_E^2 becomes a continuous random variable with probability density function (pdf) denoted by $f_d(d_E^2)$ and, consequently, the sum over d_E^2 is converted to an integral as follows

$$P_{sub}(\rho) = \int_0^{\infty} \sum_{d=d_{min}}^{N} A^c(d) Q\left(\sqrt{x\rho/2}\right) f_d(x)dx. \tag{9}$$

In order to evaluate the upper bound (9), we must determine the pdf of the squared Euclidean distance over all error sequences and channel realizations. This is the subject of the next section.

4. OUTPUT EUCLIDEAN DISTANCE DISTRIBUTION

Since all transmitted blocks undergo independent channels with the same statistics, we first evaluate the effect of the channel on a single block. By developing the squared sum in (3) and performing some algebraic computations, we can rewrite the squared Euclidean distance for the block b under the form

$$d_E^2(\mathbf{e}_b|\mathbf{h}_b) = R_0(\mathbf{h}_b)R_0(\mathbf{e}_b) + 2\sum_{\ell=1}^{L-1} R_\ell(\mathbf{h}_b)R_\ell(\mathbf{e}_b), \tag{10}$$

where $R_\ell(\mathbf{x})$ is the aperiodic autocorrelation function (AACF) at lag ℓ of the sequence $\mathbf{x}$ of length M defined by

$$R_\ell(\mathbf{x}) \triangleq \sum_{m=1}^{M} x_m x_{m-\ell}, \quad \text{for } |\ell| \leq M-1,$$

with $x_m = 0$ for $m \notin [1, M]$. Note that the first term on the right hand side of (10) is the squared Euclidean distance over an equivalent ISI-free fading channel, whereas the second term includes the effect of ISI on the Euclidean distance. In order to separate the fading effect from the ISI effect on the Euclidean distance, we rewrite (10) as the product of independent random variables as follows

$$d_E^2(\mathbf{e}_b|\mathbf{h}_b) = R_0(\mathbf{e}_b)\Gamma_b(\mathbf{h}_b)\Theta_b(\mathbf{e}_b, \mathbf{h}_b), \tag{11}$$

where

$$\Gamma_b(\mathbf{h}_b) \triangleq R_0(\mathbf{h}_b),$$
$$\Theta_b(\mathbf{e}_b, \mathbf{h}_b) \triangleq 1 + 2\sum_{\ell=1}^{L-1} \tilde{R}_\ell(\mathbf{h}_b)\tilde{R}_\ell(\mathbf{e}_b),$$

where $\tilde{R}_\ell(\cdot)$ denotes the normalized AACF by $R_0(\cdot)$. Under uniform interleaving, the error weight d is distributed randomly over the fading blocks with $\sum_{b=1}^{B} d_b = d$. Conditionally to a given block error weight d_b within the current block, we have $R_0(\mathbf{e}_b) = 4d_b$. The random variable Γ_b is the channel gain which follows a Gamma distribution (independent of b) with shape parameter $\alpha = L/2$ and scale parameter $\beta = 2/L$ with pdf denoted by $f_\Gamma(\gamma)$ given by

$$f_\Gamma(\gamma) = \frac{1}{\beta^\alpha \Gamma(\alpha)} \gamma^{\alpha-1} e^{-\gamma/\beta}, \tag{12}$$

with mean and variance given by

$$\mu(\Gamma_b) = \alpha\beta = 1, \quad \sigma^2(\Gamma_b) = \alpha\beta^2 = \frac{2}{L}. \tag{13}$$

The interference random variable Θ_b can be interpreted as an additional fading factor which quantifies the Euclidean distance fluctuations due to the selectivity of the channel which depends on d_b. To characterize the distribution of the Euclidean distance, we need to characterize the distribution of Θ_b. Unfortunately, Θ_b is a combination of related random variables whose joint probability density function is difficult to derive analytically in general [11]. Therefore, the distribution of Θ_b can only be analytically assessed through an approximation. By contrast, exact expressions for the main statistical characteristics of Θ_b including the mean and the variance, can be derived and will be used for the pdf approximation.

4.1 Mean and variance

The out-of-phase autocorrelation coefficients of the error sequence $R_\ell(\mathbf{e}_b)$ are discrete random variables. It can be easily verified that they are pairwise uncorrelated with zero mean using the fact that error elements are assumed i.i.d. with zero mean. Using this property, it can be shown (demonstration details are omitted for brevity) that the variance is given by

$$\sigma^2(R_\ell(\mathbf{e}_b)) = 16(M-\ell)\frac{d_b(d_b-1)}{M(M-1)}, \ 1 \leq \ell \leq M-1. \tag{14}$$

Similarly, autocorrelation coefficients of the channel $R_\ell(\mathbf{h}_b)$ are continuous random variables which are pairwise uncorrelated. The out-of-phase autocorrelation coefficients $R_\ell(\mathbf{h}_b)$ are zero mean with variance

$$\sigma^2(R_\ell(\mathbf{h}_b)) = (L-\ell)/L^2, \quad 1 \leq \ell \leq L-1. \tag{15}$$

Using (14), (15) and the fact that $R_\ell(\mathbf{e}_b)$ and $R_\ell(\mathbf{h}_b)$ are independent, we can show from (10) and (11) that

$$\mu(\Theta_b) = 1, \ \sigma^2(\Theta_b) = 2\frac{d_b-1}{d_b} \times \frac{L-1}{L+2} \times \frac{3M-L-1}{3M(M-1)}. \tag{16}$$

Note that variance is zero for $L = 1$ or $d_b = 1$, which corresponds respectively to an ISI-free channel or to an isolated error. Also, for $L << M$ the variance of Θ_b decreases linearly with the block size M, whereas the value of the variance is not significantly affected for increasing values the other two parameters d_b and L.

4.2 An approximation for the Euclidean distance distribution

The first step in investigating the distribution of Θ_b is to determine its support. For normalized channel gain ($\Gamma(\mathbf{h}_b) = 1$), it is shown in [12] that the squared Euclidean distance is lower bounded by a certain minimum value $d_{E,min}^2$ which depends on the channel length whatever was the channel response. This gives a lower bound on the interference variable $\theta_{min} = d_{E,min}^2/4d_b$. The maximum value θ_{max} is less important for system performance and lay out of our interests.

Additional information about the shape of the distribution of Θ_b can be obtained by noting that $\Theta_b = 1$ for all error blocks with isolated error elements separated by at least L positions regardless of the channel response, because for these sequences, all out-of-phase AACF $R_\ell(\mathbf{e}_b)$ will be identically zeros. In fact, the probability of such sequences, denoted by P_z, forms a lower bound on the conditional probability of $\Pr(\Theta_b = 1|d_b)$.We find by combinatorial enumerations the value of P_z as

$$\Pr(\Theta_b = 1|d_b) \geq P_z = \binom{M-(d_b-1)(L-1)}{d_b} / \binom{M}{d_b}.$$

which is a decreasing function of d_b. Numerically, for $N = 256$ and $L = 5$ we find $p_z = 0.97, 0.72, 0.21$ for $d_b = 2, 5, 10$, respectively. We approximate the distribution of the remaining error sequences by a truncated Gaussian distribution over $[\theta_{min}, \theta_{max}]$ with the variance $\sigma_{ISI}^2 = \sigma^2(\Theta_b)/(1-P_z)$. Finally, we deduce the conditional

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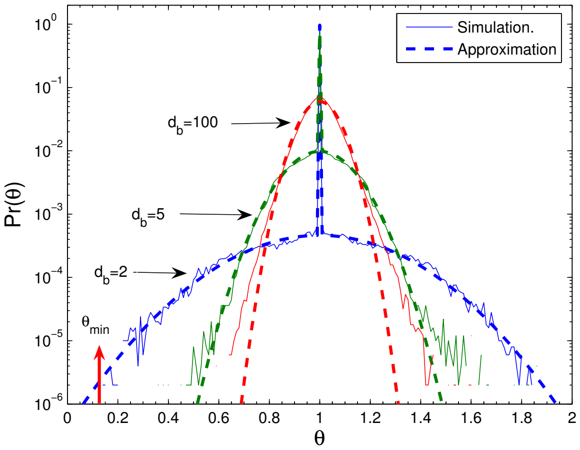

Figure 2: Distribution of the interference random variable Θ_b for $L = 5$ and $M = 256$ for different values of d_b.

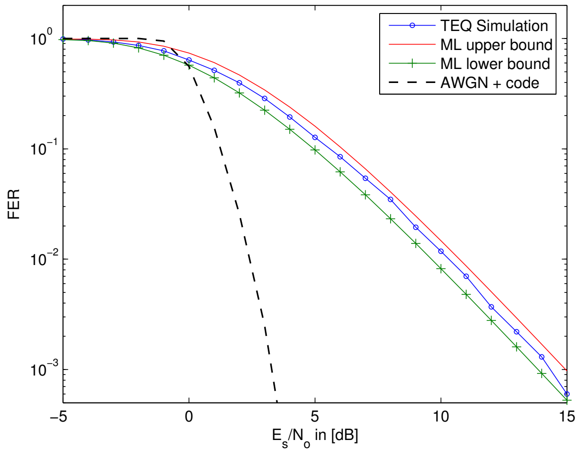

Figure 3: Upper and lower bounds on FER in the system over ISI fading channels with $B = 1$, $L = 5$, and $N = 1024$.

pdf of Θ_b:

$$f_{\Theta_b}(\theta) = P_0 \frac{1-P_z}{\sqrt{2\pi\sigma_{ISI}^2}} e^{-(\theta-1)^2/(2\sigma_{ISI}^2)} + P_z \delta(\theta - 1),$$

where P_0 is a normalization factor due to the Gaussian truncation, and $\delta(x)$ is the Dirac distribution defined as $\delta(x) = 1$ for $x = 0$, and $\delta(x) = 0$ otherwise. Figure 2 shows the curve $f_{\Theta_b}(\theta)d\theta$ for $d\theta = 0.01$ compared with a simulated pdf obtained by 10^6 error sequences over random channels with $L = 5$ for different values of d_b. From [12] (Table I), we have $d_{E,min}^2 = 0.2679$ for an error sequence of weight $d_b = 2$, which yields to $\theta_{min} = 0.134$. We observe that our approximation is very close to the actual distribution for low values of d_b. For high error weights, the tail of the actual distribution decreases more slowly than the Gaussian tail and presents a slight asymmetry around the average. Using this approximation, we can now evaluate the upper bound on FER given in (9) in the case of a single block transmission ($B = 1$). The upper bound can be rewritten using the new variables θ and γ as follows,

$$P_{sub}(\rho) = \int_\gamma \int_\theta \sum_d A^c(d) Q\left(\sqrt{2d\gamma\theta\rho}\right) f_\Theta(\theta) f_\gamma(\gamma) d\theta d\gamma. \quad (17)$$

Because of the convexity of error function $Q(.)$, performing summation over d and limiting the obtained error probability by 1 before integration over channel statistics leads to a tight upper bound as explained by Malkamaki and Leib in [13]. We evaluate the upper bound in two steps. First, we evaluate the average error probability over interference statistics for a fixed fading level as function of the instantaneous SNR $\rho' \triangleq \gamma\rho$,

$$P_{ISI}(\rho') = \int_0^\infty \left(\sum_d A^c(d) \frac{f_\Theta(t/d)}{d}\right) Q\left(\sqrt{2\rho' t}\right) dt, \quad (18)$$

where the change of variable $t = d\theta$ was used. Then we average obtained error probability over fading statistics after limiting error probability by 1 as follows

$$P_{sub}(\rho) = \int_0^\infty \min\left\{1, P_{ISI}(\rho')\right\} \frac{f_\gamma(\rho'/\rho)}{\rho} d\rho'.$$

Figure 3 shows the obtained upper bound for a rate 1/2 recursive, systematic convolutional (RSC) code with generator polynomial $(1,5/7)_8$ in octal notation. The frame length is $N = 1204$ and the channel length is $L = 5$. Simulation results were obtained by a maximum *a posteriori* turbo-equalizer after 5 iterations. The lower bound on FER shown in the same figure is the corresponding lower bound for a fading channel without ISI. We remark the obtained upper bound is within only 1dB from the lower bound over all SNR range.

4.3 Interference to fading ratio

In order to compare the impact of Θ_b on the system performance in comparison with Γ_b, we define the *interference to fading ratio* (IFR) by

$$\text{IFR} \triangleq \frac{\sigma^2(\Theta_b)}{\sigma^2(\Gamma_b)}. \quad (19)$$

By substituting (13) and (16) in (19), we have

$$\text{IFR} = \frac{d_b - 1}{d_b} \cdot \frac{L(L-1)}{L+2} \cdot \frac{3M - L - 1}{3M(M-1)}. \quad (20)$$

Typically, the IFR has a very small value for practical system parameters. For example, when $M = 256$ and $L = 5$, we find IFR ≤ 0.01 for any value of d_b.

This shows that the ML system performance is essentially dominated by channel fading rather than ISI. Therefore, the effect of the ISI on the distance distribution can be neglected ($\Theta_b \approx 1$) for the considered channel model. The system performance can be approximated by the performance of the coded system over an equivalent ISI-free flat fading channel following the Gamma distribution given in (12). For $B > 1$, the Euclidean distance analysis becomes computationally prohibitive. Consequently, we resort to the asymptotic analysis.

5. BLOCK DIVERSITY AND INTERLEAVER DESIGN

In this section, we investigate the asymptotic system performance for high SNR values and the effect of the interleaver type on the diversity gain in the system. For the equivalent flat fading channel model, the total squared Euclidean distance can be expressed using (2) and (11) by

$$d_E^2 = \sum_{b=1}^{B} 4 d_b \Gamma_b,$$

which is a linear combination of i.i.d. Gamma distributed random variables with random coefficients d_b. Obviously, the blocks with $d_b = 0$ do not contribute to the diversity in the system. We denote by $W(\mathbf{e})$ the random variable that gives the number of blocks for

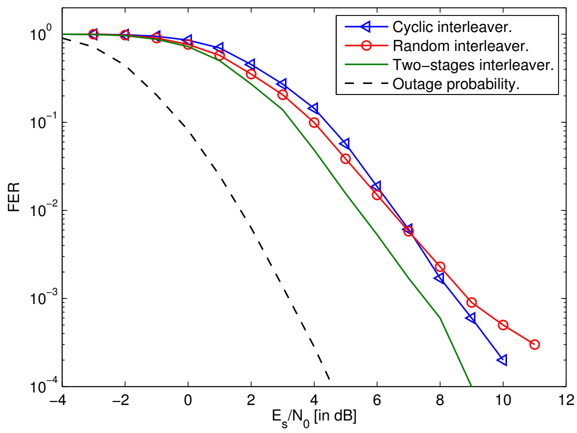

Figure 4: Comparison between FER system performance using different types of interleavers for $B = 4$, $L = 5$, and $N = 1024$.

which $d_b \neq 0$. This random variable is called *block diversity*. A higher value of W is translated into a better *diversity gain* υ defined in [14] as the asymptotic slope of the FER curve as function of the SNR in log-domain

$$\upsilon = -\lim_{\rho\to\infty} \frac{\log(\mathrm{FER}(\rho))}{\log \rho}$$

Under uniform interleaving assumption, the minimum value of block diversity W is always 1 for any value of B, and the asymptotic slope of the FER for high SNR will be the same as for $B = 1$ because there is a non-zero probability of having an error sequence with all its non-zeros error elements grouped in a single block. In fact, the weights vector $\mathbf{d} \triangleq (d_1, \cdots, d_B)$ follows a multivariate hypergeometric distribution. We can show by combinatorial analysis that the exact pmf of W is given by

$$p_W(w) = \frac{\binom{B}{w}}{\binom{N}{d}} \sum_{i=1}^{w} (-1)^{w-i} \binom{w}{i} \binom{iM}{d}.$$

The proof is omitted for the sake of space limitation. Therefore, a random interleaver is not a suitable choice for turbo-equalized systems under block diversity transmission scheme. For block fading channels, a class of linear block codes called maximum distance separable (MDS) codes, have been optimized [15, 16] using cyclic interleaving (or multiplexing) to maximize the block diversity. Cyclic interleaving alone is not sufficient to decorrelate modulated symbols as required by the turbo-equalizer. We propose to use a two-stage interleaver formed by a length-N cyclic interleaver in tandem with B random interleavers of length M. The cyclic interleaver multiplexes the codeword over the B blocks, whereas each random interleaver is performed over a single block.

In order to show the performance of the turbo-equalized system using the proposed interleaver, we have simulated the FER performance for $B = 4$ using three types of interleavers: a random interleaver, a cyclic interleaver, and a two-stage interleaver. The other system parameters are the same as previously used to obtain Figure 3. Simulations results are reported on Figure 4.

Note that the cyclic interleaver achieves the optimal diversity gain of the code given by the outage probability, but it has a poor turbo-equalization performance due to the local dependency between transmitted symbols. Conversely, the random interleaver achieves a good turbo-equalization performance but it has a poor diversity gain. Whereas, the two-stage interleaver achieves simultaneously the diversity gain of the code and a good turbo-equalization performance.

6. CONCLUSIONS

In this paper, we present a new approach to evaluate the Euclidean distance distribution at the output of a frequency-selective channel. For single block transmission, we develop an approximation of the distance spectrum for uncorrelated block-fading ISI channels. We apply this approximation to evaluate a tight upper bound on FER performance based on the uniform interleaver approach. We show that the ISI is insignificant compared to the fading for ML detection. Moreover, in the case of multiple block transmissions, we propose a two-stage cyclic-random interleaver to meet simultaneously coding diversity and turbo-equalization requirements.

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