APPLYING NON-NEGATIVE MATRIX FACTORIZATION FOR GLOBAL SIGNAL ELIMINATION FROM ELECTROMAGNETIC SIGNALS

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ABSTRACT

Anomalous environmental electromagnetic (EM) radiation waves have been reported as the portents of earthquakes. Our goal is to predict earthquakes using EM radiation waves. We have been measuring the Extremely Low Frequency (ELF) range all over Japan. However, the recorded data contain signals unrelated to earthquakes. These signals, as noise, confound earthquake prediction efforts. It is necessary to eliminate noises from observed signals in a preprocessing step. In this paper, we propose a method for global signal elimination using Non-negative Matrix Factorization (NMF) and evaluate the effectiveness of this method.

1. INTRODUCTION

Earthquakes occur frequently in Japan. Japan has suffered extensive damage from huge earthquakes many times. This gives residents reason to worry about the occurrence of giant earthquakes in the near future. The Earthquake Research Committee of Japan reported in 2001 that the probability of giant earthquakes of the Nankai and Tohnankai (Richter magnitude over 8) within 30 years is now between 40% and 50% [1]. Accurate earthquake prediction is urgently needed to minimize earthquake damage. However, most the phenomena relating to earthquakes have not been elucidated. Forecasting from report of trench survey on active faults and the occurrence cycle of past earthquakes is the traditional method of predicting earthquakes. This method is not accurate because the margin of prediction error is for several years. We are trying to more accurately predict earthquakes using different approach.

Anomalous radiations of environmental electromagnetic (EM) waves have been reported to be a precursor phenomenon of earthquakes [2, 3].In order to observe precursor EM radiation of earthquakes, we have been measuring Extremely Low Frequency (ELF) magnetic fields all over Japan since 1985 with the goal of predicting earthquakes using these signals.

Accurate earthquake prediction needs to observe accurate precursor phenomena of earthquakes. However, the ELF data contain undesired signals associated with thunderclouds, human activity, and other things. The largest signal which radiated from heat thunderclouds at lower latitudes buries earthquake precursor signals. These undesired signals distort the results of earthquake prediction. It is important to accurately extract the earthquake precursor signals. However, it is difficult because the properties of earthquake precursor signals are unknown. Therefore we remove known undesired signals from observed signals to improve SNR.

The contents of ELF data are two kinds of signals. The one is called "local signal" which have different values in each sensors. EM waves radiated from environments in the vicinity of sensors (e.g. earth's crusts, thunderclouds) become local signals. The local signals are almost unknown and there is a possibility including the earthquake precursor signals in local signals. Another one is called "global signal" which have almost same values in all sensors. The global radiations of EM waves (e.g. from heat thunderclouds) become global signals. We know the properties of global signals. The global signals are large so bulks of observed signals are global signals. Therefore, we remove the global signals from observed signals.

We used Independent Component Analysis (ICA)[4] to estimate global signal in conventional study [6, 5]. However, ICA has a not suitable possibility for accurately analyzing ELF data because the ELF data recorded absolute values. Therefore we use Non-negative Matrix Factorization (NMF)[7, 8] in place of ICA to estimate more accurate global signal and local signals.

2. SIGNAL MODEL OF GLOBAL SIGNAL ELIMINATION FROM ELF DATA

First, we define the problem that will be referred to throughout this paper. m source signals $\boldsymbol{s}(t) = [s_1(t), ..., s_m(t)]^T$ are linear-mixed and arrive at n sensors of our observation systems. Our systems record the measurement data after converting them into the absolute value. In this case, the vector of mixed signals $\boldsymbol{x}(t) = [x_1(t), ..., x_n(t)]^T$ are given by

$$\boldsymbol{x}(t) = |\boldsymbol{A}\boldsymbol{s}(t)| \tag{1}$$

where $\boldsymbol{A}$ is an $m \times n$ mixture matrix and $|\cdot|$ means absolute function.

One of the source signals is the global signal. When we describe the global signal expediently separated as $g(t)$, Eq. (1) becomes the following.

$$\boldsymbol{x}(t) = |\boldsymbol{b}g(t) + \boldsymbol{A}_L\boldsymbol{s}_L(t)| = |\boldsymbol{b}g(t) + \boldsymbol{l}(t)| \tag{2}$$

where $\boldsymbol{b}$ is sensitivity vector corresponding to global signal $g(t)$. $\boldsymbol{s}_L$ are $m-1$ source signals not including global signal and $\boldsymbol{A}_L$ is an $m-1 \times n$ mixture matrix of $\boldsymbol{s}_L$. $\boldsymbol{A}_L\boldsymbol{s}_L(t)$ mean local signals. We replace $\boldsymbol{A}_L\boldsymbol{s}_L(t)$ by $\boldsymbol{l}(t)$.

It is difficult to separate signals which are in absolute values. We simplify Eq. (2) by two assumptions.

AS1 The source signals are non-negative.

AS2 The mixture matrix is non-negative.

Under these two assumptions, we redefine Eq. (2) as following expression.

$$\boldsymbol{x}(t) = \boldsymbol{b}g(t) + \boldsymbol{l}(t) \quad \boldsymbol{b}, g(t), \boldsymbol{l}(t) \geq 0 \tag{3}$$

In this model, the global signal elimination is to subtract global signal from observed signals as following expression.

$$\boldsymbol{l}(t) = \boldsymbol{x}(t) - \boldsymbol{b}g(t) \tag{4}$$

3. NON-NEGATIVE MATRIX FACTORIZATION

Non-negative Matrix Factorization (NMF) is an algorithm to approximately factorize a given non-negative matrix under the non-negativity constraints. The input matrix $\boldsymbol{X} = [\boldsymbol{x}(1), \boldsymbol{x}(2), ..., \boldsymbol{x}(T)] \in \mathbb{R}^{n \times T}$ is approximated by NMF as

$$\boldsymbol{X} \approx \boldsymbol{AY} \quad \boldsymbol{X}, \boldsymbol{A}, \boldsymbol{Y} \geq 0 \tag{5}$$

where $\boldsymbol{A} \in \mathbb{R}_+^{n \times r}$ is a mixture matrix and $\boldsymbol{Y} = [\boldsymbol{y}(1), \boldsymbol{y}(2), ..., \boldsymbol{y}(T)] \in \mathbb{R}_+^{r \times T}$ is a component matrix. The rank of factorization, r, is chosen as $nT > nr + rT$. Eq. (5) can be written column by column as $\boldsymbol{x}(t) \approx \boldsymbol{Ay}(t)$, where $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ are the corresponding columns of $\boldsymbol{X}$ and $\boldsymbol{Y}$. NMF find $\boldsymbol{A}$ and $\boldsymbol{Y}$ by using iterative updates based on a cost function.

Some NMF algorithms have been proposed. The algorithm adopted by this paper is Image Space Reconstruction Algorithm (ISRA)[9]. The cost function of ISRA is the square of the Euclidean distance between $\boldsymbol{X}$ and $\boldsymbol{AY}$ as

$$||\boldsymbol{X} - \boldsymbol{AY}||^2 = \sum_{ik} \{X_{ik} - [\boldsymbol{AY}]_{ik}\}^2 \tag{6}$$

This is lower bounded by zero, and clearly vanishes if and only if $\boldsymbol{X} = \boldsymbol{AY}$. To be minimized the cost function, Eq. (6), ISRA applies the following update functions to $\boldsymbol{A}$ and $\boldsymbol{Y}$ enough times.

$$A_{ij} \leftarrow A_{ij} \frac{[\boldsymbol{XY}^T]_{ij}}{[\boldsymbol{AYY}^T]_{ij}}, \quad Y_{jk} \leftarrow Y_{jk} \frac{[\boldsymbol{A}^T\boldsymbol{X}]_{jk}}{[\boldsymbol{A}^T\boldsymbol{AY}]_{jk}} \tag{7}$$

$$A_{ij} = \frac{A_{ij}}{\sum_j A_{ij}} \tag{8}$$

4. METHOD OF GLOBAL SIGNAL ELIMINATION USING NMF

The procedures for eliminating global signals using NMF are as follows.

1. Estimating source signals using NMF from observed signals.
2. Identifying a global signal from among the estimated signals.
3. Subtracting the global signal from each observed signal.

4.1 Estimation of source signals

We apply the NMF algorithm to observed signals $\boldsymbol{x}(t)$, and obtain estimated source signals $\boldsymbol{y}(t)$.

4.2 Identification of the global signal

The global signal is one component of $\boldsymbol{y}(t)$. However, the components come out randomly due to permutation ambiguity. Therefore, it is necessary to identify the global signal component from the estimated signals.

Usually, the global signal is much larger than other signals that comprise the observed signals. The global signal looks like a typical observed signal. We can identify the global signal by choosing a typical signal from among the component. Therefore, we first calculate the typical observed signal $\tilde{x}(t)$ using the following expression.

$$\tilde{x}(t) = \frac{1}{N} \sum_i \frac{x_i(t) - \overline{x_i}}{\sqrt{\langle (x_i(t) - \overline{x_i})^2 \rangle}} \tag{9}$$

where N is the number of observation sites, $x_i(t)$ is the recorded signal at observation site i, and $\overline{x_i}$ is the expectation of $x_i(t)$. The operators $\langle x \rangle$ mean the time averages of x. We choose one component $y_j(t)$ which has a maximal value using the following expression.

$$r_{xy_j} = \frac{\langle \tilde{x}(t) \cdot (y_j(t) - \overline{y_j}) \rangle}{\sqrt{\langle \tilde{x}^2(t) \rangle} \sqrt{\langle (y_j(t) - \overline{y_j})^2 \rangle}} \tag{10}$$

where r_{xy_j} is the correlation coefficient between typical observed signal $\tilde{x}(t)$ and estimated component $y_j(t)$.

4.3 Subtraction of global signals

NMF estimates components $\boldsymbol{y}(t)$ and mixture matrix $\boldsymbol{A}$ at a time. The row vector $\boldsymbol{A}_j = [A_{1j}, A_{2j}, ..., A_{nj}]^T$ of $\boldsymbol{A}$ is sensitivity vector corresponding to $y_j(t)$. Therefore, we obtain estimated local signals $\hat{\boldsymbol{l}}(t)$ as:

$$\hat{\boldsymbol{l}}(\boldsymbol{t}) = \boldsymbol{x}(t) - \boldsymbol{A}_j y_j(t). \tag{11}$$

5. EFFECTIVENESS OF GLOBAL SIGNAL ELIMINATION

In this section, we discuss a method for the evaluating efficiency and reliability of the global signal elimination. In order to evaluate effectiveness of our method, we should calculate and compare the SNR before and after applying our method. However, calculating SNR from our data is impossible directly because it needs the true earthquake precursor signals. Therefore, we make the new evaluation criterion alternative of SNR.

We can use mutual information as a criterion of effectiveness. Local signals are statistically mutually independent in many cases, because few electromagnetic radiations spread far, Mutual information among the local signals mixed is relatively small. On the other hand, there is a high value of mutual information will be among observed signals because all observed signals contain the global signal. Therefore, mutual information among local signals is a good criterion how efficiently a global signal is eliminated from of all observations. However, there is sometimes a lot of mutual information among the local signals when some local signals depend on each other. Considering this problem is future work.

Mutual information between random variables X and Y is defined by the following expression

$$I(X;Y) = \int\int P(X,Y) \log\left(\frac{P(X,Y)}{P_X(X)P_Y(Y)}\right) dXdY \tag{12}$$

where $P(\cdot)$ is probability density function (pdf). In order to calculate mutual information, we need pdfs of $P_X(X), P_Y(Y)$ and joint a pdf of $P(X,Y)$. We use the quantized histograms about signals instead of pdfs. Therefore, approximate mutual information is calculated by

$$\hat{I}(X;Y) = \sum_{n_X}\sum_{n_Y} P[n_X, n_Y] \log\left(\frac{P[n_X, n_Y]}{P_X[n_X]P_Y[n_Y]}\right) \tag{13}$$

where $P[\cdot]$ denotes a discrete histogram. We usually set quantization width to 0.2 σ. The effectiveness criterion of global signal elimination is given by

$$GIC = \sum_{i,j} \frac{\hat{I}(L_i; L_j)}{N(N-1)} \quad (i \neq j) \tag{14}$$

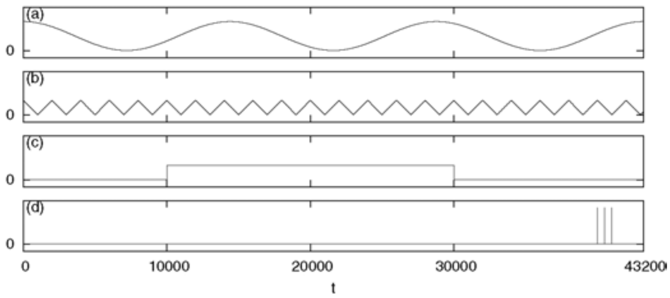

Figure 1: Generated source signals : (a) $s_1 \sim$ (d) s_4

where L_i, L_j $(i, j = 1, ..., N)$ are random variables of local signals l_i, l_j. The smaller GIC is, the more accurately the global signal is removed from the observations data and the local signals are estimated more precisely.

6. SIMULATIONS

We confirm that the proposed method eliminate global signal from observed signals which is imitated ELF data.

6.1 Processing data

We generate 4 source signals $\boldsymbol{s}(t) = [s_1(t), ..., s_4(t)]^T$ $(t = 1, 2, ...43200)$ which is shown in Figure 1. The vertical axes indicate amplitudes of signals and the horizontal axes indicate sampling index t. The signal $s_1(t)$ is a global signal. Mixture matrix $\boldsymbol{A}$ is

$$\boldsymbol{A} = \begin{bmatrix} 1.0461 & 0.1696 & 0.0011 & 2.2730 \\ 1.0011 & 0.0001 & 1.2554 & 0.3839 \\ 1.3223 & 0.0051 & 0.0000 & 0.0090 \\ 1.0359 & 2.4388 & 0.2521 & 0.0004 \\ 1.0583 & 0.0193 & 2.7609 & 0.0141 \\ 1.0204 & 0.0771 & 0.6508 & 1.9008 \\ 1.0142 & 0.0488 & 0.0095 & 0.0016 \\ 1.0796 & 0.6221 & 0.5478 & 0.4491 \\ 1.0378 & 0.6796 & 0.0138 & 0.2581 \\ 1.0087 & 0.0000 & 0.0325 & 0.0001 \\ 1.0785 & 0.9667 & 0.0001 & 0.1142 \\ 1.1592 & 0.0000 & 0.1748 & 0.1504 \end{bmatrix}. \quad (15)$$

Because a global signal is large and it recorded of almost the same value at each sensor, $\boldsymbol{A}_1$ is adjusted to large and almost the same value.

We made 12 observed signals $\boldsymbol{x}(t)$ from adding the absolute Gaussian signal to each mixture signal of $\boldsymbol{As}(t)$. Figure 2 shows a part of observed signals ($x_{01} \sim x_{06}$). The vertical axes indicate amplitudes of signals and the horizontal axes indicate sampling index t. All observed signals have shapes like 3 cycles cosine wave. This shapes are influence from the global signal $s_1(t)$. The ideal local signals corresponding to Figure 2 are shown in Figure 3. The axis, scales and orders are the same as those in Figure 2.

6.2 Results and evaluation

We apply proposed method to generated observed signals. In estimating global signal by ISRA, we set the rank of $r = 4$. Figure 4 shows estimated source signals by ISRA. The component whose r_{xyj} (Eq. (10)) is the largest is $y_1(t)$. We identify $y_1(t)$ as global signal and eliminate from $\boldsymbol{x}(t)$.

The estimated local signals corresponding to Figure 2 are shown in Figure 5. The axis, scales and orders are the same as those in Figure 2. These estimated local signals are similar to ideal local signals (Figure 3).

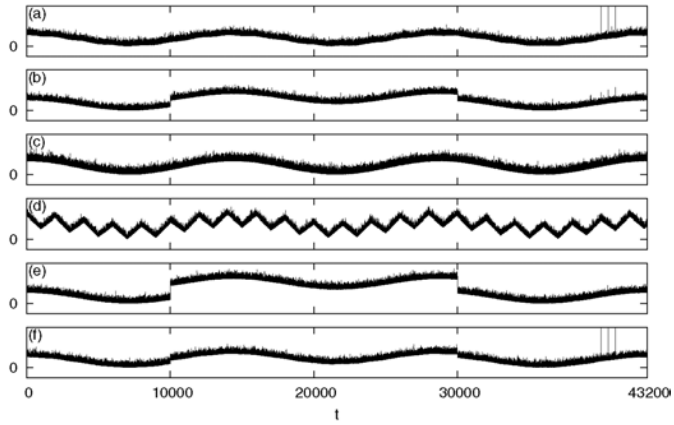

Figure 2: Generated observed signals : (a) $x_{01} \sim$ (f) x_{06}

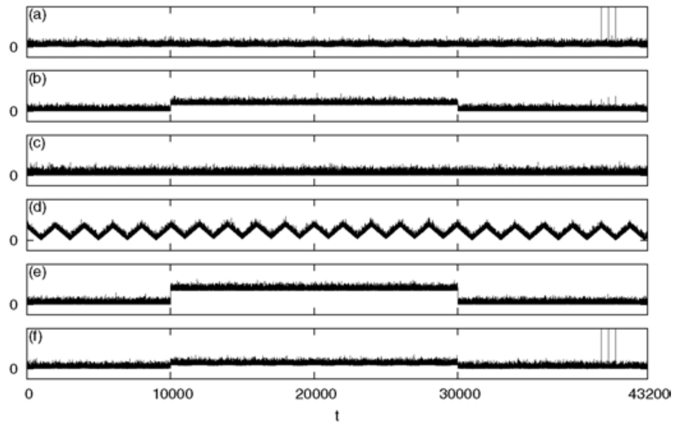

Figure 3: Ideal local signals : (a) $l_{01} \sim$ (f) l_{06}

Table 1: SNR and GIC about simulation data

Data	SNR[dB]	GIC
Observed signals	-1.71	0.524
Local signals (ideal)	—	0.058
Local signals (proposal)	13.37	0.094
Local signals (convention)	10.29	0.062

We calculate SNR and GIC from observed signals, ideal local signals, estimated local signals by proposed method and by conventional method. Table 1 shows these calculation results. The SNRs of local signals are larger than observed signals'. The local signals by proposed method are better performance than by conventional method. This reason shows that the proposed method is more accurately eliminate global signals than conventional method. Paying attention to GIC, the larger SNR is, the smaller GIC is. It shows GIC become an evaluation criterion in place of SNR.

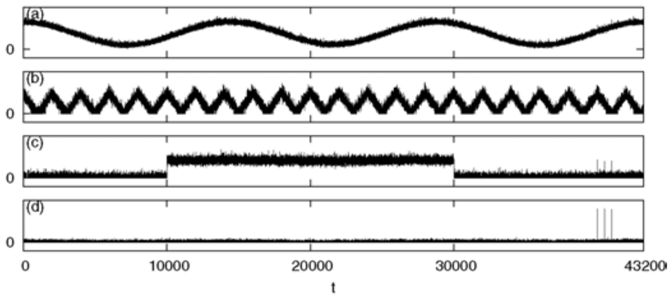

Figure 4: Estimated source signals by using ISRA : (a) y_1 ~ (d) y_4

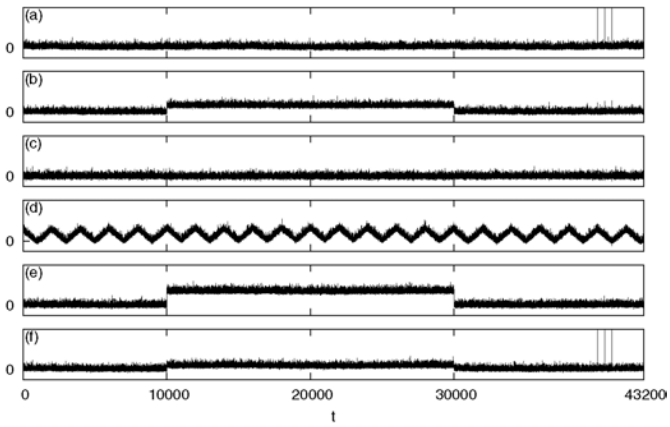

Figure 5: Estimated local signals by proposed method : (a) $\hat{l}_{01}$ ~ (f) $\hat{l}_{06}$

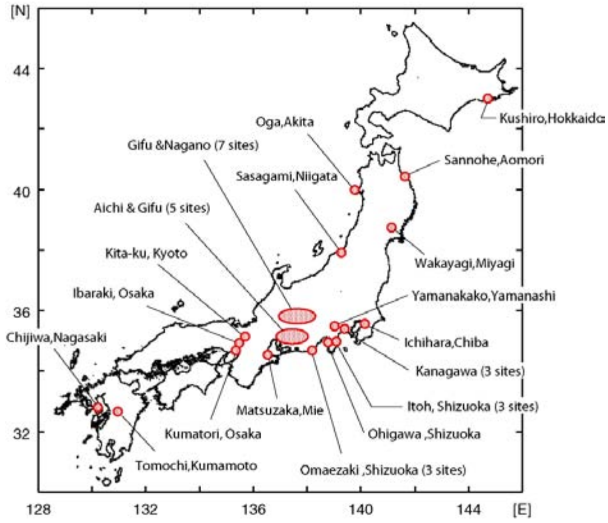

Figure 6: Arrangement of observation sites

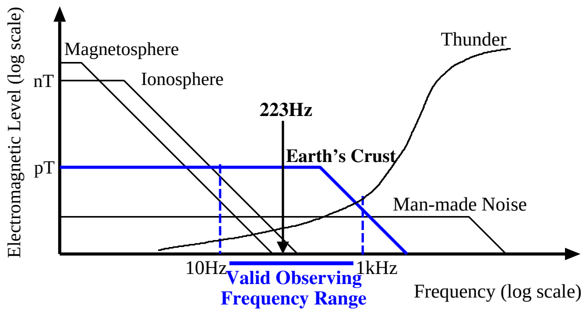

Figure 7: EM radiation levels of each source

7. GLOBAL SIGNAL ELIMINATION FROM ELF ELECTROMAGNETIC SIGNALS

7.1 Outline of ELF Band observation of EM radiation data

We have been observing power of 223Hz in EM radiation in about 40 places around the country (Fig.6). This frequency band has been a little influenced by solar activity and the global environment (Fig.7). Observation systems have three axial loop antennas with east-west, north-south, and vertical ranges. Observation devices sample EM levels (sampling frequency is 50Hz) and absolute average the signals over 6-second periods. These data are transported to our institute on the Public Telephone Network.

7.2 Processing data

We applied our proposed method to observed signals containing earthquake precursor EM radiation. An anomalous signal was observed for two days, from January 4 to 6 in 2001, at Nannoh in Gifu Prefecture (hereafter called Nannoh). We tried to obtain local signals for these days by eliminating the global signal using the proposed method. The recorded signals from Nannoh might have anomalous signals related to the earthquake, because an earthquake (magnitude 4.8) occurred at Tohnoh in Gifu Prefecture on January 6.

Figure 8 shows the ELF signals which were recorded from January 4 to 6 in 2001. (a) is observed at Sannohe in Akita Prefecture (hereafter called Sannohe), (b) is observed at Sakauchi in Gifu Prefecture (hereafter called Sakauchi), (c) is observed at Nannoh. (d) is observed at Chijiwa in Nagasaki Prefecture (hereafter called Chijiwa). The vertical axes indicate the electromagnetic levels (pT$\sqrt{\text{Hz}}$) and the horizontal axes indicate the time courses. Though the observation sites were different, both of these observed signals have high amplitudes at nighttime and have low amplitudes during the daytime. These circadian rhythms are a feature of a global signal.

We try the global signal elimination from the signal observed at the 24 observation sites including Sannohe, Sakauchi, Nannoh and Chijiwa.

7.3 Results and evaluation

We apply proposed method to generated observed signals. In estimating global signal by ISRA, we set the rank of $r = 6$. Figure 9 shows estimated global signal by ISRA. The vertical axes indicate the amplitudes and the horizontal axes indicate the time course. We eliminate this estimated global signal from ELF data.

The estimated local signals corresponding to Figure 8 are shown in Figure 10. All these signals do not have circadian rhythms like the observed signals. In addition, the obtained local signal in Nannoh has clearly anomalous signals from about 6 a.m. on the 4th to 8 a.m. on the 6th. This anomalous signal is a peculiar signal of Nannoh because it does not appear in local signals at other observation sites.

Table 2 shows *GIC*s from observed signals, estimated local signals by proposed method and estimated local signals by conventional method. The *GIC* of local signals by proposed method is the smallest. It shows the proposed method

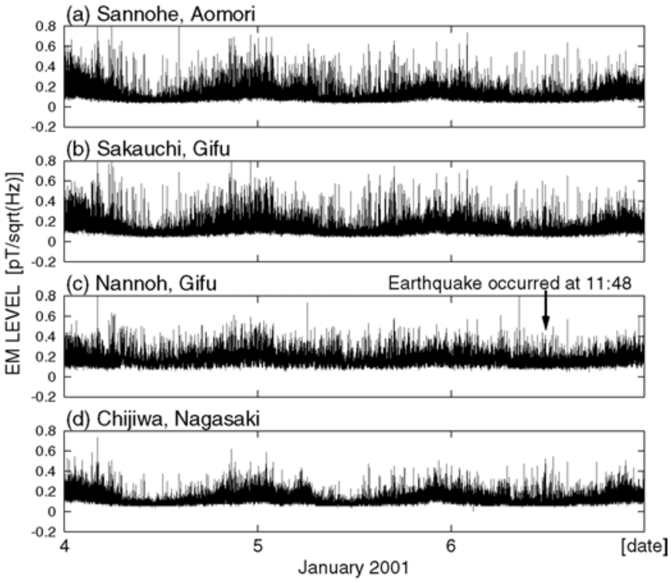

Figure 8: ELF observed signals : (a) Sannohe ; (b) Sakauchi ; (b) Nannoh ; (d) Chijiwa

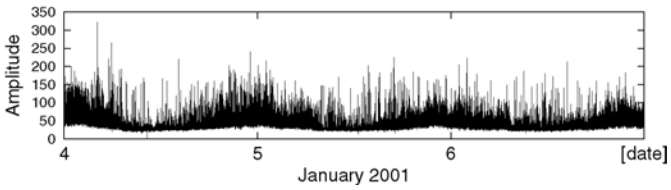

Figure 9: Estimated ELF global signal

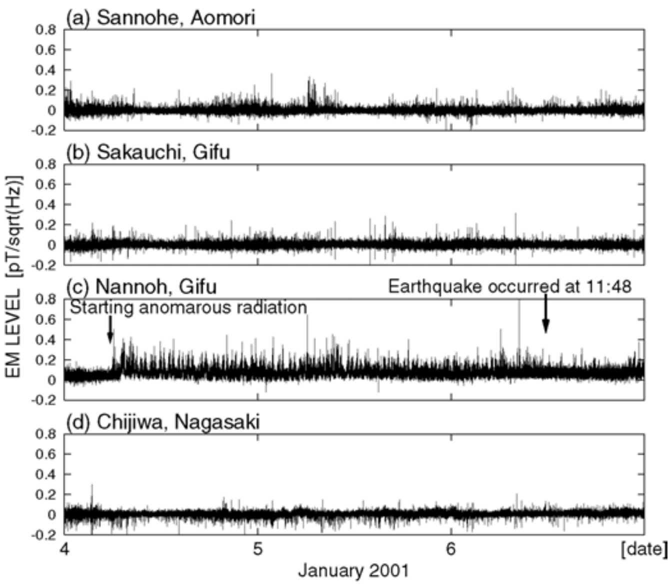

Figure 10: Estimated ELF local signals : (a) Sannohe ; (b) Sakauchi ; (b) Nannoh ; (d) Chijiwa

is accurately eliminating global signals. We think this is because the model of NMF is suitable for the model of the ELF data.

Table 2: *GIC* about ELF data

Data	*GIC*
Observed signals	0.2319
Local signals (proposal)	0.0640
Local signals (convention)	0.0781

8. CONCLUSION

In this paper, we proposed a global signal elimination method using NMF. We also proposed an effectiveness criterion *GIC* using mutual information. For generated data and ELF data, the proposed method actually estimated local signals. The *GIC* showed that NMF works effectively.

In future works, we need to modify the preprocessing and/or apply more robust algorithms. We will think about a new cost function suitable for the ELF data or absolute data. It is also necessary to examine the validity of *GIC*.

Finally, we will verify the effectiveness of the proposed method by anomalous detection and source estimation in earthquake prediction.

9. ACKNOWLEDGMENT

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