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BACKGROUND SIGNAL ESTIMATION FROM MULTI-SENSOR SIGNALS BASED ON OUTER PRODUCT AND NON-LINEAR FILTERS

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ABSTRACT

The source separation based on a statistical and computational technique is one of the most exciting topic in a multivariate signal analysis. We proposed a novel source separation technique using a tensor product expansion[1]. This technique is the signal separation that the background noise, which is observed in almost all input signals, can be estimated by using a tensor product expansion where the absolute error is used as the error function (TPE-AE). The effectiveness of TPE-AE for artificial signals and real signals is represented in [1]. However, TPE-AE has two problems; one is that a result of TPE-AE is strongly affected by Gaussian random noise, another one is that an estimated signal varies widely due to a random search. In order to solve these problems, signal estimation techniques based on a median and Modified Trimmed Mean (MTM) are proposed in this paper. These methods calculate the outer product using a non-linear filter to estimate the background noise. Results show that novel techniques can separate the background noise more accurately than the conventional TPE-AE.

1. INTRODUCTION

It is well known that the multivariate analysis is useful for extracting the features of bivariable functions. The major method of extracting the characteristics of second-order statistics like principal component analysis (PCA) produces some global feature of various data. PCA is the powerful tool for the feature extraction, however, PCA does not focus on the signal separation.

A tensor product expansion (TPE)[2][3] can approximate an *m*-variable function as the sum of the product of *m* single variable functions (SVFs). This technique is applied to nonlinear system identification[4], 3-D image processing[5], and achieve a substantial results, respectively. The tensor is calculated by minimizing a mean square error (MSE) between the input vector and the sum of the product of SVF. Assume that an input signal expressed by 2-D matrix is composed of the background noises and local signals, the signal separation using TPE is difficult since local signals are treated as outliers. Main cause is that the separated signal is strongly affected by local signals due to MSE. We have shown that a tensor product expansion with absolute error (TPE-AE)[1] is effective to separate local signals from the background noise. The local signal is observed in just few signals while the background noise is seen in most signals (Fig.1). TPE-AE is also effective to estimate the background noise from the electromagnetic wave data observed at ELF band [6]. However, a result of TPE-AE is strongly affected by Gaussian random noise. Moreover, it is difficult to estimate the background noise uniformly due to the random search on the calculation of TPE-AE. In this paper, signal estimation techniques based on a median and MTM are proposed to overcome the weakness of TPE-AE. These methods calculate the outer product using a non-linear filter to estimate the background noise without the error function.

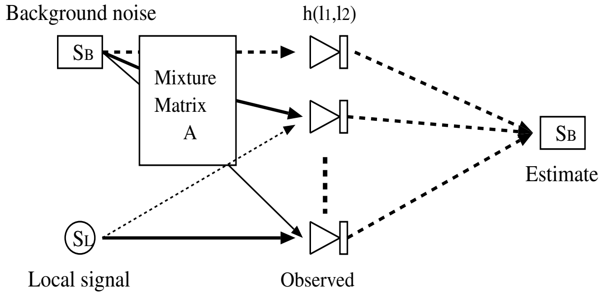


Figure 1: TPE-AE model

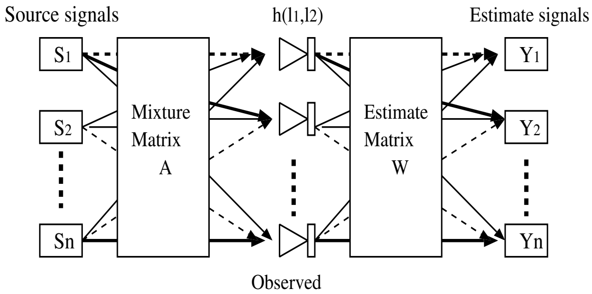


Figure 2: General model

This paper is organized as follows. Sect.2 gives the outline of TPE-AE. The background noise estimation techniques based on a median and MTM are explained in Section 3. In Sect.4, some computer simulation results using an artificial signal are presented. Finally, the conclusion is given in Section.5.

2. CONVENTIONAL METHOD

2.1 TPE-AE

Tensor product expansion (TPE) is known that the 2-D TPE is the same as singular value decomposition, hence, it can be considered that 2 vectors derived by TPE are the first principal component and its eigenvector, respectively.

Assume that the observed signal consists of two source signals, the one is observed in most signals (a background noise) while another is seen in just few signals (a local signal). In this case, the latter signal can be considered as the outlier. The separation problem becomes the simple model. It is only necessary to extract either few outliers or the background noise from the observed signals (Fig.1), while major blind source separation (i.e. independent component analysis) often estimates the n sources from n input signals [7](Fig.2).

In order to separate a background noise and a local signal, the absolute error was employed as a new criterion to calculate the tensor product[1]. It is known that an absolute error have little influence of outliers. Let $h(l_1, l_2)$ be an input 2-D matrix consists of observation signals, the tensor of a 2-D matrix calculated by TPE-AE is given as:

$$J = \sum_{l_1=1}^{q_1} \sum_{l_2=1}^{q_2} |h(l_1,l_2) - (f_1(l_1) f_2(l_2) + f_3(l_2))| \tag{1}$$

where J is the error function, $f_1(l_1)f_2(l_2)$ and $f_3(l_2)$ are approximation terms of AC and DC components, respectively. q_1 is the length of the signal, q_2 is the number of observation signals, $f_1(l_1)$ and $f_2(l_2)$ are l_1th and l_2th element of a vector, respectively. $f_1(l_1), f_2(l_2)$ and $f_3(l_2)$, which yield minimum J, gives us the global noise included in the input matrix $h(l_1, l_2)$.

The simple and reasonable method based on Monte Carlo Simulation (MCS) was applied to calculate the optimal $f_1(l_1), f_2(l_2), f_3(l_2)$. MCS produces optimal solutions by extensive trials using random numbers. The feasibility and separability conditions of this method for the background noise estimation are confirmed in [1].

By using TPE-AE, an estimated background noise is represented by $f_1(l_1)f_2(l_2) + f_3(l_2)$. Thus, a local signal can be calculated by subtracting $f_1(l_1)f_2(l_2) + f_3(l_2)$ from $h(l_1, l_2)$.

3. BACKGROUND NOISE ESTIMATION USING NON-LINEAR FILTER

TPE-AE calculates the tensor product which minimizes the sum of an absolute error between an input data $h(l_1, l_2)$ and a tensor product $f_1(l_1)f_2(l_2) + f_3(l_2)$. However, we need to define the optimal variance of a random number in MCS since the large variance does not give us correct solutions while small numbers yield the local minimum. Moreover, MCS cannot estimate the background noise uniformly due to the random search. In order to solve these problems, we propose the estimation techniques of the background noise based on the outer product calculated by the nonlinear filter.

3.1 Median method

The median is applied to calculate the outer product for avoiding the influence of the random search. The median filter is generally used to reduce an impulsive noise in an image data. The median is calculated by sorting all the values from the neighborhood into numerical sequence and then finding the middle value of the input sequence (Fig.3).

Assume that $f_2(l_2), f_3(l_2)$ is a constant value, respectively. We calculate an optimal $f_1(l_1)$ to minimize the criterion (1). The absolute error J_{f_1} at $l_1 = m$ is gives as:

$$J_{f_1} = \sum_{l_2=1}^{q_2} |h(m,l_2) - (f_1(m) f_2(l_2) + f_3(l_2))|. \tag{2}$$

$f_1(m)$, which yields $J_{f_1} = 0$ at $l_2 = n$, is expressed as:

$$f_1(m) = (h(m,n) - f_3(n))/f_2(n). \tag{3}$$

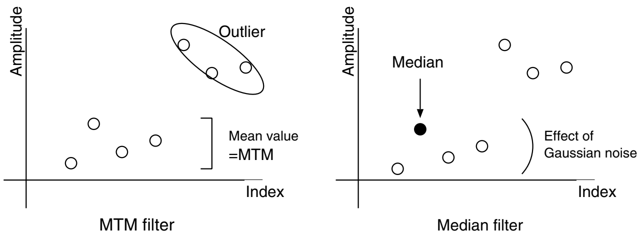


Figure 3: Non-linear filters

Then we get q_2 solutions from (2) and (3). Since the median minimizes the sum of an absolute error in data, the middle value of $f_1(m)$ gives us the minimal J_{f_1} in (2). Therefore, we have following update formula:

$$f_1(l_1) = \mathrm{med}_{l2}((h(l_1,l_2) - f_3(l_2))/f_2(l_2)). \tag{4}$$

where $\mathrm{med}_{l2}()$ returns a median in the window l_2 of width q_2. Similarly, update algorithms for $f_2(l_2), f_3(l_2)$ are obtained by the following expressions.

$$f_2(l_2) = \mathrm{med}_{l1}((h(l_1,l_2) - f_3(l_2))/f_1(l_1)) \tag{5}$$

$$f_3(l_2) = \mathrm{med}_{l1}(h(l_1,l_2) - f_1(l_1) f_2(l_2)) \tag{6}$$

(4) - (6) produces background noises by extensive trials using a median. At first, $f_1(l_1), f_2(l_2), f_3(l_2)$ are initialized by a small random number. The outer product to estimate the background is given as following steps:

I. Estimate the $f_3(l_2)$ using (6)
II. Estimate the $f_2(l_2)$ using (5)
III. Estimate the $f_1(l_1)$ using (4)

The termination criteria for above iterative steps should be decided. The iteration time is referred as K_{MED}. This method calculates the outer product indirectly without the error function by using a median.

3.2 MTM method

In TPE-AE, the background noise, which is observed concurrently with local signals, is not estimated accurately due to an effect of Gaussian signal. In order to improve the estimation performance, MTM [8][9] is applied to calculate the outer product corresponding to a background noise. TPE-AE and a median method focus on the minimization problem of the sum of the absolute error between an input matrix and its outer product. MTM method is not focused on any criteria.

The running median calculates the median of the observations in the window of width $2k+1$.

$$\mathrm{MED}(x_{l_1}) = \mathrm{med}(x_{l_1-k}, \dots, x_{l_1+k}), l_1 \in Z. \tag{7}$$

Trimmed means were proposed as a compromise between moving averages and medians. MTM defines the amount of trimming signals depending on the current time window. Input data, which is closer than a given distance p_{l_1} from the middle value, is averaged by MTM filter (Fig.3). The calculation is given as:

$$\mathrm{MTM}(x_{l_1}) = \frac{1}{|I_{l_1}|} \sum_{i \in I_{l_1}} x_{l_1+i} \tag{8}$$

$$I_{l_1} = i = -k, .., k : |x_{l_1+i} - \hat{\mu}_{l_1}| \le p_{l_1}$$

$$\hat{\mu}_{l_1} = \mathrm{med}(x_{l_1-k}, \dots, x_{l_1+k}), l_1 \in Z.$$

If $p_{l_1} = 0$, (8) outputs the median while $p_{l_1} = \infty$ gives us the moving average. In order to obtain p_{l_1}, the median absolute derivation (MAD) or a standard validation is often used to achieve a robust estimation. This paper applies the MAD method expressed as:

$$\hat{\sigma}^M_{l_1} = c_n \cdot \mathrm{med}(|x_{l_1-k} - \hat{\mu}_{l_1}|, ..., |x_{l_1+k} - \hat{\mu}_{l_1}|). \quad (9)$$

where $p_{l_1} = 2\hat{\sigma}^M_{l_1}$, c_n is the important factor to define the threshold for a trimming. c_n depends on the window widths q_1 and q_2 of input vectors. MTM filter provides a reliable and an accurate estimation since outliers, which are at the extremes of the sorted list, are not averaged.

Assume that $\mathrm{MTM}_{li}()$ outputs the MTM from (8) in the window l_i of width q_i. The update formula for $f_1(l_1), f_2(l_2), f_3(l_2)$ based on MTM is expressed as:

$$f_1(l_1) = \mathrm{MTM}_{l2}((h(l_1,l_2) - f_3(l_2))/f_2(l_2)) \quad (10)$$
$$f_2(l_2) = \mathrm{MTM}_{l1}((h(l_1,l_2) - f_3(l_2))/f_1(l_1)) \quad (11)$$
$$f_3(l_2) = \mathrm{MTM}_{l1}(h(l_1,l_2) - f_1(l_1)f_2(l_2)) \quad (12)$$

The outer product, which approximates the background noise, is calculated by the iterative algorithm using (10) - (12). The iteration time of this algorithm is referred as K_{MTM}. We call this iterative algorithm a MTM method.

4. SIMULATION RESULTS

4.1 Artificial signal

The simulation results of a median and a MTM method are shown to demonstrate its effectiveness for the background noise reduction. A simple artificial signal based on known functions is applied to the conventional and proposed methods. The artificial signal composed of a background noise component and a local signal are generated by a sine wave and a block pulse, respectively. The separability condition of TPE-AE was confirmed in [1]. Assume that a background noise is observed at almost input signals while a local signal is included in a few signals. Table 1 lists the conditions of the input matrix $h(l_1,l_2)$ shown in Fig.4, where l_1 indicates the index of the time courses, l_2 is the index of the artificial signal A-E (0 to 4). The vertical axis in Fig.4 shows the amplitude of an artificial signal, the horizontal axis indicates the cycle of sine wave ($l_1/576$). In the input matrix, 2 signals include a local signal in cycles 7 to 10.

According to an empirical result, K_{MED}, K_{MTM} are defined as 10, c_n is 1.7. The averaged absolute errors between the estimated signal ($f_1(l_1)f_2(l_2) + f_3(l_2)$) and an actual background noise (sine wave) are shown in Fig.5. This figure indicates errors of 100 trials using different initial numbers. The vertical axis shows the averaged absolute error, the horizontal axis indicates the index of each trial. From Fig.5, the averaged absolute error of TPE-AE is distributed from 0.04 to 0.12 while the errors of a median and a MTM method is found in 0.05 and 0.03, respectively. This is attributed to the fact that TPE-AE often gives us the poor results since the MCS is strongly affected to the initial number and the random number of the update algorithm.

From Fig.5, the absolute error of a MTM method is smaller than its median method. In order to identify a specific cause of this fact, the local signals, which are the result of subtracting an estimated signal from an input signal, are shown in Fig.6 and Fig.7. The horizontal and the vertical axis are the same as Fig.4. From results of TPE-AE and median method, the local signal constructed of a block pulse is shrunk during 7 - 10 cycle. On the other hand, the local signal calculated by a MTM method does not yield the decay of signals. The main cause is that if the local signal is included less than just half of observed signals, a median filter outputs the largest value of the Gaussian noise while MTM filter outputs its mean value (Fig.3). It means that a MTM method does not shrink local signals compared to other methods. Thus, a MTM method is more effective in eliminating the background noise than a median method and TPE-AE.

Table 1: Conditions of the artificial signal

	Background noise	Block pulse
A	sin(2π/576)/3.0+N(0.8,0.05²)	none
B	sin(2π/576)/3.0+N(1.2,0.05²)	0.3
C	sin(2π/576)/4.0+N(1.0,0.05²)	none
D	sin(2π/576)/5.0+N(0.8,0.05²)	0.6
E	sin(2π/576)/5.0+N(1.2,0.05²)	none

:N(μ, σ^2) means Gaussian noise μ is mean, $\sigma^2$is variance

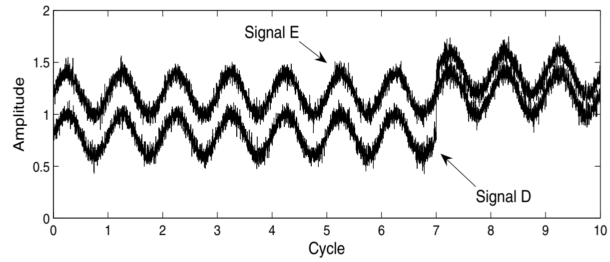


Figure 4: Input signal

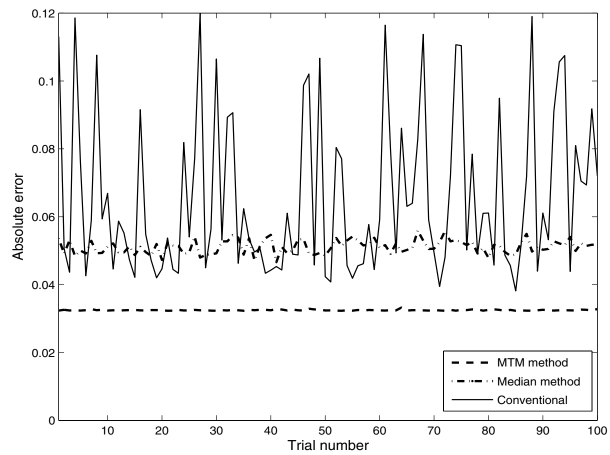


Figure 5: Absolute errors for conventional and proposed methods

4.2 Sound data

In order to demonstrate the separability of the proposed method, the artificial signal composed of two real sounds is applied to three methods. Real sounds shown in Fig.8 are the recorded vowel /a/ and a sound of clap hands sampled at 8000Hz. The vowel and a clap are treated as the background signal and a local signal, respectively. Table 2 indicates the conditions of the input matrix $h(l_1,l_2)$ without a time delay,

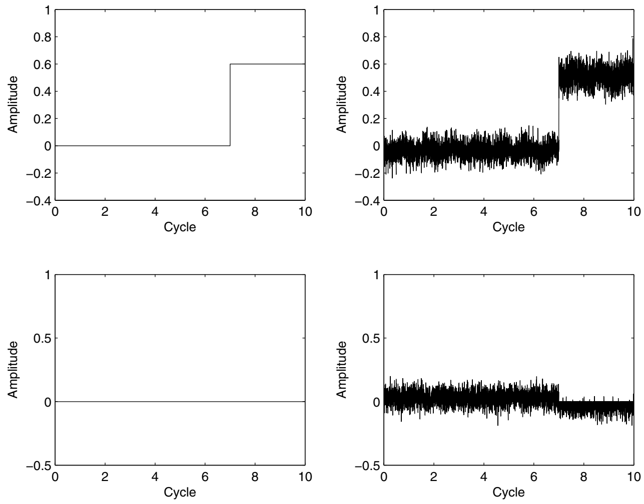


Figure 6: The result of subtracting estimated signals from an input signal: Original signal (left hand side), TPE-AE (right hand side)

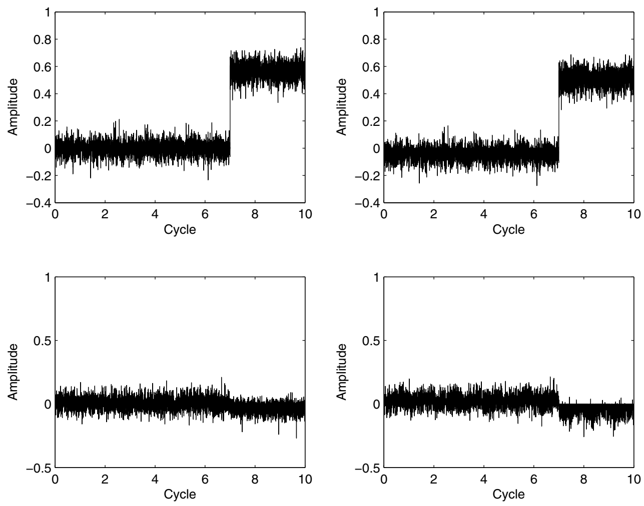


Figure 7: The result of subtracting estimated signals from an input signal: MTM method (left hand side), Median method (right hand side)

where S_B is the background signal (Fig.8(a)), S_L means the local signal (Fig.8(b)). The purpose of this simulation is to separate the vowel and a clap from the input signal shown in Fig.9.

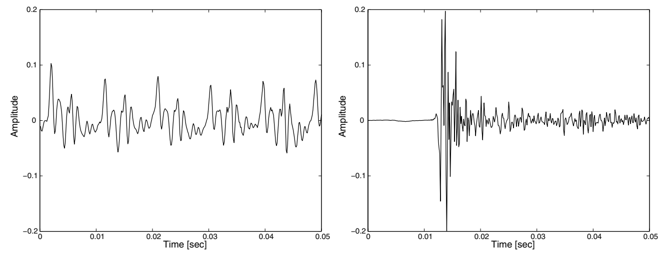
K_{MED}, K_{MTM} are 10, c_n is 1.7. The averaged absolute errors between the estimated background signal and an actual background signal (vowel) are plotted to Fig.10. This figure includes results of 100 trials using different initial numbers. The vertical and horizontal axes are the same as Fig.5. The separated background signals and local signals are shown in Fig.11 and Fig.12. The vertical line indicates the amplitude, the horizontal line is the time (sec).

From Fig.11 and 12, the background signal and the local signal, which are composed of a vowel and a clap respectively, are separated by using three methods. Moreover, a few difference between local signals estimated by each techniques is found in these figures.

Table 2: States of the input signal

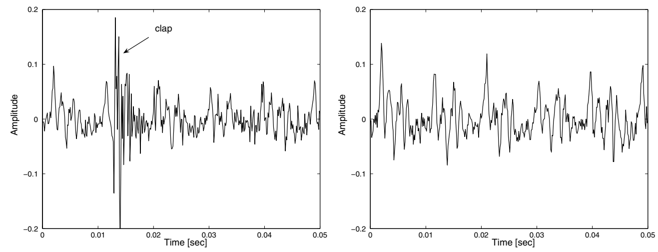
	Background signal	Local signal
A	$S_B \cdot 1.0$+N(0,0.01^2)	$S_L/2$
B	$S_B \cdot 0.8$+N(0,0.01^2)	S_L
C	$S_B \cdot 1.2$+N(0,0.01^2)	none
D	$S_B \cdot 1.0$+N(0,0.01^2)	none
E	$S_B \cdot 0.9$+N(0,0.01^2)	none

:N(μ,σ^2) means Gaussian noise μ is mean, $\sigma^2$is variance



(a) (b)

Figure 8: Sound data: (a)Vowel /a/, (b) Clap



(a) (b)

Figure 9: Input signals: (a)Signal B, (b) Signal C

From Fig.10, the mean of absolute errors for TPE-AE is distributed about 23.5×10^{-3} while the result of median and a MTM method is found in 5×10^{-3} and 4.5×10^{-3}, respectively. Compared to other methods, TPE-AE often gives us the poor estimation results, the median method does not produce the estimation result uniformly.

From these results, we confirmed that a MTM method is more effective in signal separation from the artificial signal composed of real data than median method and TPE-AE.

5. CONCLUSION

In this paper, we proposed signal estimation methods based on the nonlinear filters that calculate the outer product. The conventional method using TPE-AE is strongly affected by Gaussian random noise when the anomalous signal is included. Moreover, it is difficult to estimate the background noise uniformly due to a difficulty on the calculation of TPE-AE. Simulation results have shown that a median and a MTM method can estimate the background noise more uniformly than a conventional method. Other results have shown that a MTM method can estimate a background noise and a local signal without the shrinkage of each component.

The separability conditions of TPE-AE had shown in a conventional research. However, the separability of a MTM method is not clarified yet. Remaining problems are to extend the proposal to the signal separation with time delay and to show the separability condition of a MTM method.

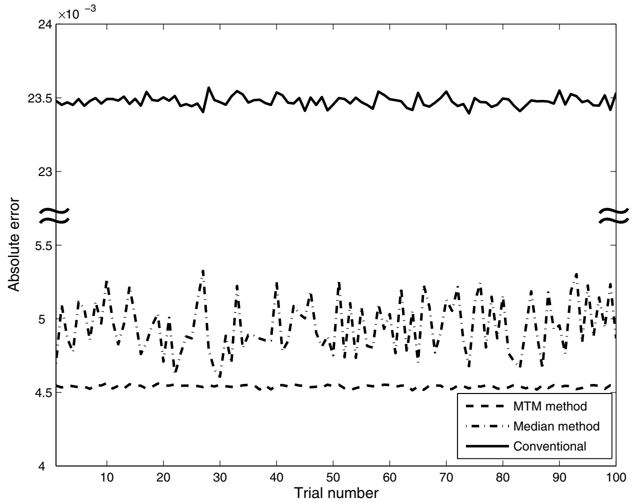


Figure 10: Absolute errors for conventional and proposed methods

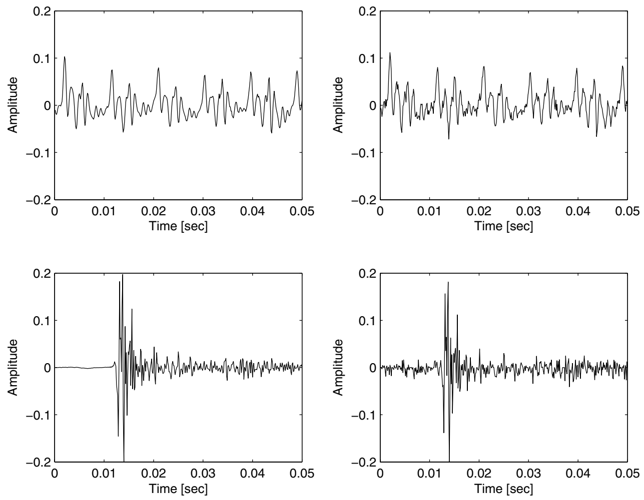


Figure 11: Estimated background signal and local signal: Original signal (left hand side), TPE-AE (right hand side)

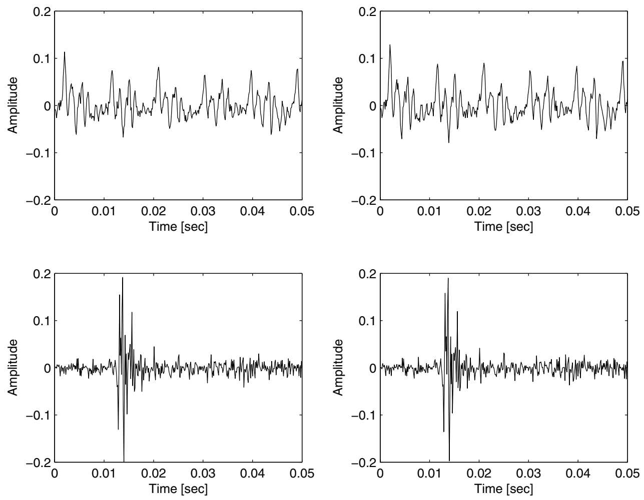


Figure 12: Estimated background signal and local signal: MTM method(left hand side), Median method (right hand side)

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