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RANDOM DISCRETE MEASURE OF THE PHASE POSTERIOR PDF IN TURBO SYNCHRONIZATION

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ABSTRACT

In this paper we consider the iterative decoding of channels with strong phase noise. We propose to use a random discrete measure to estimate the phase posterior pdf given the past observations (forward pdf) and another random discrete measure to estimate the phase posterior pdf given the future observations (backward pdf). The particle filter algorithm is used to recursively generate the supports in the relevant phase space area and recursively update the weights associated to these supports. An estimation of the phase posterior pdf given all the past and future observations is then derived from the forward and backward measures. The relevance of our proposal is finally illustrated through simulation of binary LDPC codes and QPSK modulation over a severe Wiener-Levy phase noise with a standard deviation of $\sigma_\Delta = 6$ degrees. Our algorithm is compared with a forward-backward message passing algorithm performed over a trellis resulting from the discretization of the phase. The proposed algorithm leads to a a slight performance degradation compared to the optimal treillis based method.

1. INTRODUCTION

Phase tracking has attracted an increasing interest in coherent digital communications. Since estimating jointly the phase and the data is generally intractable, a number of suboptimal algorithms have been proposed using a phase estimator followed by a data detector.
In order to improve the performance, one can use iterative estimation : at each iteration, a phase estimation is performed using the soft channel decoder output from the previous estimation, and then the soft channel decoder input is calculated using the phase estimation.
If the channel phase is constant during one frame, the Expectation Maximisation (EM) algorithm [1] can be performed. When the phase is varies during the frame, the EM algorithm cannot be directly applied and we need to perform a sliding window version of the EM algorithm.
Assuming that the phase variation statistics are known by the receiver (Bayesian approach), it becomes possible to give analytical expressions of the time evolution of the phase posterior pdf given the past observations (forward pdf) and the time evolution of the phase posterior pdf given the future observations (backward pdf). These two pdf directly lead to the complete phase posterior pdf (given the past and the future observations). Yet, the expressions giving the time evolution of the forward and backward posterior pdf involve inextricable integral calculations, and they cannot be directly exploited. Two types of suboptimal methods exist to estimate these pdf. One consists in assuming that the pdf belongs to a parameterized family of functions [2]: the family of functions is chosen so that an analytical solution of the integrals can be reached.
The other type of method, based on the phase discretization, is more efficient but requires a greater complexity: it consists in dividing the phase state $[0 \quad 2\pi)$ into Q equispaced intervals and transforming the integrals into discrete sums [3]. The phase posterior pdf are then replaced by probability mass functions on the discretized phase space and are recursively computed. Using a factor graph representation, such methods can be seen as a forward backward message passing algorithm [4]. If Q is the number of phase states and M the modulation size, the complexity of the trellis algorithm is in $\mathrm{O}(MQ^2)$. One solution to reduce this complexity is to assume that only a few number of phase state transition R are possible between two successive sampling times. The complexity of this "reduced Trellis" algorithm is then in $\mathrm{O}(MQR)$.
In this paper, we propose to use two random discrete measures (RDM) to estimate the phase posterior pdf given the past observations and the phase posterior pdf given the future observations. These RDM are estimated only in the relevant zone, using two Sequential Importance Sampling Resampling (SISR) algorithms, also known as Particle Filter (PF) algorithms [5]. The PF algorithm recursively generates the RDM supports and update the RDM weights. An estimation of the phase posterior pdf given all the past and future observations is then derived from the forward and backward measures.
This study follows some preliminary work presented in [6], where the particle filter was used to calculate the forward and backward messages of the message-passing algorithm. The use of particle filter for strong phase noise synchronization was also proposed in [7], but the authors only used the PF to calculate a phase estimation, while we propose here to estimate the phase posterior pdf.
The rest of the paper is organized as follow: the system model is given in section 2, and the symbol posterior probability (for the decoder input) is calculated in section 3, as a function of the forward and backward phase posterior pdf. The

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trellis method and the particle filter method are respectively described in subsections 3.1 and 3.2. Section 4 presents and discusses the simulation results and section 5 draws the concluding remarks of this paper.

2. SYSTEM MODEL

Let $\{b_l\}_{l=1...L}$ be a sequence of coded bits and $\{a_n\}_{n=1...N}$ its mapping to a M-PSK constellation $\mathscr{X}$. Let $\{c_k\}_{k=1...K}$ be the transmitted sequence built from $\{a_n\}$ after N_p pilot symbols have been inserted ($K = N + N_p$). We consider the following equivalent baseband complex system model:

$$r_k = c_k e^{j\theta_k} + n_k \qquad k = 1,\dots,K, \tag{1}$$

where n_k is a complex circular gaussian noise with variance σ^2. We assume that the phase noise θ_k can be modelled by the following Wiener-Levy (random walk) process:

$$\theta_k = (\theta_{k-1} + \Delta_k) \mod 2\pi, \tag{2}$$

where Δ_k is a white Gaussian noise with variance σ_Δ^2. This phase evolution model is a first order Markov process:

$$p(\theta_k|\theta_{k-1},\dots,\theta_0) = p(\theta_k|\theta_{k-1}). \tag{3}$$

This evolution model is symmetrical and one can write:

$$\theta_k = (\theta_{k+1} - \Delta_{k+1}) \mod 2\pi, \tag{4}$$

$$p(\theta_k|\theta_{k+1},\dots,\theta_K) = p(\theta_k|\theta_{k+1}). \tag{5}$$

(2) and (4) respectively represent the forward and backward phase evolution model.

3. SYMBOL POSTERIOR PROBABILITY

The decision problem is given by:

$$\begin{aligned}\hat{b}_l &= \arg\max_{b_l} p(b_l|\mathbf{r}) \\ &= \arg\max_{b_l} \int_\theta p(b_l,\theta|\mathbf{r})\mathrm{d}\theta \\ &= \arg\max_{b_l} \int_\theta \sum_{\mathbf{b}=\{b_m\}_{m\neq l}} p(\mathbf{b},\theta|\mathbf{r})\mathrm{d}\theta. \end{aligned} \tag{6}$$

This optimal solution is generally intractable and a suboptimal solution is to use an iterative estimation. At each iteration, a phase estimation is performed using the soft information bits provided by the channel decoder, and then the soft information bits are calculated using the phase estimation. The soft information bit is given by $\lambda(b_l) = \log(\frac{p(b_l=1|\mathbf{r})}{p(b_l=0|\mathbf{r})})$. $\lambda(b_l)$ can be directly calculated from the set of symbol posterior probabilities $\{p(c_k = S|\mathbf{r})\}_{S\in\mathscr{X}}$, where c_k is the symbol which contains b_l. To simplify the notation, symbol S is omitted and $p(c_k = S|\mathbf{r})$ is simply noted $p(c_k|\mathbf{r})$. Marginalizing $p(c_k|\mathbf{r})$ on the phase parameter θ_k and applying the Bayes relation leads to:

$$\begin{aligned} p(c_k|\mathbf{r}) &= \int_{\theta_k} p(c_k,\theta_k|\mathbf{r})\mathrm{d}\theta_k \\ &= \int_{\theta_k} p(c_k,\theta_k|\mathbf{r}_{1:k-1},\mathbf{r}_{k+1:K},r_k)\mathrm{d}\theta_k \\ &\propto \int_{\theta_k} p(r_k|c_k,\theta_k,\mathbf{r}_{1:k-1},\mathbf{r}_{k+1:K}) \\ &\qquad \times p(c_k,\theta_k|\mathbf{r}_{1:k-1},\mathbf{r}_{k+1:K})\mathrm{d}\theta_k \\ &\propto \int_{\theta_k} p(r_k|c_k,\theta_k) \\ &\qquad \times p(c_k|\theta_k,\mathbf{r}_{1:k-1},\mathbf{r}_{k+1:K}) \\ &\qquad \times p(\theta_k|\mathbf{r}_{1:k-1},\mathbf{r}_{k+1:K})\mathrm{d}\theta_k. \end{aligned} \tag{7}$$

The first term of the integral in (7) is the complex circular gaussian pdf:

$$p(r_k|c_k,\theta_k) = \mathscr{N}_\mathbb{C}(r_k, c_k e^{j\theta_k}, \sigma), \tag{8}$$

where $\mathscr{N}_\mathbb{C}(x,m,\sigma) \stackrel{\Delta}{=} \frac{1}{2\pi\sigma^2}\exp\{-\frac{1}{2\sigma^2}\|x-m\|^2\}$. The second term in (7) is equal to the symbol prior probability

$$p(c_k|\theta_k,\mathbf{r}_{1:k-1},\mathbf{r}_{k+1:K}) = \frac{1}{M}. \tag{9}$$

(7) therefore becomes:

$$p(c_k|\mathbf{r}) \propto \int_{\theta_k} \mathscr{N}_\mathbb{C}(r_k, c_k e^{j\theta_k},\sigma) p(\theta_k|\mathbf{r}_{1:k-1},\mathbf{r}_{k+1:K})\mathrm{d}\theta_k. \tag{10}$$

$p(\theta_k|\mathbf{r}_{1:k-1},\mathbf{r}_{k+1:K})$ can be expressed as a function of the forward posterior pdf $p(\theta_k|\mathbf{r}_{1:k-1})$ and the backward posterior pdf $p(\theta_k|\mathbf{r}_{k+1:k})$:

$$\begin{aligned} p(\theta_k|\mathbf{r}_{1:k-1},\mathbf{r}_{k+1:K}) &\propto p(\mathbf{r}_{1:k-1},\mathbf{r}_{k+1:K}|\theta_k) \\ &\propto p(\mathbf{r}_{k+1:K}|\theta_k,\mathbf{r}_{1:k-1})p(\mathbf{r}_{1:k-1}|\theta_k) \\ &\propto p(\mathbf{r}_{k+1:K}|\theta_k)p(\mathbf{r}_{1:k-1}|\theta_k) \\ &\propto p(\theta_k|\mathbf{r}_{1:k-1})p(\theta_k|\mathbf{r}_{k+1:K}), \end{aligned} \tag{11}$$

since given θ_k, $\mathbf{r}_{k+1:K}$ is independent on $\mathbf{r}_{1:k-1}$, $p(\theta_k)$ is a uniform pdf and $p(\mathbf{r}_{1:k-1})$, $p(\mathbf{r}_{k+1:K})$, $p(\mathbf{r}_{1:k-1},\mathbf{r}_{k+1:K})$ do not depend on θ_k.

Equation (10) gives an analytical relationship between the continuous pdf $p(\theta_k|\mathbf{r}_{1:k-1},\mathbf{r}_{k+1:K})$ and the symbol posterior probability. The phase discretization approach consists in approximating the continuous pdf $p(\theta_k|\mathbf{r}_{1:k-1},\mathbf{r}_{k+1:K})$ by a discrete probability measure: the phase space $[0 \quad 2\pi)$ is divided into Q equal intervals ϕ_q of width $\frac{2\pi}{Q}$ and center $\psi_q = \frac{(2q-1)\pi}{Q}$. Consequently, the continuous pdf $p(\theta_k|\mathbf{r}_{1:k-1},\mathbf{r}_{k+1:K})$ is replaced by the discrete probabilities $\{p_u(\theta_k \in \phi_q)\}_{q=1,\dots,Q}$ defined by:

$$p_u(\theta_k \in \phi_q) \stackrel{\Delta}{=} p(\theta_k \in \phi_q|\mathbf{r}_{1:k-1},\mathbf{r}_{k+1:K}). \tag{12}$$

Now $p(c_k|\mathbf{r})$ can be derived from p_u with a discrete sum on q:

$$p(c_k|\mathbf{r}) \propto \sum_{q=1}^{Q} \mathscr{N}_\mathbb{C}(r_k, c_k e^{i\phi_q},\sigma) p_u(\theta_k \in \phi_q). \tag{13}$$

As in (11), $p_u(\theta_k \in \phi_q)$ can be expressed as a function of the forward posterior probability $p_f(\theta_k \in \phi_q) \triangleq p(\theta_k \in \phi_q|\mathbf{r}_{1:k-1})$ and the backward posterior probability $p_b(\theta_k \in \phi_q) \triangleq p(\theta_k \in \phi_q|\mathbf{r}_{k+1:K})$:

$$p_u(\theta_k \in \phi_q) \propto p_f(\theta_k \in \phi_q) p_b(\theta_k \in \phi_q). \quad (14)$$

According to (11) and (13), the knowledge of $p_f(\theta_k \in \phi_q)$ and $p_b(\theta_k \in \phi_q)$ directly leads to the symbol posterior probability. Two methods are now presented to estimate $p_f(\theta_k \in \phi_q)$ and $p_b(\theta_k \in \phi_q)$. In subsection 3.1 the classical trellis approach is presented and in subsection 3.2 our approach based on the RDM approximation is described. In these two sections $\{p(c_k = x)\}_{x\in\mathscr{X}}$ is the symbol probability provided by the decoder output (from the previous iteration).

3.1. trellis

In the trellis approach [3], $p_f(\theta_k \in \phi_q)$ and $p_b(\theta_k \in \phi_q)$ are recursively computed for all q using the following relationship:

$$\begin{aligned} p_f(\theta_k \in \phi_q) &= p(\theta_k \in \phi_q|\mathbf{r}_{1:k-1}) \\ &= \sum_{r=1}^{Q} p(\theta_k \in \phi_q, \theta_{k-1} \in \phi_r|\mathbf{r}_{1:k-1}) \\ &= \sum_{r=1}^{Q} p(\theta_k \in \phi_q|\theta_{k-1} \in \phi_r) \\ &\qquad \times p(\theta_{k-1} \in \phi_r|\mathbf{r}_{1:k-2}, r_{k-1}) \\ &\propto \sum_{r=1}^{Q} p(\theta_k \in \phi_q|\theta_{k-1} \in \phi_r) p(r_{k-1}|\theta_{k-1} \in \phi_r) \\ &\qquad \times p_f(\theta_{k-1} \in \phi_r), \end{aligned} \quad (15)$$

where the discrete transition probability $p(\theta_k \in \phi_q|\theta_{k-1} \in \phi_r)$ can be obtained by matching the moments of the discrete pdf and the Gaussian phase difference pdf associated to the model (2). $p(r_{k-1}|\theta_{k-1} \in \phi_r)$ is the observation likelihood, proportional to $\sum_{x\in\mathscr{X}} p(c_k = x) \times \mathscr{N}_{\mathbb{C}}(r_{k-1}, xe^{j\psi_r}, \sigma)$.
Equivalently, in the backward direction:

$$\begin{aligned} p_b(\theta_k \in \phi_q) &= p(\theta_k \in \phi_q|\mathbf{r}_{k+1:K}) \\ &= \sum_{r=1}^{Q} p(\theta_k \in \phi_q|\theta_{k+1} \in \phi_r) \\ &\quad \times p(r_{k+1}|\theta_{k+1} \in \phi_r) p_b(\theta_{k+1} \in \phi_r). \end{aligned} \quad (16)$$

The complexity of recursively computing the forward and backward probabilities using (15) and (16) is in $\mathrm{O}(MQ^2)$. To reduce it, it is often assumed that the phase state transition probabilities $p(\theta_k \in \phi_q|\theta_{k-1} \in \phi_r)$ and $p(\theta_k \in \phi_q|\theta_{k+1} \in \phi_r)$ are null when $\min\{(r-q)[Q], (q-r)[Q]\} > R$ where R is the maximal number of considered paths. The number of non-null terms in (15) and (16) is equal to R and the complexity is reduced to $\mathrm{O}(MQR)$.

3.2. random discrete measure

We propose to approximate the two continuous pdf $p(\theta_k|\mathbf{r}_{1:k-1})$ and $p(\theta_k|\mathbf{r}_{k+1:K})$ with two random discrete measures (RDM) $\{\theta^{(i)}_{f,k}, \tilde{w}^{(i)}_{f,k-1}\}$ and $\{\theta^{(j)}_{b,k}, \tilde{w}^{(j)}_{b,k+1}\}$ respectively. $\{\theta^{(i)}_{f,k}, \tilde{w}^{(i)}_{f,k-1}\}$ is also called a set of particles, $\theta^{(i)}_{f,k}$ being the particle support and $\tilde{w}^{(i)}_{f,k-1}$ the particle weight [5]. The difference between the support index (k) and the weight index ($k-1$) (resp. ($k+1$)) in the forward (resp. backward) RDM comes from the fact that we estimate a one-step prediction posterior pdf $p(\theta_k|\mathbf{r}_{1:k-1})$ (resp. $p(\theta_k|\mathbf{r}_{k+1:K})$) and not the online posterior pdf $p(\theta_k|\mathbf{r}_{1:k})$ (resp. $p(\theta_k|\mathbf{r}_{k:K})$).

$$p(\theta_k|\mathbf{r}_{1:k-1}) \approx \sum_{i=1}^{N_{part}} \tilde{w}^{(i)}_{f,k-1} \delta(\theta_k - \theta^{(i)}_{f,k}), \quad (17)$$

$$p(\theta_k|\mathbf{r}_{k+1:K})) \approx \sum_{j=1}^{N_{part}} \tilde{w}^{(j)}_{b,k+1} \delta(\theta_k - \theta^{(j)}_{b,k}). \quad (18)$$

N_{part} is the number of particles, $\tilde{w}^{(i)}_{f,k-1}$ is the normalized importance weight at time k associated with the particle i, $\delta(\theta_k - \theta^{(i)}_{f,k})$ denotes the Dirac function in $\theta_k = \theta^{(i)}_{f,k}$ and the subscripts f and b stand for "forward" and "backward".
We generate the supports and update the weights using the Sequential Importance Sampling Resampling (SISR) algorithm, also known as Particle Filter (PF) algorithm [5]. For the forward (resp. backward) algorithm, the particle supports are recursively generated using the first order Markov evolution model (2) (resp. (4)). It corresponds to the prior importance function. Consequently, the particle weights are updated using the observation likelihoods:

$$\text{forward PF:} \quad w^{(i)}_{f,k} = w^{(i)}_{f,k-1} p(r_k|\theta^{(i)}_{f,k}), \quad (19)$$

$$\text{backward PF:} \quad w^{(j)}_{b,k} = w^{(j)}_{b,k+1} p(r_k|\theta^{(j)}_{b,k}), \quad (20)$$

where the observation likelihoods $p(r_k|\theta^{(i)}_{f,k})$ and $p(r_k|\theta^{(j)}_{b,k})$ are calculated by marginalization on the transmitted symbol probabilities :

$$p(r_k|\theta_k) = \sum_{x\in\mathscr{X}} p(c_k = x) \mathscr{N}_{\mathbb{C}}(r_k, xe^{j\theta_k}, \sigma). \quad (21)$$

A resampling step is added to avoid the particle degeneracy and maintain the set of particles in the region of interest. The resampling is performed when the number of efficient particles is inferior to $N_{threshold} = N_{part}/3$.
Once the forward and backward RDM have been obtained,the two RDM are used to calculate $p_f(\theta_k \in \phi_q)$ and $p_b(\theta_k \in \phi_q)$:

$$
\begin{aligned}
p_f(\theta_k \in \phi_q) &= p(\theta_k \in \phi_q | \mathbf{r}_{1:k-1}) \\
&= \int_{\theta_k \in \phi_q} p_f(\theta_k) \mathrm{d}\theta_k \\
&\approx \int_{\theta_k \in \phi_q} \sum_{i=1}^{N_{part}} \tilde{w}_{f,k-1}^{(i)} \delta(\theta_k - \theta_{f,k}^{(i)}) \mathrm{d}\theta_k \\
&\approx \sum_{i|\theta_{f,k}^{(i)} \in \phi_q} \tilde{w}_{f,k-1}^{(i)}. \qquad (22)
\end{aligned}
$$

For the backward way:

$$
\begin{aligned}
p_b(\theta_k \in \phi_q) &= p(\theta_k \in \phi_q | \mathbf{r}_{k+1:K}) \\
&\approx \sum_{j|\theta_{b,k}^{(j)} \in \phi_q} \tilde{w}_{b,k+1}^{(j)}. \qquad (23)
\end{aligned}
$$

The complexity order of the particle filter is in O(MN_{part})+O(Q_{PF}), where Q_{PF} is the number of phase states considered in (22) and (23).

4. SIMULATION RESULTS

The performance of the proposed algorithm is compared to the discretized phase approach. The considered code is a (3,6)-regular LDPC code with codewords of length 4000. The chosen modulation is QPSK. A pilot symbol is inserted every 20 transmitted symbols, plus one at the end of the frame. As in [2], a severe phase noise model with $\sigma_\Delta = 6$ degrees is assumed. In each method we perform 10 global iterations between the channel decoder and the phase estimator. In each global iteration, 20 iterations over the LPDC graph are performed. Figures 1 and 2 show examples of the backward and forward random discrete measures and the reconstructed global posterior pdf at the first global iteration and the fourth global iteration respectively. For these figures, 32 particles are used. The pdf are observed at sampling time $k = 410$ and the true phase was equal to 6.26 radians. The forward, backward and complete pdf estimated using the two filters are close to the pdf given by the trellis (figure 1). After four global iterations, the estimated complete pdf is composed of only one peak centered on the true phase value.

Figure 3 and Figure 4 give respectively the frame error rate and bit error rate for different values of E_b/N_0. For each simulation point, at least 50 frame-errors have been received. $N_{part} = 8, 16, 32, 64$ particles have been used for the particle filters algorithm with $Q_{PF} = 16$. For the phase discretization (trellis) method, 8, 16 and 32 phase states have been considered. The performance obtained for $Q = 32$ can be considered as the maximum achievable performance [3], but no performance degradation have been observed for the reduced treillis with $Q = 16$. Compared to the treillis methods (full or reduced), a degradation of less than 0.1 dB is observed for the 48 particles proposed algorithm (target BER = 0.01). A degradation of 0.15 dB (resp. 0.3 dB) is observed for the 16 particles (resp. 8 particles) proposed algorithm.

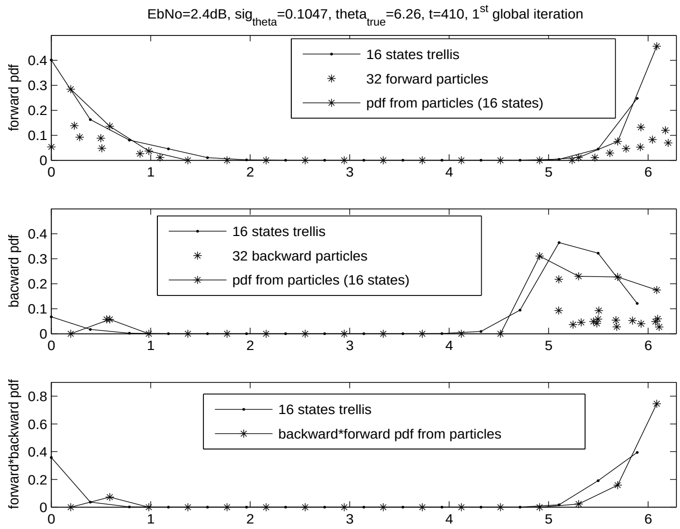

Fig. 1. Illustration of the two random measure before the first decoding

5. CONCLUSIONS

In this paper we have considered the iterative decoding of channels with strong phase noise. We propose to estimate the phase forward (resp. backward) posterior pdf by using a random discrete measure which is recursively updated with the particle filter algorithm. The complete phase posterior pdf (given the past and future symbols) is the product of the two pdf. The two RDM are therefore used to evaluate the forward and the backward pdf on a common quantification of $[0 \quad 2\pi)$ and the product of these two discretized pdf gives an estimation of the complete phase posterior pdf. The relevance of our proposal was evaluated through simulations, showing only a slight degradation compared to the optimal treillis based method. The proposed approach could also be generalized to track the joint posterior pdf of several parameters such as the phase, the frequency shift and the channel gain [11].

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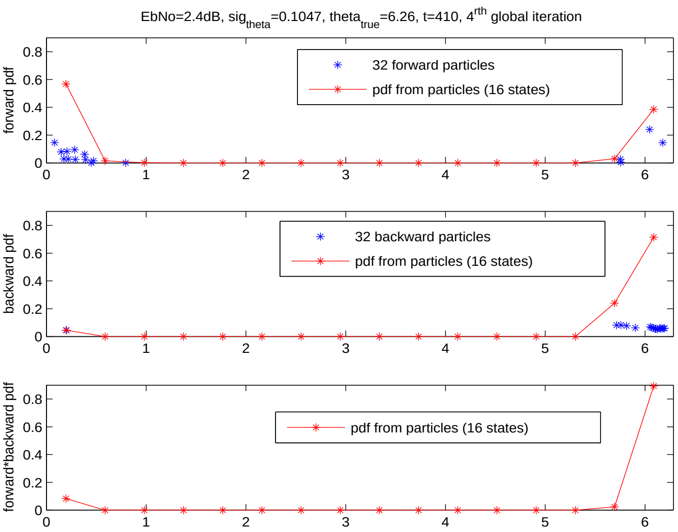

Fig. 2. Illustration of the two random discrete measure after 4 global iteration

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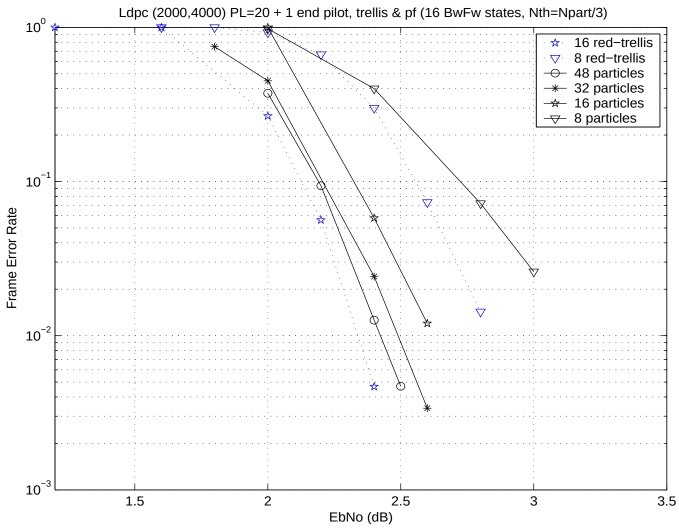

Fig. 3. Frame Error Rate performances of the trellis and proposed algorithms

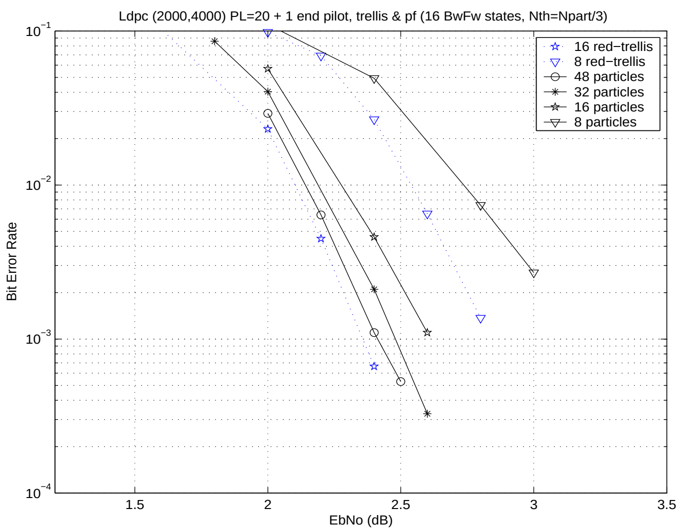

Fig. 4. Bit Error Rate performances of the trellis and proposed algorithms