# DETECTION AND PULSE COMPRESSION IN PASSIVE RADAR WITH OFDM RADIO SIGNALS

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# **ABSTRACT**

By exploring the orthogonal frequency-division multiplexing (OFDM) signal structure to build a frequency model, a generalized likelihood ratio test is presented, together with simulations of probabilities of detection of an OFDM based passive radar. The new analytical model gives insight into system components inter-dependence. It allows both the analyzis of Doppler shift rotation approximation and the study of its effect on detection performance.

#### 1. INTRODUCTION

The low-cost and undetectability properties of passive radar systems render them very popular topics of research and development. Capabilities of such radar systems based on commercial telecommunications have been demonstrated for frequency modulation (FM) [1], for global system for mobile communication (GSM) [2] and for orthogonal frequency-division multiplexing (OFDM) [3, 4]. Herein and elsewhere, they have been shown to be alternatives and/or complements of active radars dedicated to airports or cities air spaces surveillance.

This paper brings further insight in OFDM based radar by giving a frequency formulation of the signals received by an array of sensors. This new compact model is used to develop a Generalized Likelihood Ratio Test and to analyze the loss incurred by pulse compression [5].

Let us remind that Pulse Compression (PC) is a typical radar approximation which consists in assuming that the phase shift is constant along a pulse length. Then, target detection relies on Doppler changes from one pulse to another. PC results in a tradeoff between computational cost reduction and performance capabilities degradation.

For OFDM based radar, radar pulses can be related to the OFDM symbols. In this paper, we show that pulse compression consists in approximating a Dirichlet function (a.k.a. aliased sinc) at its central points. This Dirichlet function represents the spread of information between adjacent frequency channels due to the Doppler shift.

In [5], study of the OFDM ambiguity function shows that there is a range of allowed target speed limiting the compression loss. Our frequency formulation of the model allows to complete this analysis. We extend and illustrate this result with simulations based on Digital Audio Broadcasting [6, 7]. In this DAB context, five frequency channels are required to

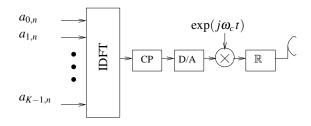


Figure 1: Transmitter block diagram of an OFDM system. CP denotes the cyclic prefix and  $\mathbb{R}$  is the real part.

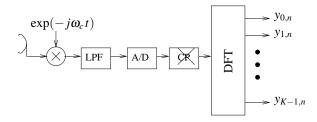


Figure 2: The receiver block diagram. LPF is a low pass filter.

represent the Dirichlet function. Thus, in this radar scenario, PC should reduce each OFDM symbol to five points to keep a good tradeoff between computational burden and detection performance.

Section 2 presents the signals mathematical models. Section 3 describes the DAB based radar characteristics: resolutions, ambiguity functions and pulse compression. Finally, section 4 details target detection simulations with pulse compression.

Let us begin with the OFDM radar signal representation.

### 2. SYSTEM DESCRIPTION

In this section, we give the general formulation of OFDM signals and introduce our compact frequency formulation of the OFDM based radar received signal.

#### 2.1 OFDM model

Let us recall the basic OFDM model in figures 1 and 2. We assume that the use of a cyclic prefix both preserves the orthogonality of the K tones and eliminates intersymbol interference between consecutive OFDM symbols. Further, the channel is supposed to be slowly varying so that it is considered constant during  $N_s$  consecutive OFDM symbols.

Then, we can describe the system as a set of K parallel Gaus-

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sian channels, with attenuation  $h_k$ ,

$$y_{k,n} = h_k a_{k,n} + b_{k,n}, \quad 0 \le k \le K - 1, \quad 1 \le n \le N_s.$$

The OFDM information symbols, the  $a_{k,n}$ , are assumed to be QPSK sequences. Each frequency component of the noise  $b_{k,n}$  is assumed independent identically distributed complex zero-mean Gaussian, uncorrelated with the channel.

Next, we detail the time and frequency representations of the delayed and Doppler shifted version of an OFDM symbol without its cyclic prefix.

## 2.2 Doppler signal model

In this paragraph, we introduce the frequency representation of the  $n^{\text{th}}$  OFDM symbol of a Doppler echo. The general case, for  $N_s$  OFDM symbols and M sensors, is given in the next subsection.

OFDM signals are encoded and decoded with Discrete Fourier Transform (DFT). We note

$$\mathbf{t} = \left[ egin{array}{c} 0 \ dots \ K-1 \end{array} 
ight] T_e \quad ext{and} \quad \boldsymbol{\omega} = rac{2\pi}{K} \left[ egin{array}{c} 0 \ dots \ K-1 \end{array} 
ight] {F_e} \, ,$$

where  $T_e = 1/F_e$  is the sampling period. Then,  $\mathbf{W} = \exp\{j\mathbf{t}\boldsymbol{\omega}^T\}$  is the DFT matrix, where  $\exp\{.\}$  is the elementwise exponential function. Also, when we sum up a vector and a scalar, we assume that the scalar is summed with each element of the vector.

The time representation of the  $n^{th}$  OFDM symbol without its cyclic prefix is given by  $\mathbf{W}\mathbf{a}_n/K$ , where  $\mathbf{a}_n$  is a column vector collecting K information symbols.

Now, when this signal is delayed by  $\tau_D$  smaller than the cyclic prefix duration, it becomes

$$\mathbf{W}\,\mathscr{D}\left(\exp\left\{-j\tau_D\left[\boldsymbol{\omega}+\boldsymbol{\omega}_c\right]\right\}\right)\,\boldsymbol{a}_n/K\,,$$

where  $\omega_c$  is the carrier angular frequency and where  $\mathcal{D}\{.\}$  is diagonalization operator.

To take into account the Doppler effect, this vector has to be element-wise multiplied by the following phase shift vector

$$\exp\{j\omega_D(\mathbf{t}-\tau_D+[P+(n-1)N]T_e)\},$$

where  $\omega_D$  is the Doppler angular frequency, P is the number of points of the cyclic prefix, and N = K + P is the number of points of an OFDM symbol plus its cyclic prefix. So,  $(n-1)NT_e$  is the duration of n-1 previous OFDM symbols with their cyclic prefix.

Then, taking the DFT of the element-wise product of the two preceding vectors, we get the frequency representation

$$\frac{1}{K} \exp\{-j(\omega_c + \omega_D) \tau_D + j\tilde{\omega}_D (P + (n-1)N)\}$$

$$\mathbf{W}^H \mathcal{D}\{\exp\{j\omega_D \mathbf{t}\}\} \mathbf{W} \mathcal{D}\{\exp\{-j\tau_D \omega\}\} \boldsymbol{a}_n,$$

where  $\tilde{\omega}_D = \omega_D/Fe$ . Finally, the  $k^{\text{th}}$  frequency bin,  $0 \le k \le K-1$ . is

$$e^{-j(\omega_c+\omega_D)\,\tau_D}\,e^{j\tilde{\omega}_D\,(P+(n-1)N)}\,\mathbf{d}_k\,\mathcal{D}\{\exp\{-j\tau_D\boldsymbol{\omega}\}\}\,\boldsymbol{a}_n$$

where  $\mathbf{d}_k$  is the  $(k+1)^{\text{th}}$  line of  $\mathbf{W}^H \mathcal{D}\{\exp\{j\omega_D \mathbf{t}\}\}\mathbf{W}/K$ . We rewrite the  $k^{\text{th}}$  frequency bin as

$$e^{j\tilde{\omega}_D P} \exp\left\{-j\tau_D(\boldsymbol{\omega}^T + \boldsymbol{\omega}_D + \boldsymbol{\omega}_c)\right\} \mathscr{D}\{\mathbf{d}_k\}\boldsymbol{a}_n e^{j\tilde{\omega}_D(n-1)N}$$
 (1)

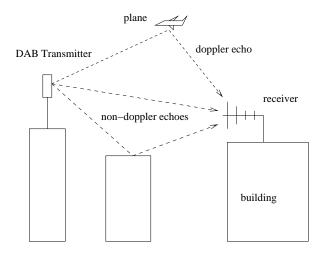


Figure 3: Radar scenario and OFDM signals received on an array of M sensors.

to find the general model given in equation (3).

Next, we summarize those results to give the frequency model of  $N_s$  OFDM symbols received on an array of M sensors.

## 2.3 Radar model and pulse compression

Our scenario is sketched on figure 3. We suppose that  $N_s$  OFDM symbols are received by a uniform linear array of M sensors. We consider the presence of a moving target whose delay is less than the cyclic prefix duration. After removing the cyclic prefixes, we obtain the frequency matrix formulation

$$\mathbf{Y}_k = \mathbf{h}_k \, \mathbf{a}_k^T + \alpha_D \, \mathbf{S}_k + \mathbf{N}_k \,, \tag{2}$$

where  $0 \le k \le K - 1$ , and

- $\mathbf{Y}_k \in \mathbb{C}^{M,N_s}$  is the received signal at frequency k,
- $\mathbf{h}_k \in \mathbb{C}^{M,1}$  contains the channel transfer function at frequency k,
- $\mathbf{a}_k \in \mathbb{C}^{N_s,1}$  contains the OFDM symbols at frequency k,
- $\alpha_D$  is the complex amplitude of the Doppler signal,
- $\mathbf{S}_k \in \mathbb{C}^{M,N_s}$  is the target Doppler shifted signal at frequency k,
- and  $\mathbf{N}_k \in \mathbb{C}^{M,N_s}$  is the additive noise, assumed Circular Complex Gaussian, i.e.,

$$\operatorname{vec}\{\mathbf{N}_k\} \sim \mathbb{CNC}\left(\mathbf{0}, \mathbf{I}_{N_s} \otimes \mathbf{R}_k\right),$$

where  $\mathbf{R}_k \in \mathbb{C}^M$  is the noise spatial covariance matrix at frequency k.

Note that the DFT length is assumed much longer than the noise correlation length, so that the noise is uncorrelated from one frequency to another frequency. Moreover, the model assumes that the channel and the Doppler parameters are constant during the illumination time.

Now, let us describe the Doppler signal at frequency k. From equation (1), we have

$$\mathbf{S}_k = \exp\{j\tilde{\boldsymbol{\omega}}_D P\} \mathbf{B} \mathbf{D}_k \mathbf{A}^T \mathbf{D}_D, \tag{3}$$

where

- $\omega_D$  is the Doppler angular frequency,  $\tilde{\omega}_D = \omega_D/F_e$ , and  $F_e$  is the sampling frequency,
- *P* is the number of points of the cyclic prefix,
- $\mathbf{B} \in \mathbb{C}^{M,K}$  is the wideband beamforming matrix, expressed by

$$\mathbf{B} = \exp\{-j\mathbf{r}_D(\boldsymbol{\omega}^T + \boldsymbol{\omega}_D + \boldsymbol{\omega}_c)\},\,$$

where  $\exp\{.\}$  is the elementwise exponential function,

-  $\mathbf{r}_D \in \mathbb{R}^{M,1}$  is the vector of delays of the Doppler echo,

$$\mathbf{r}_D = \tau_D + \mathbf{c}_M \frac{d}{c} \sin \theta_D, \quad \mathbf{c}_M = [0, \dots, M-1]^T,$$

where  $\tau_D$  is the delay on the first sensor, d is the inter-sensor spacing, c is the speed of light,  $\theta_D \in [-\pi/2, \pi/2]$  is the angle of arrival,

- $\omega$  is the DFT angular frequency vector,  $\omega = [\omega_0, \dots, \omega_{K-1}]^T$ , and  $\omega_k = 2\pi k F_e/K$ ,
- $\omega_{\rm c}$  is the carrier angular frequency,
- $\mathbf{D}_k \in \mathbb{C}^K$  is a diagonal matrix,  $\mathbf{D}_k = \mathcal{D}\{\mathbf{d}_k\}$ , containing the Dirichlet function defined by: for  $0 \le p \le K 1$

$$d_k(p+1) = \frac{1}{K} \sum_{q=0}^{K-1} \exp\{j q \left(\tilde{\omega}_D + \tilde{\omega}_p - \tilde{\omega}_k\right)\},\,$$

where  $\mathcal{D}\{.\}$  is the diagonalization operator,

- $\mathbf{A} \in \mathbb{C}^{N_s,K}$  is the matrix of the OFDM symbols,
- $\mathbf{D}_D \in \mathbb{C}^{N_s}$  is a diagonal matrix containing the Doppler shift from one symbol to the other, *i.e.*, its entries are

$$\exp\{j\tilde{\omega}_D(n-1)N\}, \quad 1 \leq n \leq N_s$$

where N = K + P is the number of points of an OFDM symbol with its cyclic prefix.

The frequency dependence of the Doppler matrix  $\mathbf{S}_k$  relies only on  $\mathbf{D}_k$ , which itself varies through a circular rotation along frequencies (because  $\mathbf{d}_k$  is  $2\pi$ -periodic). Thus, we limit our study of the Dirichlet function to  $\mathbf{D}_0 = \mathcal{D}\{\mathbf{d}\}$  that we rewrite as: for  $0 \le p \le K-1$ 

$$d(p+1) = \frac{1}{K} e^{j(K-1)(\tilde{\omega}_p + \tilde{\omega}_D)/2} \frac{\sin[K(\tilde{\omega}_p + \tilde{\omega}_D)/2]}{\sin[(\tilde{\omega}_p + \tilde{\omega}_D)/2]}, \quad (4)$$

and d(p+1)=1 by continuous prolongation if  $\tilde{\omega}_p+\tilde{\omega}_D$  is a  $2\pi$  multiple. This vector represents the energy spreading over adjacent Gaussian channels, see figure 4 for an illustration. Pulse compression to L terms approximates the Dirichlet function to its L central points (and we note  $\mathbf{d}_L$  the associated Dirichlet function).

Finally, remember that this model has been written for a Doppler delay<sup>1</sup> shorter than the cyclic prefix duration. Its extension to a more general case is straightforward but burdensome.

Next, our frequency model is used to study the effect of pulse compression on DAB based radar performance.

## 3. DAB RADAR AMBIGUITY FUNCTION

In this section, we present the attainable radar resolutions with a passive system based on DAB. Then, we use the frequency model defined by equations (2) and (3) to study pulse compression (PC) loss on the radar ambiguity function.



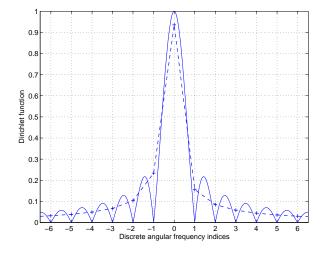


Figure 4: Absolute value of a Dirichlet function with a Doppler effect ('+' markers on the dashed line) and without (in this case, the central point is one while other points are zeros).

### 3.1 Radar resolutions with DAB

Based on the DAB characteristics, we consider an illumination signal made of

- $N_s = 32$  OFDM symbols,
- K = 1024 tones,
- $B = F_e = KW = 1.024$ MHz bandwidth,
- P = K/4 points for the cyclic prefix,
- and  $F_c = 200$ MHz carrier frequency.

Received on M = 10 sensors, this signal consists of 320 thousand sample. Such a system allows

- a spatial resolution of  $c/F_e \simeq 290 \,\mathrm{m}$ ,
- a temporal resolution of  $1 \mu$  s,
- an angular resolution of 2/(M-1) rad  $\simeq 12.7^{\circ}$  (with sensor spacing  $d = \lambda/2 \simeq 0.75$ m),
- and a Doppler resolution of  $W/1.25/N_s = 25$ Hz.

Each OFDM symbol has 1.25ms duration. So, with one point per OFDM symbol, *i.e.* L=1 in PC, the Doppler band under analyzis is [-400,400]Hz.

The total signal duration is 40ms. It corresponds to a distance of 14m traveled by a plane at Mach 1. This validate our hypothesis that the geometry of the experimental configuration does not change. It also validates the first order approximation of the Doppler effect. Finally, let us remark that a plane at Mach 1 creates a maximum Doppler shift of  $F_D=226{\rm Hz}$ . Next, we study pulse compression influence on the radar ambiguity function.

# 3.2 Ambiguity function and PC

The general form of the ambiguity function takes into account the Doppler frequency shift: it is the correlation between the Doppler signal at a reference point S and the Doppler signal along the parameter space  $\widehat{S}$ ,

$$A(\widehat{\tau}_D, \widehat{\theta}_D, \widehat{F}_D) = \operatorname{tr}\left(\mathbf{S}^H \widehat{\mathbf{S}}\right) = \sum_{k=0}^{K-1} \operatorname{tr}\left(\mathbf{S}_k^H \widehat{\mathbf{S}}_k\right).$$

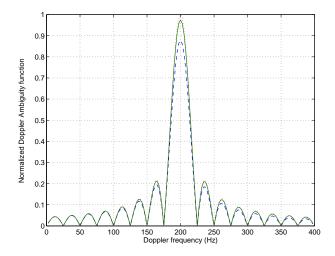


Figure 5: Doppler cut of the ambiguity diagram of an OFDM signal for three values of L (pulse compression): L=1 blue dashed line, L=5 green line and L=K red dotted line.

If we assume that the Doppler signals can be approximated with only L non-zero terms in the Dirichlet function, the associated components are noted  $\widehat{\mathbf{d}}_L$ ,  $\widehat{\mathbf{D}}_{k,L}$  and  $A_L$  (if L=K, there is no approximation). Let note  $\mathbb{E}\{.\}$  the expectation operator. Now, assuming the OFDM information symbols are independent unit power random processes, our frequency formulation leads to

$$|\mathbb{E}\{A_L\}| = \left| \operatorname{tr} \left( \widehat{\mathbf{D}}_D \mathbf{D}_D^H \right) \operatorname{tr} \left( \mathbf{B}^H \widehat{\mathbf{B}} \sum_{k=0}^{K-1} \widehat{\mathbf{D}}_{k,L} \mathbf{D}_k^H \right) \right|$$
$$= \left| \operatorname{tr} \left( \widehat{\mathbf{D}}_D \mathbf{D}_D^H \right) \operatorname{tr} \left( \mathbf{B}^H \widehat{\mathbf{B}} \right) \mathbf{d}^H \widehat{\mathbf{d}}_L \right|.$$

This expression offers many possibilities in waveform design and pulse compression analysis. The most influent factor is the Dirichet function

$$\left| \operatorname{tr} \left( \widehat{\mathbf{D}}_D \mathbf{D}_D^H \right) \right| = \left| \frac{\sin[N_s N(\hat{\hat{\omega}}_D - \tilde{\omega}_D)/2]}{\sin[N(\hat{\hat{\omega}}_D - \tilde{\omega}_D)/2]} \right|,$$

which shapes the Doppler cut ambiguity function of figure 5. There, zeros are modulo  $F_e/(NN_s) = W/(1.25N_s) = 25$ Hz. Also, defining the loss due to pulse compression, noted  $L_{PC}$ , as the ratio of the ambiguity functions with and without compression, we get

$$L_{PC} = \frac{|\mathbb{E}\{A\}|}{|\mathbb{E}\{A_L\}|} = \left| \frac{\mathbf{d}^H \widehat{\mathbf{d}}}{\mathbf{d}^H \widehat{\mathbf{d}}_L} \right|.$$

At the main lobe of the ambiguity function, we have  $\mathbf{d} = \hat{\mathbf{d}}$ . As  $\mathbf{d}^H \mathbf{d} = 1$ , the loss at the main lobe is  $1/|\mathbf{d}^H \mathbf{d}_L|$ . Thus, if L = 1, the loss at the main lobe is the inverse of the central point of the Dirichlet function of equation (4), *i.e.*,

$$\left| K \frac{\sin(\tilde{\omega}_D/2)}{\sin(K\tilde{\omega}_D/2)} \right|^2 = \left| K \frac{\sin(\omega_D/(2KW))}{\sin(\omega_D/2/W)} \right|^2.$$

Using Taylor series representation, we get  $F_D \simeq W/4$  for the limit of 1dB loss  $(20 \log |L_{PC}|)$ , or 90% conservation, at the main lobe. Thus, with W = 1kHz, the 1dB limit is 250Hz. In conclusion, reducing the PC loss requires to increase the tones bandwidth W. To do so while keeping a good Doppler resolution, one has to increase  $N_s$ , the number of OFDM symbols.

In the next section, we present the detector built from generalized likelihood and we study its performance as a function of the Doppler parameters and of the pulse compression.

# 4. PULSE COMPRESSION AND DETECTION

We assume that the noise cross spectral matrices, the  $\mathbf{R}_k$ 's, are known. Moreover, we suppose that OFDM information symbols are known, which requires an error-free demodulation of the DAB radio signal.

#### 4.1 Maximum likelihood test

After estimating the channel coefficients and the Doppler amplitude, the generalized likelihood ratio leads to

$$T_L(\tau_D, \theta_D, F_D) = \frac{\left| \sum_{k=0}^{K-1} \operatorname{tr} \left( \mathbf{Y}_k^H \mathbf{R}_k^{-1} \mathbf{S}_{k,L} \mathbf{P}_{\mathbf{a}_k^*}^{\perp} \right) \right|^2}{\sum_{k=0}^{K-1} \operatorname{tr} \left( \mathbf{S}_{k,L}^H \mathbf{R}_k^{-1} \mathbf{S}_{k,L} \mathbf{P}_{\mathbf{a}_k^*}^{\perp} \right)}, \quad (5)$$

where  $\mathbf{P}_{\mathbf{a}_{k}^{*}}^{\perp} = \mathbf{I}_{N_{s}} - \mathbf{a}_{k}^{*} \mathbf{a}_{k}^{T} / \|\mathbf{a}_{k}\|^{2}$ , is the orthogonal projection onto the subspace perpendicular to the subspace defined by the column vector  $\mathbf{a}_{k}^{*}$ .

The whitened noise  $\mathbf{R}_k^{-1/2}\mathbf{N}_k$  is distributed as  $\mathbb{CNC}(\mathbf{0},\mathbf{I}_{MN_s})$ . Thus, the argument of the sum of the numerator of equation (5) is

$$\begin{array}{lcl} \operatorname{tr}\left(\mathbf{Y}_{k}^{H}\,\mathbf{R}_{k}^{-1}\,\mathbf{S}_{k,L}\,\mathbf{P}_{\mathbf{a}_{k}^{*}}^{\perp}\right) & \sim & \mathbb{CNC}(m_{k,L},v_{k,L}) \\ & \text{with } m_{k,L} & = & \operatorname{tr}\left(\boldsymbol{\alpha}_{D}^{*}\mathbf{D}_{k}^{H}\,\mathbf{R}_{k}^{-1}\,\mathbf{D}_{k,L}\,\mathbf{P}_{\mathbf{a}_{k}^{*}}^{\perp}\right) \\ & v_{k,L} & = & \operatorname{tr}\left(\mathbf{D}_{k,L}^{H}\,\mathbf{R}_{k}^{-1}\,\mathbf{D}_{k,L}\,\mathbf{P}_{\mathbf{a}_{k}^{*}}^{\perp}\right). \end{array}$$

and, frequency independence leads to

$$2T_L: \chi^2\left(2, \frac{2|m_L|^2}{v_L}\right),\,$$

where  $m_L = \sum_{k=0}^{K-1} m_{k,L}$  and  $v_L = \sum_{k=0}^{K-1} v_{k,L}$ . Note that if L = K, the bias parameter becomes  $2|m|^2/v = 2|\alpha_D|^2v$ . Finally, choosing  $\mu$  such that PFA  $= \int_{\mu}^{\infty} p_{\chi_2^2}(x) dx$ , we have

a detector while realizing the test  $T_L \underset{H_0}{\gtrless} \frac{\mu_1}{2}$ .

Next, we use this result to compute the detection probability for different configuration parameters.

## 4.2 Detection probability (DP) simulation

For PFA =  $10^{-2}$ , we analyze the DP relatively to the noise power and the Doppler parameters for three values of L: 1, 5 et K (no pulse compression).

OFDM information symbols are QPSK and independent while the additive noise is white Gaussian complex circular. The channel coefficients are simulated from at least 10 echoes normalized so that the maximal amplitude is one.

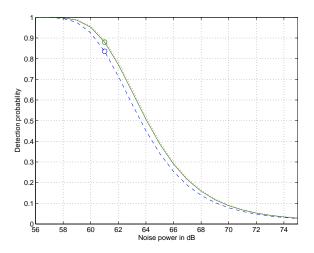


Figure 6: Detection probability versus noise power in dB for three values of L (Dirichlet function approximation): L = 1 blue dashed line, L = 5 green line and L = K red dotted line.

Thus, the total non-Doppler channel response has a minimum power of  $10\log_{10}(KMN_s) = 55.15$ dB. The Doppler amplitude is  $\alpha_D = 10^{-2}$  so that the total Doppler signal contribution is  $10\log_{10}(KMN_s|\alpha_D|^2) = 15.15$ dB.

#### 4.2.1 DP versus white noise power

For  $F_D = 200$ Hz,  $\tau_D = 3/4T_{PC}$ , and  $\theta_D = \pi/4$ , we plot on figure 6 the detection probability versus the noise power: the blue dashed line corresponds to L = 1 (one point in the pulse compression), the green line corresponds to L = 5 and the red dotted line corresponds to L = K (no pulse compression). For a noise power of 61dB, we add circles on the graph. Here, the pulse compression represents a detection loss of 5%, which can be regain if the noise is one half dB lower.

# 4.2.2 DP versus Doppler parameters

For a noise power of 61dB, we plot on figure 7 the detection probability obtained with different values of  $F_D$ ,  $\tau_D$ , and  $\theta_D$ . Again, the blue dashed line corresponds to L=1 (one point in the pulse compression), the green line corresponds to L=5 and the red dotted line corresponds to L=K (no approximation). For a Doppler frequency 200Hz, we add circles on the graph. Here, we still find that the pulse compression implies a loss of 5%. This loss goes up to 30% when the Doppler frequency reaches 400Hz. The other Doppler parameters, angle and time of arrival, have almost no influence, which confirms the result of subsection 3.2.

## 5. CONCLUSION

We have presented a new frequency formulation of the signals received by an array of sensors in OFDM based radar. These equations have enlightened that the Doppler frequency affects the pulse compression ambiguity function through a Dirichlet function. We have shown that to limit pulse compression loss, one may increase the bandwdith between carriers. But, this decreases the prefix cyclic duration. Thus, one

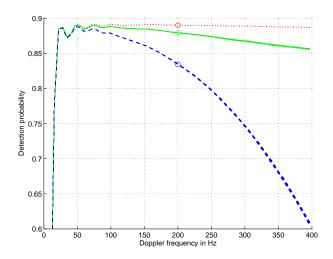


Figure 7: Detection probability versus  $F_D$  in Hz for three values of L (Dirichlet function approximation): L=1 blue dashed line, L=5 green line and L=K red dotted line. These results are obtained for different values of  $\tau_D$ ,  $\theta_D$  and a noise power of 61dB.

might keep the same waveform and limit the radar to use one over two (or four) tones (while divising every OFDM symbol in two or four). Our model assumes that the Doppler delay is included in the cyclic prefix, which has for consequence that this parameter does not influence the Doppler signal detection. Future work will endorse the evaluation of a model taking into account a Doppler delay bigger than the cyclic prefix duration. We will also evaluate the Doppler parameters influence on adaptive detectors.

#### REFERENCES

- [1] P.E. Howland, D. Maksimiuk and G. Reitsma, "FM radio based bistatic radar," *Radar, Sonar and Navigation, IEE Proceedings*, vol. 152, No. 3, pp. 107–115, June 2005.
- [2] D.K.P. Tan, H. Sun, Y. Lu, M. Lesturgie and H. Chan, "Passive radar using Global System for Mobile communication signal: theory, implementation and measurements," *Radar, Sonar and Navigation, IEE Proceedings*, vol. 152, pp. 116–123, June 2005.
- [3] C. Langton, "OFDM tutorial," in http://www.complextoreal.com/chapters/ofdm2.pdf.
- [4] D. Poullin, "Passive detection using digital broadcasters (DAB, DVB) with COFDM modulation," *Radar, Sonar and Navigation, IEE Proceedings*, vol. 152, pp. 143–152, June 2005.
- [5] G.E.A. Franken, H. Nikookar and P. van Genderen, "Doppler Tolerance of OFDM-coded Radar Signals," in *3rd European Radar Conf.*, Sept. 2006, pp. 108–111.
- [6] D. H. Layer, "Digital radio takes to the road," *Spectrum*, *IEEE*, vol. 38, pp. 40–46, Jul. 2001.
- [7] H.J. Yardley, "Bistatic Radar Based on DAB Illuminators: The Evolution of a Practical System," in *Proc. Radar Conference*, 2007 IEEE, Boston, USA, April 17-20. 2007, pp. 688–692.