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A SIMPLE CONSTRAINED BASED ADAPTIVE NULL STEERING ALGORITHM

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ABSTRACT

This paper presents and investigates a simple constrained based adaptive null steering algorithm. In the environment where the desired signal is stronger or not significantly weaker than the interferences, nulling in the direction of interest becomes a problem for conventional zero tracking algorithm. With the proposed algorithm in this paper, the null positions can be selectively tracked one after another without affecting other null positions so that the interference signals can be rejected while the desired signal preserves. The main advantage of the proposed algorithm is its simplicity. When compared with conventional LMS based Linear Constrained Minimum Variance (LCMV) beamforming algorithm, the proposed method converges in a faster manner; and compared with QR-RLS based Minimum Variance Distortionless Response (MVDR) beamformer, the proposed algorithm has lower complexity. Furthermore, due to its cascade configuration, the steering vector error problem can be resolved by using the new method.

1. INTRODUCTION

An adaptive array is a system consisting of an array of sensor elements and a real-time adaptive signal receiver-processor that automatically adjusts the array beam pattern by weighting the input of the individual elements. These weights can be chosen based on various algorithms [1]-[5]. One capability provided by the adaptive array techniques is to respond to an unknown interference environment by steering nulls and reducing sidelobe levels in the direction of the interference.

The multiple sidelobe canceller (MSC) proposed by Applebaum [1] consists of a main antenna with high gain and one or more auxiliary antenna elements. With properly chosen auxiliary antenna weights, it is then possible for the auxiliary channels to cancel the main channel interference component. The overall system then has a response of zero in the direction of interference sources. This algorithm requires the absence of the desired signal from auxiliary channels for weight determination. Widrow et al. [2] proposed the use of a reference signal and the array weights could be chosen to minimize the mean square error between the adaptive array output and this reference signal. The weights can also be chosen to directly maximize the signal to interference-plus-noise ratio (SINR) [3]. A general solution for the weights requires the knowledge of both the desire signal and noise covariance matrices. Frost [4] has suggested a least mean-square (LMS) based Linear Constrained Minimum Variance (LCMV) beamforming algorithm, where the desire signal is preserved while minimizing contributions to the output due to interfering signals and noise arriving from directions other than the direction of interest. The main advantage of this algorithm is its flexibility and simplicity. However, the algorithm exhibit slow rate of convergence towards the optimal solution.

To improve the speed of the LMS algorithm without sacrificing too much of its implementation simplicity, null-steering Davies beamformers [5] can be employed as the underlying array processing structure. By using Davies beamformer, the null positions can be tracked one after another without affecting other null positions. A fast zero-tracking algorithms based on the use of the LMS algorithm [6] has been investigated. Using this algorithm, the complex zeros of the power inversion array [7] are adjusted individually in a cyclical manner to track individual interferences. Since look direction constraint is not used, this algorithm is of great advantage in scenarios where the desired signal is weak compared with the interference signals. However, in the environment where the desired signal is stronger or not significantly weaker than the interferences, a deep null will be placed in the direction of interest resulting in desired signal loss. A constrained null steering algorithm [8] was presented to ensure that the look direction response remain unchanged when a particular null is being updated. However, in this algorithm, the look direction constraint must be satisfied in each null update process.

In order to further improve the convergence behavior, QR-recursive least-squares (QR-RLS) based beamformers [9,10] have been proposed. The potential advantages of QR-RLS based method include numerical stability and pipelined structure in implementation. However, the complexity becomes high since RLS is used here instead of LMS.

In this paper, a simple constrained based adaptive null steering algorithm based on the algorithm introduced in [6] will be presented and analyzed. Using the proposed algorithm, the zeros can be selectively tracked so that the interferences can be rejected while the desired signal preserves. Furthermore, due to its cascade configuration, steering vector error problem can be resolved. Throughout this paper, the direction of desired signal is assumed to be roughly known.

The remainder of this paper is organized as follows. The next section explains the theory of power inversion array. Section III briefly summarize the fast zero-tracking algorithm presented in [6]. The proposed algorithms are presented in Section IV and V. Section VI gives the simulation results. Finally, the conclusion is given in Section VII.

2. POWER INVERSION ARRAY

Fig. 1 shows the structure of linear uniformly spaced narrowband power inversion array with $N+1$ elements.

As can be seen, the zeroth element has a unity gain and the other elements are weighted by the coefficients $a_1,\ldots,a_N$. The directional pattern of the array expressed in the polynomial form is given by:

$$D(\theta) = 1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \ldots + a_N z^{-N} \tag{1}$$

where

$$z = exp(j\frac{2\pi d}{\lambda}\sin\theta) \tag{2}$$

d is the interelement spacing, λ is the wavelength and θ is the direction relative to the array normal.

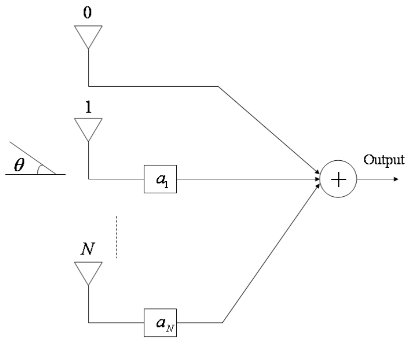

Figure 1: An $(N+1)$-element power inversion array

Such a polynomial may be represented as the product of N factors giving the N zeros of the directional pattern.

$$D(\theta) = (1 - b_1 z^{-1})(1 - b_2 z^{-1}) \dots (1 - b_N z^{-1}) \tag{3}$$

where

$$b_N = exp(j\phi_N) = exp(j\frac{2\pi d}{\lambda}\sin\theta_N) \tag{4}$$

Thus an $(N+1)$-element array has N degrees of freedom in its pattern and places maximum of N perfect nulls in certain directions.

Assume that there exist $N+1$ signals, one weak desired signal and N strong interference signals. We wish to suppress the interference signals. Suppose that the coefficients a_N in the array are adjusted to minimize the array output power. Clearly, the output power will be minimized when the array has directed its nulls at the strong interference signals. As a result, all the interference signals will be rejected while the desired signal will not be in a null. This explains why the SINR can be improved by using power inversion array.

3. NARROWBAND ZERO-TRACKING ALGORITHM

The fast zero-tracking algorithm [6] for narrowband arrays is briefly summarized in this section. Using this algorithm, the complex zeros of the array are repetitively updated and thus the nulls are placed in the direction of interference signals.

Suppose zero b_1 is to be updated to track the strongest interference signal. From (3), the directional pattern of the array is given by:

$$D(\theta) = (1 - b_1 z^{-1})\hat{D}(\theta) \tag{5}$$

where

$$\begin{aligned} \hat{D}(\theta) &= (1 - b_2 z^{-1})(1 - b_3 z^{-1}) \dots (1 - b_N z^{-1}) \\ &= 1 + c_1 z^{-1} + c_2 z^{-2} + \dots + c_{N-1} z^{-(N-1)} \end{aligned} \tag{6}$$

can be seen as the directional pattern of an N-element array with the coefficients $1, c_1, c_2, \dots, c_{N-1}$.

From (5) and (6), $D(\theta)$ can be expressed as

$$D(\theta) = \hat{D}(\theta) - b_1 \hat{\hat{D}}(\theta) \tag{7}$$

where

$$\hat{\hat{D}}(\theta) = z^{-1} + c_1 z^{-2} + \dots + c_{N-1} z^{-N} \tag{8}$$

Therefore, the processing of (7) can be implemented by using the structure in Fig. 2.

The output of the array is given by

$$y = L_1 - b_1 R_1 \tag{9}$$

Using the theory of power inversion array, the zero b_1 can be tracked by minimizing the array output power.

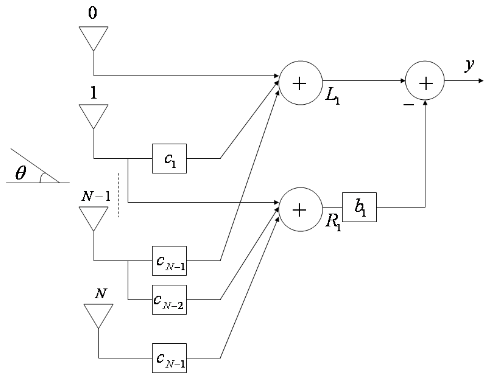

Figure 2: Array structure for updating b_1

From (9), the output power is given by

$$E[|y|^2] = E[|L_1|^2] - E[L_1 R_1^*]b_1^* - E[L_1^* R_1]b_1 + E[|R_1|^2]b_1 b_1^* \tag{10}$$

Note that in (10), $*$ denotes complex conjugation. By carrying out derivative operation with respect to b_1^*

$$\frac{\partial E[|y|^2]}{\partial b_1^*} = E[|R_1|^2]b_1 - E[L_1 R_1^*] \tag{11}$$

From (11), the average output power will be minimized if b_1 is given by its optimal value

$$b_{1opt} = \frac{E[L_1 R_1^*]}{E[|R_1|^2]} \tag{12}$$

Thus, the zero b_1 is eventually updated and a null is placed in the direction of the strongest interference signal. Now suppose zero b_2 is to be updated in the next cycle to track the second strongest interference signal. The same updating procedure described above for b_1 can be applied to the adjustment of b_2, except that the coefficients in Fig. 2 need to be changed accordingly. The update of the second zero will not affect other null positions, which is the key feature of Davies beamformer. The zeros can also be iteratively updated using LMS algorithm with the arrival of each new data sample [6].

4. PROPOSED NULL STEERING ALGORITHM WITH SINGLE CONSTRAINT

The look direction constraint is not used in the algorithm above. Thus, in the environment where the desired signal is stronger or not significantly weaker than the interference sources, one zero corresponding to the desired signal will be updated. A null will, therefore, be placed in the direction of interest by the array.

The algorithm proposed in this section adds a look direction constraint to the system, so that the desired signal will preserve while the nulls in the direction of interference signals will remain. The main advantage of this algorithm is its simplicity.

Fig. 3 shows the block diagram of the proposed algorithm. The array structure in Block I is same as that presented in last section while a new block (Block II) is added. The purpose of Block I is to track the desired signal as well as the interference sources in the environment after collecting enough number of data samples. The number of training samples is chosen such that the zero being updated will close to its optimal value as discussed in [6]. Since it is assumed that the direction of desired signal is roughly known, we can easily figure out the zero closest to direction which corresponds to the signal of interest.

With the information of zeros obtained from Block I, the array coefficients in Block II can be adjusted using the proposed method to suppress the interference signals while maintain a fixed gain in the look direction.

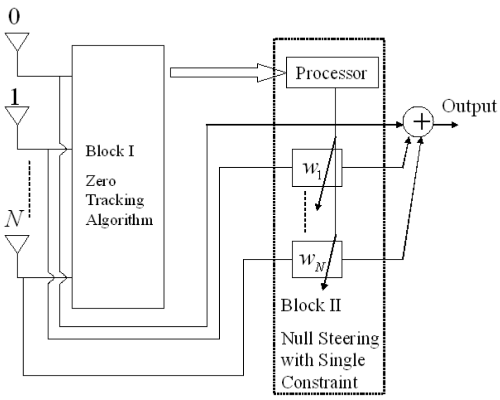

Figure 3: Block diagram of proposed null steering algorithm with single constraint

To illustrate how the coefficients in Block II can be easily adjusted, we assume that there exist one desired signal and two interferences in the environment, and a 4-element array is used. However, the algorithm proposed can be easily applied to the situation where more signals exist and more array elements are used.

From (3), the directional pattern of the system in Block I is

$$D_I(\theta) = (1 - b_s z^{-1})(1 - b_1 z^{-1})(1 - b_2 z^{-1}) \tag{13}$$

where b_s, b_1, b_2 correspond to the zeros for the desired signal, interference I and interference II, respectively.

Let the directional pattern of Block II be

$$\begin{aligned} D_{II}(\theta) &= (1 - \gamma z^{-1})(1 - b_1 z^{-1})(1 - b_2 z^{-1}) \\ &= \mathbf{w}^T \mathbf{z} \end{aligned} \tag{14}$$

where

$$\mathbf{w} = \begin{bmatrix} 1 \\ w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -(\gamma + b_1 + b_2) \\ \gamma b_1 + \gamma b_2 + b_1 b_2 \\ -\gamma b_1 b_2 \end{bmatrix} \tag{15}$$

is the array coefficients to be adjusted and

$$\mathbf{z} = [1 \quad z^{-1} \quad z^{-2} \quad z^{-3}]^T \tag{16}$$

Note that in (14), a new unknown parameter γ is introduced and to be calculated based on the look direction constraint. The zeros b_1 and b_2 remain in the directional pattern so that the two interference signals can be suppressed. T represents transpose.

In order to recover the desired signal with no amplitude and phase distortion, we define a unity gain in the look direction, that is

$$\mathbf{w}^H \mathbf{v}(\theta_s) = 1 \tag{17}$$

where $\mathbf{v}(\theta_s)$ is the steering vector in the direction of desired signal and H denotes Hermitian transpose. For uniform linear array, the steering vector takes the form

$$\mathbf{v}(\theta_s) = [1 \quad exp(-j2\pi u_s) \quad exp(-j2\pi 2 u_s) \quad exp(-j2\pi 3 u_s)]^T \tag{18}$$

where

$$u_s = \frac{d \sin\theta_s}{\lambda} \tag{19}$$

From (15), (17) and (18), the value of parameter γ can be easily obtained, since γ is the only unknown. With γ and from (14), we can adjust the array coefficients in Block II. Thus, we obtain the directional pattern with two nulls in the direction of interferences and a unity gain in the direction of interest.

The computational complexity in terms of multiplications is calculated and compared to the complexity of the LMS based LCMV method and adaptive QR-RLS method. Table 1 shows the comparison of computational complexity for different methods using M elements and K data samples:

Table 1: Comparison of computational complexity for LCMV, QR-RLS and new method with single constraint

Method	Number of real multiplications
LMS based LCMV	$2KM + 2K$
QR-RLS	$10KM + 30KM^2$
New method w/ single constraint	$2KM + 3K + 6M - 8$

It is evident that the proposed algorithm here becomes convenient in terms of computational complexity with respect to the QR-RLS scheme. The new method with single constraint and LMS based LCMV method have almost equal computational complexity.

5. PROPOSED NULL STEERING ALGORITHM WITH MULTIPLE CONSTRAINTS

The proposed algorithm in the last section above is designed for use as a fast null steering algorithm. In this section, we will show that, by incorporating more constraints, the same basic structure can also function like a beamformer, where the output SINR of the mainbeam is enhanced with a decreased sidelobe level.

The idea of proposed method in this section is to relax the unity gain constraint in the look direction. Consider the 4-element array discussed in last section, we still define the same directional pattern as (14) for the array in Block II. Now instead of single constraint in the look direction, we impose multiple constraints which consist of:

- A flexible gain α in the look direction, that is

$$\mathbf{w}^H \mathbf{v}(\theta_s) = \alpha \tag{20}$$

- A flexible gain β in a randomly chosen sidelobe direction θ_n, that is

$$\mathbf{w}^H \mathbf{v}(\theta_n) = \beta \tag{21}$$

- A predefined ratio between α and β,

$$\frac{\alpha}{\beta} = 20 \tag{22}$$

Note that in (22), 20 is defined as the ratio. The reason why we set the three constraints is to lower the sidelobe level and improve the mainlobe to sidelobe ratio. From the simulation results given later, if the random sidelobe direction is chosen adequately, the overall sidelobe level will be significantly reduced.

From (15), (18), (20), (21) and (22), we have the following equation

$$[1 \quad w_1 \quad w_2 \quad w_3] \begin{bmatrix} 1 & 1 \\ exp(-j2\pi u_s) & exp(-j2\pi u_n) \\ exp(-j2\pi 2u_s) & exp(-j2\pi 2u_n) \\ exp(-j2\pi 3u_s) & exp(-j2\pi 3u_n) \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ \frac{1}{20} \end{bmatrix}^T \tag{23}$$

Since the array coefficients w_N are functions of unknown parameter γ, equation (23) can be easily solved for the values of γ and α.

Note that in this case, the gain α obtained in the look direction may be a complex number, resulting amplitude and phase distortion of the desired signal. Thus, the new structure is shown in Fig. 4, where a new block (Block III) is added to compensate for the desired signal distortion.

For the proposed algorithm here using M elements and K data samples, we have the following:

- Number of real multiplications:$2KM + 3K + 10M - 12$

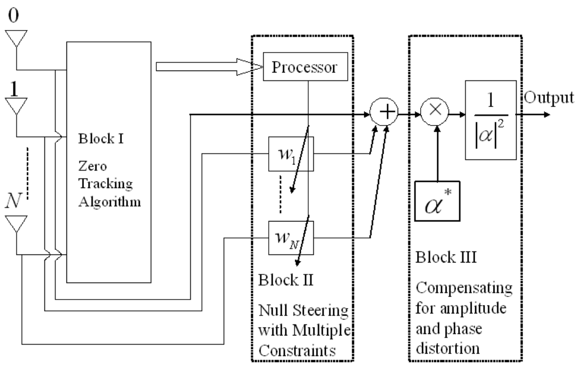

Figure 4: Block diagram of proposed null steering algorithm with multiple constraints

Although adding more constraints, the complexity only increases by a small amount compared with proposed method with single constraint and is still lower than that of QR-RLS based method.

A summary of the proposed algorithms in sections 4 & 5 is given in Table 2.

Table 2: Summary of the proposed algorithms

with single constraint	with multiple constraints
Track desired signal and interferences using method in [6] to get corresponding zeros in (13)	
• Calculate parameter γ using equations (15) (17) and (18)	• Choose random sidelobe direction in (21)
• Adjust array coefficients using equation (15)	• Define ratio in (22)
	• Calculate parameters γ, α and β using equations (15), (18), (20), (21), (22)
	• Adjust array coefficients using equation (15)
	• Form Block III in Fig. 4 for compensation of signal distortion

6. SIMULATION RESULTS

Some simulation results will now be presented to verify the above theoretical results. The simulated array is assumed to be a four-element linear array with interelement spacing of $\lambda/2$.

The simulated environment consists of a desired signal of power 20 dB arriving from $-30°$, and two independent interference signals of power 40 dB and 40 dB, assumed to be located at 24° and 27° to the array normal. Finally, independent white Gaussian noise of power 0 dB is added to each sensor element to account for the presence of ambient isotropic noise and other broadband sources of noise.

Fig. 5 shows the directional patterns for C.C. Ko's method [6] and proposed null steering algorithm with single constraint. As seen in the figure, by using C.C. Ko's method, a deep null is placed at $-30°$, i.e. in the direction of desired signal, and two deep nulls are placed in the directions of interference signals. Thus, this array system will reject all data signals, including the desired signal and the interferences. Our objective is to get rid of the null in the direction of interest and impose a unity gain to it.

By using proposed method with single constraint, the two deep nulls steered in the directions of interference signals remain unchanged. A unity gain in the look direction is achieved so that the desired signal will preserve.

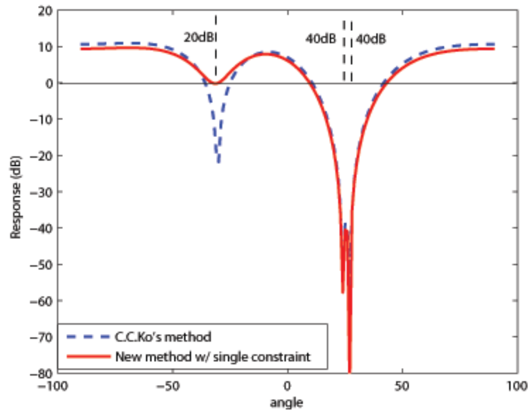

Figure 5: Directional pattern for C.C.Ko's method and new method with single constraint

Note that with the three zeros obtained in Block I and using the proposed method, the parameter γ in (14) can be easily calculated. This γ does not relate to the zeros of the directional pattern of the array in Block II, since we use the degree of freedom to impose a gain in the look direction.

Figure 6 compares the average output power convergence curves of using LCMV, QR-RLS and the proposed algorithm. The desired misadjustment for all algorithms is 5%. Since the processing in Block II in the proposed method does not need to be involved in the update process, the convergence behavior of the whole proposed system will be totally controlled by Block I, therefore the proposed method has almost the same convergence behavior as the one in [6]. As can be seen, the new method converges in a faster manner compared with LCMV beamformer. The QR-RLS and proposed method have very similar convergence behavior. The theoretical convergence analysis for the proposed algorithm will be provided in the journal version of the paper.

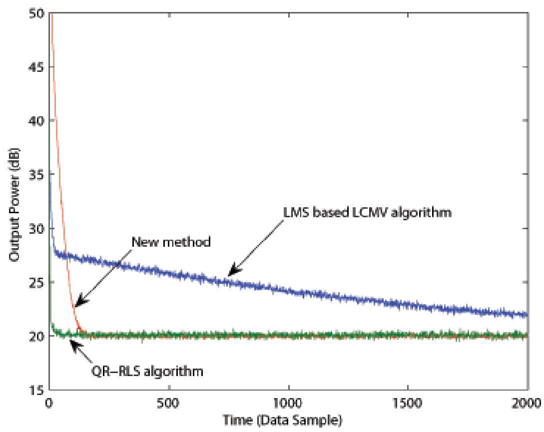

Figure 6: Stochastic output power convergence behavior of LCMV, QR-RLS and new method

Now some simulation results for the proposed method with multiple constraints will be presented. Consider the same simulated array and environment discussed above, we impose the gain β in (21) to sidelobe direction of 50°. The explanation of the choice of random sidelobe direction in (21) will be provided in the journal version of the paper.

Fig. 7 shows the directional pattern for QR-RLS MVDR beamformer [10] and the proposed algorithm with multiple constraints.

As can be seen, the proposed method with multiple constraints has overall lower sidelobe level compared with QR-RLS MVDR beamformer. The null depths for the new method reach deeper levels, resulting in better performance in terms of interferences rejection capabilities.

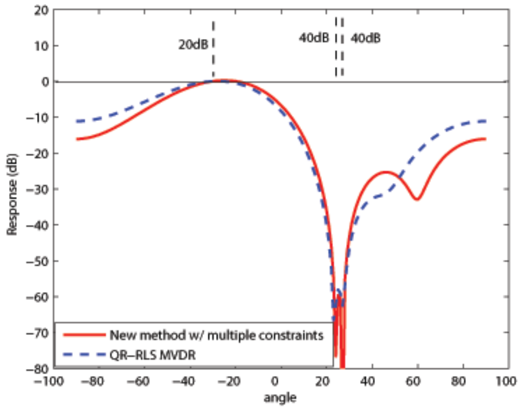

Figure 7: Directional pattern for QR-RLS MVDR and new method with multiple constraints

Fig. 8 shows the same set of curves as Fig. 7 but in a different scenario where two interferences of power 70 dB and 55 dB are located at 40° and $-30°$ to the array normal and the desired signal of power 40 dB arrives from the broadside direction. The gain β in (21) is imposed to sidelobe direction of 65° for the proposed method with multiple constraints.

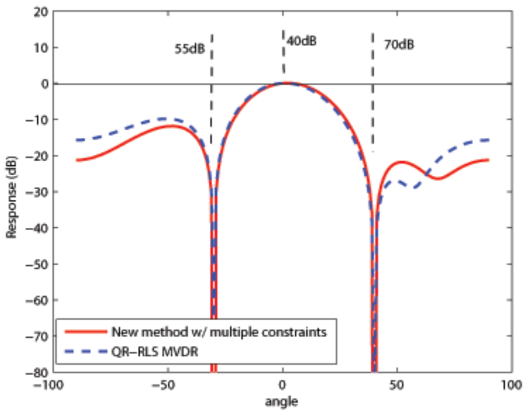

Figure 8: Directional pattern for QR-RLS MVDR and new method with multiple constraints in another scenario

Now consider the same scenario as Fig. 8 with steering vector assumed to be located at 0°. The desired signal is coming from 1°, hence resulting in a steering vector error problem. Fig. 9 shows the directional pattern for LCMV beamformer and the new method with multiple constraints under this steering vector error scenario. Since the LCMV beamformer try to minimize the total output power while maintain a fixed gain in the look direction, it will produce a null in the actual desired signal direction. For the proposed method, the desired signal direction will be first obtained in Block I. Therefore, the system can still steer to the actual data signal direction.

7. CONCLUSION

In this paper, a simple constrained based adaptive null steering algorithm has been proposed and investigated. In this algorithm, the

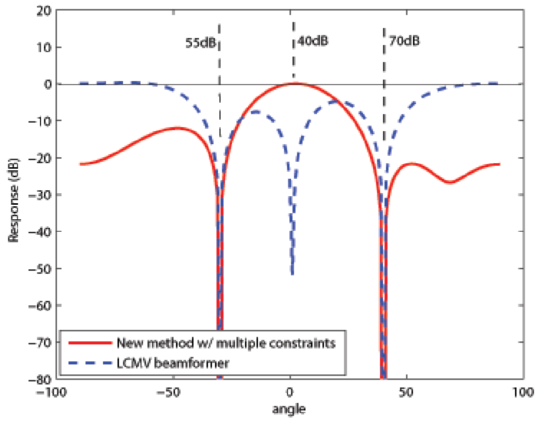

Figure 9: Directional pattern for LCMV and new method with multiple constraints under steering vector error scenario

zeros can be selectively tracked one after another without affecting other null positions so that the interference signals can be rejected while the desired signal preserves. As has been shown analytically and by using simulation results, the new method has a significantly faster convergence behavior compared with LCMV beamformer. An overall lower sidelobe level can be achieved by using the proposed method with multiple constraints. The complexity required to implement the new method is far less than that for QR-RLS MVDR beamformer. Furthermore, the steering vector error problem can also be resolved by using the proposed method due to its cascade configuration.

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