"P300 SPELLER" BRAIN-COMPUTER INTERFACE: ENHANCEMENT OF P300 EVOKED POTENTIAL BY SPATIAL FILTERS

Bertrand Rivet[1], Antoine Souloumiac[2], Guillaume Gibert[3], Virginie Attina[3]

[1] GIPSA-lab, CNRS-UMR 5216,
Grenoble Institute of Technology
46 avenue Félix Viallet,
38000 Grenoble, France

[2] CEA, LIST, Laboratoire Processus
Stochastiques et Spectres
CEA, Saclay,
Gif sur Yvette, F-91191, France

[3] INSERM, U821,
Lyon, F-69500, France;
IFR 19, Institut Fédératif des Neurosciences, Lyon, F-69000, France;
Univ Lyon 1, Lyon, F-69000, France

ABSTRACT

Brain-Computer Interfaces (BCI) are communication systems that use brain activity to control a computer or other devices. The BCI system described in this study is based on the P300 speller BCI paradigm designed by Farwell an Donchin in 1988 [1]. A new unsupervised algorithm is proposed in this paper[1]. It is based on the projection of the raw EEG signal into the estimation of the P300 subspace. In this algorithm, brain responses to the target stimuli and to the non target stimuli are taken into account. They provide a better estimation of the P300 subspace main components. Data recorded on three subjects were used to evaluate the proposed method. The results are presented using a Bayesian linear discriminant analysis (BLDA) classifier.

1. INTRODUCTION

Brain-Computer Interfaces (BCI) are new kinds of devices which enable direct communication between the user's brain and a computer [2, 3]. Such kind of human-computer interfaces analyze electroencephalographic (EEG) or other brain activities. BCIs, which provide a new non-muscular powerful channel for communicating with the external world, are thus suitable for people that are incapable of any motor functions (e.g. people with severe neuromuscular disorders or "locked in" people). Present-day BCIs determine the intent of the user from different electrophysiological signals : for instance, the user may control some brain waves (e.g. mu or beta rhythms) or the BCI may exploit natural responses of the brain to external stimuli (e.g. event-related potentials) [2]. The BCI studied in this paper is the P300 speller [1, 4]: it enables people to spell a text on a computer. It is based on natural responses of the brain to external visual stimuli: the oddball paradigm consists of discriminating between common non-target stimuli and rare target stimuli which evoke a P300 potential. Unfortunately, the signal-to-noise ratio (SNR) of EEG signals is very low, and moreover the recorded EEG signals may also contained muscular and/or ocular artefacts. Several methods, based on independent component analysis (ICA), were thus proposed to enhance the SNR and to remove the artefacts [5, 6]. However, the major drawback of such methods is that they are not specifically designed to separate brain waves and they are supervised. Indeed, after the decomposition in independent components (IC) it is necessary to select (manually or thanks to spatio-temporal prior)

the ICs which contained the evoked potentials. In this paper, we propose a new unsupervised algorithm to automatically estimate P300 subspace from raw EEG signals. It is based on a previously proposed method [7, 8]. In the present study, the new method, to estimate spatial filters which enhance the P300 potentials, does not only take into account the target stimuli as in [7, 8] but it also uses the non-target stimuli.

This paper is organized as follows. Section 2 presents the new proposed method to estimate the spatial filters to enhance the P300 evoked potentials, while Section 3 presents the method to classify the stimuli. Section 4 resumes the experiments used to test the proposed method before conclusions and perspectives in Section 5.

2. ENHANCEMENT OF P300 POTENTIALS

The aim of this study is to provide a simple and unsupervised estimation of the P300 subspace so that the classification between target/non-target epochs is simplified which thus leads to a faster spelling device.

2.1 P300 speller Brain Computer Interface

The BCI addressed in this paper is the P300-speller introduced by Farwell and Donchin [1]. It enables users to spell a text: a 6×6 matrix, that includes all the alphabet letters as well as other symbols, was presented to the user on a computer screen (Fig. 1(a)). A sequence is thus defined as the intensification of each of the 6 rows and of the 6 columns in a random order. To spell a character, the users had to mentally count the number of times the letter/symbol, they wish to communicate, is intensified. In response to this counting, a P300 evoked potential was elicited in the brain (i.e. a positive deviation around 300ms after the stimulus). The desired character hence appears on 2 out of the 12 intensifications in a sequence, since a character is defined as the intersection of a given row and a given column. The task is thus to detect the oddball stimuli (row/column intensifications) which lead to a P300 evoked potential (Fig. 1(b)). To produce a more robust BCI, each character was spelled several consecutive times. However, this repetition decreases the number of characters spelled per minute: e.g. with 15 repetitions, only 2 characters were spelled per minute [1, 4].

The aim of the proposed method is thus to correctly predict a character with as low as possible sequence repetitions leading to increase the information rate.

2.2 P300 subspace estimation

The raw EEG recorded from the user's scalp not only contains the desired P300 evoked potentials but also ongoing ac-
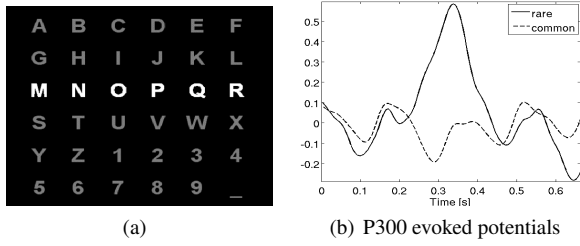

(a)

(b) P300 evoked potentials

Figure 1: Brain-Computer Interface "P300 speller". Fig. 1(a): screen display as was shown to the subjects with an highlighted row. Fig. 1(b): time course of the actual average signal waveforms at C_z.

tivity of the brain and muscular and/or ocular artefacts. As a result, the SNR is very low and the classification task (i.e. the character prediction) is not easy. We thus proposed to enhance the P300 potentials by projecting the raw recorded EEG on the P300 subspace (i.e. the subspace which contains most of the P300 potentials) before the classification.

The proposed method is based on two main ideas. Firstly, there exists a typical response synchronized with the target stimuli superimposed with a evoked response by all the stimuli (target plus non-target). Secondly, the synchronized response with the target stimuli might be enhanced by a spatial filtering.

Let $x_j(t)$ denote the EEG signal recorded by the j-th sensor at time index t and let $X \in \mathbb{R}^{N_t \times N_s}$ be the matrix of recorded EEG signals whose (i,j)-th entry is $x_j(i)$. N_s is the number of sensors and N_t the number of temporal samples. Let $a_j^{(1)}(t)$ denote the ERP signal for the j-th sensor at time index t by the target stimuli, and let $a_j^{(2)}(t)$ denote the superimposed signal for the j-th sensor at time index t related to each stimuli. The first main idea is thus expressed as

$$X = D_1 A_1 + D_2 A_2 + N, \qquad (1)$$

where $A_1 \in \mathbb{R}^{N_e^1 \times N_s}$ and $A_2 \in \mathbb{R}^{N_e^2 \times N_s}$ are the matrices of ERP signals whose (i,j)-th entries are $a_j^1(i)$ and $a_j^2(i)$, respectively. $D_2 \in \mathbb{R}^{N_t \times N_2}$ is the Toeplitz matrix whose first column is defined such that $D_2(\tau_k, 1) = 1$ where τ_k is the stimulus onset of the k-th stimulus ($1 \le k \le K$, with K the total number of stimuli) and such that all the other elements are null. $D_1 \in \mathbb{R}^{N_t \times N_1}$ is the Toeplitz matrix whose first column is defined such that $D_2(\tau_k', 1) = 1$ where τ_k' is the stimulus onset of k-th target stimulus ($1 \le k \le K' \le K$, with K' the total number of target stimuli) and such that all the other elements are null. N_1 and N_2 are the number of temporal samples of the target and superimposed ERPs, respectively. Typically, N_i is chosen to correspond to 600 milliseconds or one second. Finally, $N \in \mathbb{R}^{N_t \times N_s}$ is the on going brain activity which is not related to the stimuli.

Let us rewrite model (1) as

$$X = DA + N, \qquad (2)$$

where $D = [D_1, D_2]$ and $A = [A_1^T, A_2^T]^T$, with $\cdot^T$ the transpose operator. The least square estimation of A is given by

$$\hat{A} = \arg\min_A \|X - DA\|_2^2, \qquad (3)$$

whose solution is

$$\hat{A} = \begin{bmatrix} \hat{A}_1 \\ \hat{A}_2 \end{bmatrix} = (D^T D)^{-1} D^T X. \qquad (4)$$

Let B_1 and B_2 be two matrices such that

$$\hat{A}_1 = B_1^T X, \qquad (5)$$
$$\hat{A}_2 = B_2^T X. \qquad (6)$$

Note that in [7, 8] the model was $X = D_1 A_1 + N$, leading to a slightly different estimation of A_1: $A_1^\dagger = (D_1^T D_1)^{-1} D_1^T X$.

The second main idea leads to estimate N_f spatial filters $\mathbf{u}_i^{(1)}$ ($1 \le i \le N_f \le N_s$) such that the synchronized response with target stimuli are enhanced by this spatial filtering

$$X U_1 = D_1 A_1 U_1 + D_2 A_2 U_1 + N U_1, \qquad (7)$$

where $U_1 \in \mathbb{R}^{N_s \times N_f}$ is the matrix of spatial filters whose i-th column is $\mathbf{u}_i^{(1)}$. We proposed to estimate U_1 such that the signal to signal plus noise ratio is maximum:

$$\hat{U}_1 = \arg\min_{U_1} \frac{\text{Tr}(U_1^T \hat{A}_1^T D_1^T D_1 \hat{A}_1 U_1)}{\text{Tr}(U_1^T X^T X U_1)}. \qquad (8)$$

Replacing $\hat{A}_1$ by its expression (5) and thanks to the QR decomposition [9] of $D_1 = Q_1 R_1$, $D = Q_d R_d$ and $X = Q_x R_x$, one can expressed (8) as

$$\hat{V}_1 = \arg\min_{V_1} \frac{\text{Tr}(V_1^T Q_x^T B_1 R_1^T R_1 B_1^T Q_x V_1)}{\text{Tr}(V_1^T V_1)}, \qquad (9)$$

where $V_1 = R_1 U_1$. Matrix V_1 is thus obtained from the Rayleigh quotient (9) whose solution is the concatenation of N_f eigenvectors associated with the N_f largest eigenvalues of matrix $Q_x^T B_1 R_1^T R_1 B_1^T Q_x$ [9]. These vectors can be efficiently obtained from the singular value decomposition (SVD) of $R_1 B_1^T Q_x = \Phi \Lambda \Psi^T$, where Φ and Ψ are two unitary matrices and Λ is a diagonal matrix with nonnegative diagonal elements in decreasing order. The solution of criterion (8) is thus obtained by

$$\hat{U}_1 = R_x^{-1} [\psi_1, \cdots, \psi_{N_f}], \qquad (10)$$

where ψ_i is the i-th column vector of Ψ. The enhanced signals are finally computed by

$$\hat{S} = X \hat{U}_1 \qquad (11)$$

Basic linear algebra manipulations lead to rewrite $\hat{A}_1$ as

$$\hat{A}_1 = R_1^{-1} \Phi \Lambda \Psi^T R_x$$
$$\simeq A_1' W_1^T, \qquad (12)$$

where $A_1' = R_1^{-1} [\phi_1, \cdots, \phi_{N_f}] \text{diag}(\lambda_1, \cdots, \lambda_{N_f})$ and $W_1 = R_x^T [\psi_1, \cdots, \psi_{N_f}]$. $\text{diag}(\cdot)$ is the diagonal matrix composed of elements in parentheses. In (12), the synchronised response $\hat{A}_1$ is thus approximated by the main components A_1' and W_1 whose columns are the main timecourses of the evoked potentials and the corresponding spatial weights, respectively. The P300 subspace is then defined by the couples $\{(\hat{\mathbf{u}}_i^{(1)}, \hat{\mathbf{a}}_i'^{(1)})\}_{1 \le i \le N_f}$.

The proposed algorithm to enhance the evoked potentials related to the target stimuli is summarized in Algorithm 1.

Algorithm 1 Algorithm to enhanced synchronized responses with target stimuli.

1: Compute matrix $D = [D_1, D_2]$
2: Compute matrices B_1 and B_2 from D (5) and (6)
3: Compute QR factorization of $X \Rightarrow X = Q_x R_x$
4: Compute QR factorization of $D_1 \Rightarrow D_1 = Q_1 R_1$
5: Compute SVD of $R_1 B_1^T Q_x \Rightarrow R_1 B_1^T Q_x = \Phi \Lambda \Psi^T$
6: Select the N_f couples of singular vectors (ϕ_i, ψ_i) associated with the N_f largest singular values λ_i
7: Finally, $\forall\, 1 \leq i \leq N_f$, $\left(\hat{\mathbf{u}}_i^{(1)}, \hat{\mathbf{a}}_i^{\prime(1)}\right) = \left(R_x^{-1}\psi_i, R_1^{-1}\phi_i\lambda_i\right)$
8: Estimate enhanced signals $\forall\, 1 \leq i \leq N_f$:

$$\hat{s}_i(t) = \hat{\mathbf{u}}_i^{(1)T} \mathbf{x}(t)$$

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3. BCI CLASSIFICATION

In the P300 speller BCI problem, the spelled character is identified by the detection of a P300 evoked potential related to a given row and to a given column illuminations for each sequence. Feature vector $\mathbf{p}_j$ corresponding to the j-th illumination is given by the concatenation of N_f epochs of estimated sources $f_i^{(j)} = [f_i^{(j)}(0), \cdots, f_i^{(j)}(N_e)]^T$ defined by

$$f_i^{(j)}(t) = \left(s_i(t) \times \Pi_{N_e}(t - \tau_j)\right) * \delta(t + \tau_j), \qquad (13)$$

where $s_i(t)$ is the estimated sources (11), τ_j is the time-code of the j-th illumination and $\Pi_{N_e}(t)$ is the boxcar function equal to 1 on its support $[0, Ne]$ and equal to 0 elsewhere (typically N_e is chosen to correspond to 600 milliseconds or one second). Thus $\mathbf{p}_j$ is defined by

$$\mathbf{p}_j = \left[\mathbf{f}_1^{(j)T}, \cdots, \mathbf{f}_{N_f}^{(j)T}\right]^T. \qquad (14)$$

Moreover, let t_j denote the associated class with j-th illumination, $t_j = 1$ if the j-th illumination contains the spelled symbol and $t_j = 0$ otherwise.

Among the proposed classifiers for BCIs, Bayesian linear discriminant analysis (BLDA) [10, 11] is chosen since it proved to be efficient and was fully automatic (i.e. no hyperparameters to adjust) [10]. It aims at finding, thanks to a Bayesian framework, a discriminant vector $\mathbf{w}$ such that $\mathbf{w}^T \mathbf{p}$ is as closed as possible to the class t associated with the corresponding feature vector $\mathbf{p}$. This discriminant vector $\mathbf{w}$ is thus estimated from the set of couples $\{\mathbf{p}_j, t_j\}_{1 \leq j \leq N_c}$ obtained from the N_c symbols in the training database.

4. EXPERIMENTAL RESULTS

In this section, the database used to test the proposed algorithm and the pre-processing stages are first described in Subsection 4.1. The estimation results of P300 evoked potentials are described in Subsection 4.2 while the results of BCI prediction are presented in Subsection 4.3.

4.1 Data acquisition and pre-processing

4.1.1 Data acquisition

Three healthy male subjects participated voluntarily in the experiment. They are all free of neurological diseases and had no previous experience with the P300 speller paradigm.

EEG activity was recorded with a BrainAmp amplifier (BrainProduct GmBH, Munich), from 29 Ag/AgCl scalp electrodes placed at standard positions of an extended 10-20 international system referenced to the nose and grounded to the forehead. The EEG was collected and stored using the BCI2000 system with P300 speller scenario [12].

Each run corresponds to one word and each word is composed of two to six characters. Before each run, the entire word to spell was indicated at the top of the display. Subjects were asked to focus on the current symbol (which was shown after the word in parentheses) and to mentally count the number of times this letter was intensified. Each row and column in the spelling matrix was randomly intensified for 100ms and the delay between two consecutive intensifications was 80ms, leading to an interstimulus interval (ISI) of 180ms. For each symbol, the 12 columns and rows were intensified 15 times. There was a 2.5s period between each character of a run. In total there were 75, 65 and 68 symbols for the first, second and third subject, respectively.

4.1.2 Pre-processing

The EEG signals are sampled at 500Hz. Before estimating the spatial filters by the proposed method, the following pre-processing stages are applied. The data are first filtered by a fourth order forward-backward Butterwoth bandpass filter. Cut-off frequencies are set to 1Hz and 20Hz. For each sensor, the bandpass filtered signals were then normalised so that they had a zero mean value and a standard deviation equal to one. The temporal lengths of the synchronised responses (5) and (6) were chosen equal to one second.

4.2 Enhancement of P300 evoked potentials

In this set of experiments (Fig. 2), the proposed 'multi stimuli' algorithm to enhance P300 evoked potentials is applied on the database for each subject separately. For each subject, all the available symbols were used to estimate the P300 subspace defined by $\left\{(\hat{\mathbf{a}}_i^{\prime(1)}, \hat{\mathbf{u}}_i^{(1)}, \hat{\mathbf{w}}_i^{(1)})\right\}_{1 \leq i \leq N_f}$, where the length of synchronised responses A_1 and A_2 is chosen equal to 1 second.

The results obtained by the proposed 'multi stimuli' method (Subsection 2.2), which takes into account the target stimuli as well as the non-target stimuli, are compared with the results provided by 'mono stimuli' method previously described in [7] which only considers the target stimuli. The results are illustrated for one subject. As one can see, the proposed method slightly improves the estimation quality of the synchronised response with the target stimuli A_1. Indeed, with the 'mono stimuli' method [7], one can mainly see on the rear sensors (Fig. 2(a)) that there exist undulations at the same frequency as the rows/columns intensifications (5.6Hz equal to the inverse of the ISI). A contrario, with the proposed 'multi stimuli' method, these undulations are reduced (Fig. 2(e)).

The major difference appears in the P300 subspace estimation. Let us compare the three first components $\left\{(\hat{\mathbf{a}}_i^{\prime(1)}, \hat{\mathbf{u}}_i^{(1)}, \hat{\mathbf{w}}_i^{(1)})\right\}$, for $1 \leq i \leq 3$, obtained by the 'multi stimuli' method (Fig. 2(f), 2(g) and 2(g)) with those provided by 'mono stimuli' method [7] (Fig. 2(b), 2(c) and 2(d)). One can see on the first component obtained by 'mono stimuli' method [7] (Fig. 2(b)) a strong component at 5.6Hz, which is mainly due to the rows/columns intensifications, superimposed with the evoked potential (between 200ms – 400ms). The second and third components (Fig. 2(c) and 2(d)) seem

(a) Synchronised response $A_1^\dagger$ (b) First component   (c) Second component   (d) Third component

(e) Synchronised response $\hat{A}_1$ (f) First component   (g) Second component   (h) Third component
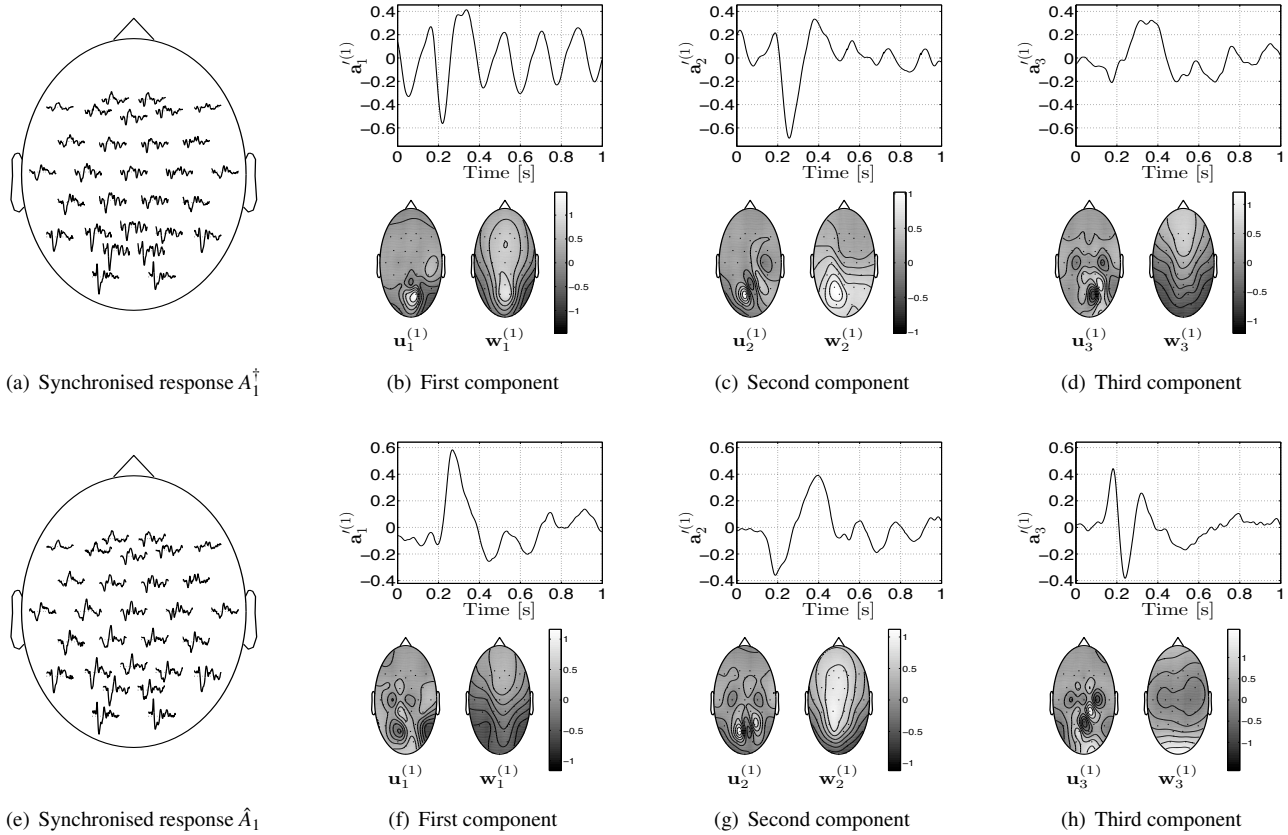
Figure 2: Results of P300 enhancement. Fig. 2(a), 2(b), 2(c), 2(d) refer to results obtained by method described in [7] which only takes into account the target stimuli. Fig. 2(e), 2(f), 2(g), 2(h) refer to results obtained by the proposed method (Algo. 1) which takes into account the target as well as the non-target stimuli. Fig. 2(a) and 2(e) show the synchronised response with the target stimuli $A_1^\dagger$ and $\hat{A}_1$, respectively. Fig. 2(b), 2(c), 2(d) and Fig. 2(f), 2(g), 2(h) show the estimation of the three first dimensions of P300 subspace.

to contain more significant evoked response. A contrario, one can see that the 'multi stimuli' method provides better estimation of the synchronised response since the component at 5.6Hz is drastically reduced compared with the results of the 'mono stimuli' method.

4.3 BCI classification

In this set of experiments, we compared the results of BCI classification obtained by different methods: spatial filters U_1 are estimated by different methods while the classifier linear $\mathbf{w}$ is estimated by BLDA for each case. In each experiment, N_c symbols are used to train spatial filters U_1 and to train linear classifier $\mathbf{w}$. Note that after epoching estimated sources $\hat{s}_i(t)$ (11), $\mathbf{f}_i^{(j)}$ is decimated with a 10 factor before computing parameter vector $\mathbf{p}_j$ (14).

The first method uses no spatial filter (*i.e.* U_1 is chosen equal to the identity matrix) and is denoted 'reference'. The second method denoted 'mono stimuli' estimates spatial filters U_1 by method [7] while the third method denoted 'multi stimuli' estimates spatial filters U_1 by the proposed method (Algo. 1). The averaged results (for the three subjects) of BCI classification are presented in Fig. 3, which shows the percentage of success versus the number of repetitions of the same symbol for different couples (N_f, N_c).

As expected, for each configuration the number of repetitions increases the performance. The same positive correlation is found with respect to the number of training symbols N_c. It is quite interesting to consider the cases where only the first dimension of the P300 subspace is used (Fig. 3(a), 3(c) and 3(e)) since it reveals the effect of accounting for the non-target stimuli A_2 in addition to the target stimuli A_1. The proposed 'multi stimuli' method is seen to outperform the earlier proposed 'mono stimuli' method [7]. Therefore it highlights the fact that the 'multi stimuli' method provides more P300 potential in the first component than the 'mono stimuli' method. Note also that only a limited number of components are needed to provide good performance since using more than four dimensions does not improve the prediction performance. Furthermore, even if using the first four components of P300 subspace improves the BCI classification, it cancels the benefit provided by the 'multi stimuli' method compared to 'mono stimuli' method [7], as one can see in Fig. 2(f), 2(g) and 2(h). Indeed, preliminary investigations show that the first four dimensions span more or less the same P300 subspace using the 'multi stimuli' method or 'mono stimuli' method. However, the enhancement of P300 potentials by spatial filters improve the quality compared to the 'reference' method. Moreover, the dimension of parameter vector $\mathbf{p}_j$ is smaller with 'multi stimuli' method than

(a) $N_f=1$, $N_c=2$

(b) $N_f=4$, $N_c=2$

(c) $N_f=1$, $N_c=5$

(d) $N_f=4$, $N_c=5$
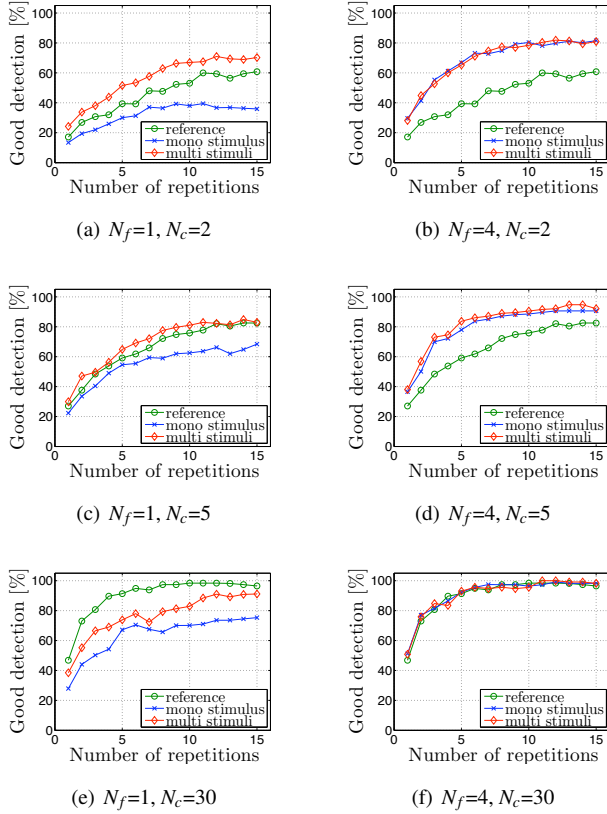
(e) $N_f=1$, $N_c=30$

(f) $N_f=4$, $N_c=30$

Figure 3: The percentage of success as a function of the number of trial repetitions. Each figure title indicates the number of components N_f of the P300 subspace and the number of symbols N_c used to train the spatial filters $\hat{U}_1$ and the linear classifier $\mathbf{w}$. The legend refers to the P300 enhancement algorithm used: 'reference' means no P300 enhancement (*i.e.* $\hat{U}_1$ is the identity matrix), 'mono stimulus' algorithm is described in [7] and 'multi stimuli' is the proposed method (Algo. 1).

with 'reference' method by ratio N_x/N_f (equal to 7.25 using $N_x = 29$ sensors and $N_f = 4$ components).

Finally, to obtain the best performance, one would choose to use more training symbols. However, this solution has the drawback to need more time before using the BCI online. Spelling one symbol with 15 repetitions takes about 30 seconds: this means that about 15 minutes are requested to record 30 training symbols while only about 2.5 minutes are requested to record 5 training symbols with a slight loss of performance (Fig. 3(d) and 3(f)). Thus the proposed 'multi stimuli' method increases the BCI ergonomics.

5. CONCLUSIONS AND PERSPECTIVES

In this paper, a new unsupervised method to enhance evoked response by target stimuli in an oddball paradigm was presented. Given the time indexes of rows/columns intensifications, the proposed algorithm estimates the main components of the P300 subspace. It was shown to efficiently improve the quality of the evoked subspace thanks to the assumption that each stimulus induces a response A_2 while target stimuli induce, in addition, a typical response A_1. Using this method

before the classification BCI task allows to speed up the BCI since less words are required to train the spatial filters and the linear classifier given . Moreover, using this spatial enhancement significantly reduces the dimension of the parameter vector used to the prediction word.

In further work, it would be interesting to consider that the latency of synchronised response between different stimuli is not constant and to further investigate the consequence of two consecutive target stimuli in the synchronised response.

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