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# AN ADAPTIVE BLIND CHANNEL SHORTENING ALGORITHM FOR MCM SYSTEMS

Cenk Toker and Gökhan Altın
Hacettepe University, Department of Electrical and Electronics Engineering, Ankara, Turkey
cenk.toker@ee.hacettepe.edu.tr


## ABSTRACT

*In this paper, we propose an adaptive blind channel shortening algorithm for MCM systems such as ADSL. The algorithm is composed of two phases. In the first phase, a stochastic gradient descent algorithm is utilized to search the minimum of the proposed cost function. It is demonstrated that the cost surface is multimodal and not all minima attain good performance. In the second phase of the procedure, genetic algorithms are employed to find the best solution according to a pilot deviation criterion among all solutions to the cost function.*

*The major advantage of the proposed algorithm is, it inherently provides shortened channel information in contrast to similar algorithms in the literature in which channel estimation has to be performed separately after shortening, employing either training sequences or blind channel estimation.*


## 1. INTRODUCTION

Multicarrier Modulation (MCM) has recently been deployed as the modulation technique for several communication systems, e.g., IEEE 802.16 (WiMAX), ADSL, ADSL/2+, etc. The reason for this choice is the ease of combatting intersymbol interference. However, this easiness comes in the expense of appending a cyclic prefix to the MCM frame. For proper operation of MCM, the cyclic prefix has to be longer than the order of the communication channel which is related to the time duration of the impulse response of the channel. Therefore, for a relatively long channel, the cyclic prefix has to be long too. However, this is undesirable because the cyclic prefix does not convey any usable information and increasing its length will decrease the throughput of the system.

In practice the length of the cyclic prefix is set to a fixed value, for example for ADSL it is 32 samples. However, the length of the channel can be much longer than this figure as in the CSA loop models. This causes severe inter-carrier interference (ICI) in the operation of MCM hence significantly decreases the system performance. In this case, the common practice is to employ a channel shortening equalizer to effectively shorten the channel 'visible' to MCM to a length less than the length of the cyclic prefix.

Channel shortening filtering is a widely examined topic in the literature. A very thorough investigation of algorithms can be found in [1] which joins most of these algorithms in a common framework. Although they do not give optimum solutions, among these algorithms the Minimum Mean Squared Error (MMSE) [2] and the Maximum Shortening Signal-to-Noise Ratio (MSSNR) [3] are possibly the most widely considered ones due to their analytical tractability.

Most of the channel shortening equalizer proposals in the literature depend on perfect channel state information (CSI). However, this information can be attained by performing channel estimation at the receiver, and this may not be possible for several reasons. Channel estimation requires the transmission of training signals which reduces the valuable channel capacity. Similarly, the receiver may not possess the knowledge of the training signal.

In cases where channel estimation is not desirable or possible, equalization can still be performed by blind signal processing techniques. Although blind equalization is a well-studied field, blind channel shortening is a rather unexplored field in the literature [4]. MERRY is a technique depending on the equality of the cyclic prefix and the signal at the end of the MCM frame. SAM [5], SLAM [6] and [7] can be considered as the members of a family of blind equalizers. They basically depend on the fact that if the span of the autocorrelation of the shortened channel impulse response has the same length as the shortened channel itself.

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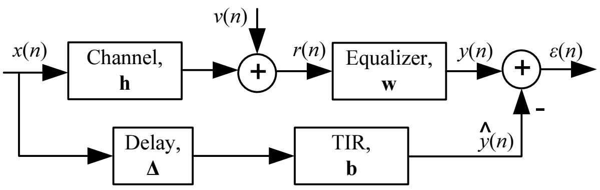


Figure 1: Block diagram of MMSE channel shortening equalizer.

The algorithms mentioned above have a major shortcoming. The algorithms operate as the Time domain EQualizer (TEQ) front-end of the MCM receiver [8]. In order to properly combat the ISI/ICI, TEQ has to jointly operate with a Frequency domain EQualizer (FEQ) which requires the CSI of the shortened channel. However, although the algorithms mentioned above shorten the channel, this information is not explicitly provided and has to be obtained by further channel estimation. Clearly, this is in contrast to the philosophy of blind signal processing.

In this paper we propose a novel blind channel shortening technique which also provides the CSI of the shortened channel and is named as BACS-SI (Blind, Adaptive Channel Shortening Algorithm with shortened channel State Information). The basic idea of BACS-SI is a combination of the MMSE and SAM channel shortening methods. Roughly speaking, MMSE aims at minimizing the difference between the shortened channel impulse response (which is the convolution of the actual channel and the equalizer) and a Target Impulse Response (TIR). Here, we aim at minimizing the difference between the *autocorrelation* of the shortened channel impulse response and the *autocorrelation* of the TIR in an iterative manner. At the end of the iterations, both the shortening equalizer and the corresponding TIR are obtained, and this TIR can later be used in the FEQ.

Some of the notation used throughout the paper is as follows. A scalar, a vector and a matrix will respectively be denoted by a lower-case italic, a lower-case boldface and an upper-case boldface letter. The convolution operator is represented by $(\cdot) * (\cdot)$, and $(\cdot)^T$ is used for matrix transpose and transpose. All signals and coefficients are assumed to be real valued.

## 2. MMSE CHANNEL SHORTENING

The block diagram of the MMSE channel shortening algorithm is given in Figure 1. The main purpose of the MMSE channel shortening equalizer is to reduce the length of the shortened channel, i.e. the convolution of the actual channel and the equalizer, to a predetermined shorter length by satisfying the MSE criterion.

On the upper branch, the signal arriving at the receiver, $r(n)$, can be modeled as the convolution of the zero mean, unit variance data symbols, $x(n)$, with the actual channel, $h(n)$, of length $n_h$ which is represented by the vector $\mathbf{h} = [h_0 \quad h_1 \quad \cdots \quad h_{n_H-1}]^T$ and zero-mean additive white Gaussian noise $v(n)$ with variance $\sigma_v^2$ added on top. Data symbols and noise samples are assumed to be statistically independent .

Next, this signal is processed by the equalizer $\mathbf{w} = [w_0 \quad w_1 \quad \cdots \quad w_{n_w-1}]^T$ of length $n_w$ generating the output signal $y(n)$

$$\begin{aligned} y(n) &= \mathbf{w}^T \mathbf{r}_n = \mathbf{w}^T \mathbf{H} \mathbf{x}_n + \mathbf{w}^T \mathbf{v}_n \\ &= \mathbf{c}^T \mathbf{x}_n + \mathbf{w}^T \mathbf{v}_n \end{aligned} \tag{1}$$

where $\mathbf{r}_n = [r(n) \quad r(n-1) \quad \cdots \quad r(n_w-1)]^T$, $\mathbf{x}_n = [x(n) \quad x(n-1) \quad \cdots \quad x(n_h+n_w-1)]^T$, $\mathbf{v}_n = [v(n) \quad v(n-1) \quad \cdots \quad v(n_w-1)]^T$, and $\mathbf{H}$ is the Toeplitz convolution matrix, e.g. [9].

As can be infered from (1), the convolution of the communication channel and the equalizer, i.e. the $n_c \times 1$ vector $\mathbf{c}$ can be considered as the shortened channel, where $n_c = n_h + n_w - 1$.

The lower branch of Figure 1 can be visualized as a virtual one which does not physically exist. Together with the delay $\Delta$ the $n_b \times 1$ TIR vector $\mathbf{b} = [b_0 \quad b_1 \quad \cdots \quad b_{n_b-1}]^T$ can be included in an $(n_h+n_w-1) \times 1$ augmented TIR vector $\tilde{\mathbf{b}} = \left[\mathbf{0}_{1\times\Delta} \quad \mathbf{b}^T \quad \mathbf{0}\right]^T$ where $\mathbf{0}$ is the all-zeros vector of proper dimensions. Therefore the output of the augmented TIR is

$$\hat{y}(n) = \tilde{\mathbf{b}}^T \mathbf{x}_n.$$

The error term $\varepsilon(n)$ is defined as the difference between the output of the equalizer and the TIR, and the MSE criterion can be written as

$$J = E\left\{(\varepsilon(n))^2\right\} = E\left\{(y(n) - \hat{y}(n))^2\right\} \tag{2a}$$

$$= E\left\{\left(\left(\mathbf{c}^T - \tilde{\mathbf{b}}^T\right)\mathbf{x}_n + \mathbf{w}^T \mathbf{v}_n\right)^2\right\} \tag{2b}$$

which is minimized over $\mathbf{w}$ and $\mathbf{b}$ under some constraint, for example the unit norm constraint ($\mathbf{b}^T\mathbf{b} = 1$), in order to avoid the trivial solution.

In other words, the MMSE criterion minimizes the difference between the impulse response of the shortened channel and TIR while considering noise enhancement also. Evidently, optimization of the MMSE criterion requires the channel coefficients $\mathbf{h}$ which has to be obtained by channel estimation.

## 3. PROBLEM FORMULATION

If channel estimation is not possible or desirable, one can opt for blind channel shortening equalization. For this purpose, we follow the framework of the MMSE algorithm, however, we change the signals $y(n)$ and $\hat{y}(n)$ with their autocorrelation functions $R_{yy}(l)$ and $R_{\widehat{y}\widehat{y}}(l)$, respectively, in (2a).

Then, refering to Figure 1, the error sequence is defined as

$$e(l) = R_{yy}(l) - R_{\widehat{y}\widehat{y}}(l). \tag{3}$$

Hence, the proposed cost function, $J$, for BACS-SI is defined as

$$J = \sum_{l=0}^{n_c-1} \left(R_{yy}(l) - R_{\widehat{y}\widehat{y}}(l)\right)^2. \tag{4}$$

The auto-correlation function of the sequence $y(n)$ is

$$R_{yy}(l) = E\{y(n)y(n-l)\} \tag{5}$$

$$= E\left\{\left(\mathbf{c}^T\mathbf{x}_n + \mathbf{w}^T\mathbf{v}_n\right)\left(\mathbf{x}_{n-l}^T\mathbf{c} + \mathbf{v}_{n-l}^T\mathbf{w}\right)\right\}$$

$$= \mathbf{c}^T\mathbf{R}_{xx}(l)\,\mathbf{c} + \mathbf{w}^T\mathbf{R}_{vv}(l)\,\mathbf{w} \tag{6}$$

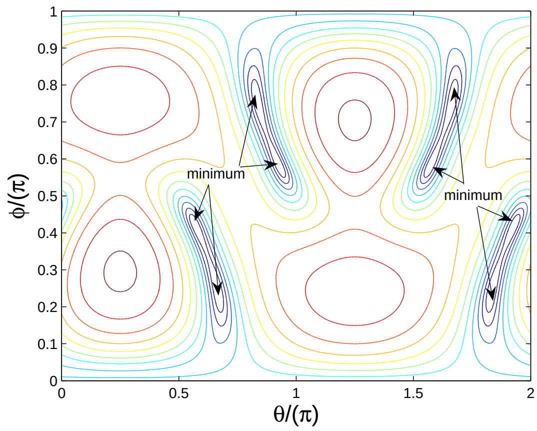


Figure 2: Cost surface of the proposed blind channel shortening equalization algorithm.

where the $(n_h+n_w-1) \times (n_h+n_w-1)$ signal and the $n_w \times n_w$ noise covariance matrices respectively are $\mathbf{R}_{xx}(l) = E\left\{\mathbf{x}(n)\mathbf{x}^T(n-l)\right\}$ and $\mathbf{R}_{vv}(l) = E\left\{\mathbf{v}(n)\mathbf{v}^T(n-l)\right\}$. If both $x(n)$ and $v(n)$ are assumed i.i.d., it can be shown that both $\mathbf{R}_{xx}(l)$ and $\mathbf{R}_{vv}(l)$ are Toeplitz matrices with only one (off-)diagonal of nonzero entries determined by the lag $l$.

Then, $R_{yy}(l)$ becomes

$$\begin{aligned} R_{yy}(l) &= \sum_{k=0}^{n_c-1} c(k)c(k-l) + \sigma_n^2 \sum_{k=0}^{n_w-1} w(k)w(k-l) \\ &\triangleq R_{cc}(l) + \sigma_v^2 R_{ww}(l) \end{aligned} \tag{7}$$

Similarly, the autocorrelation of the sequence $\widehat{y}(n)$ is

$$R_{\widehat{y}\widehat{y}}(l) = E\left[\widehat{y}(n)\widehat{y}(n-l)\right]$$

$$= E[\left(\tilde{\mathbf{b}}^T\mathbf{x}_n\right)\left(\mathbf{x}_{n-l}^T\tilde{\mathbf{b}}\right)] \tag{8}$$

$$= \tilde{\mathbf{b}}^T\mathbf{R}_{xx}(l)\,\tilde{\mathbf{b}} \tag{9}$$

, or, it is straightforward to show that

$$R_{\widehat{y}\widehat{y}}(l) = \sum_{k=0}^{n_b-1} b(k)\,b(k-l) \triangleq R_{bb}(l). \tag{10}$$

Hence the cost function, $J$, can be rewritten as

$$\begin{aligned} J &= \sum_{l=0}^{n_c-1} \left(R_{yy}(l) - R_{\widehat{y}\widehat{y}}(l)\right)^2 \\ &= \sum_{l=0}^{n_c-1} \left(R_{cc}(l) + \sigma_n^2 R_{ww}(l) - R_{bb}(l)\right)^2. \end{aligned} \tag{11}$$

As usual with other blind signal processing algorithms, the cost surface in (11) is multimodal which is demonstrated in Figure 2. Here, the channel is $\mathbf{h} = [1 \quad 0.3 \quad 0.2]^T$ and for demonstration purposes, the TIR is kept fixed at $\mathbf{b} = [1 \quad 0.5]^T$. The equalizer has three taps $\mathbf{w} = [w_1 \quad w_2 \quad w_3]^T$ and the norm of the equalizer taps is assumed to be unity, i.e. $\mathbf{w}^T\mathbf{w} = 1$. Therefore these coefficients can be represented by two axes defined as the horizontal axis $\theta = \tan^{-1}\left(\sqrt{w_1^2 + w_2^2}/w_3\right)$ and the vertical axis $\phi = \tan^{-1}(w_2/w_1)$. It is interesting to observe that the minima of the cost function are related in a certain way.

**Proposition 1:** An autocorrelation function (e.g. $R_{yy}(l)$, $R_{cc}(l)$ and $R_{bb}(l)$ in (11)) is invariant to taking the conjugate-reciprocal of any zero of the related sequence with respect to the unit circle.

**Proof:** The outline of the proof is as follows. The Fourier transform of an autocorrelation function is the power spectral density (PSD) of the related sequence. Since the PSD provides only the squared-magnitude information, a zero in the z-plane or its conjugate-reciprocal with respect to the unit circle has the same magnitude response on the unit circle. Hence there are $2^{n_z}$ different combinations of the zeros of a sequence which result in the same PSD where $n_z$ is the number of zeros.

**Proposition 2:** The autocorrelation function of the convolution of two sequences is equal to the convolution of the two seperate autocorrelation functions of these sequences.

**Corollary 1:** A direct result of Propositions 1 and 2 is, there are $2^{n_w}$ combinations for $\mathbf{w}$ which give the same autocorrelation function $R_{cc}(l)$, including the negatives. Similarly, there are $2^{nb}$ combinations for $\mathbf{b}$ which give the same autocorrelation function $R_{bb}(l)$, including the negatives. Therefore, there are $2^{n_w+n_b}$ minima of (4) with the same value, and all these minima can be generated from a single minimum by simple zero reciprocal and order-reversal operations. This property will later be used to find the optimum $\mathbf{w}$ and $\mathbf{b}$ pair which give the maximum bit rate amongst all combinations.

## 4. ADAPTIVE ALGORITHM

In order to find a minimum of the cost surface defined in (11), we will employ a stochastic gradient descent algorithm. In order to avoid an all-zeros solution, a constraint must be imposed on this cost function. Therefore, for the present work, because of the reason explained in Proposition 1, we assume that the equalizer has unit norm, i.e. $\mathbf{w}^T\mathbf{w} = 1$.

The variables of this optimization problem are the equalizer and TIR coefficients, $\mathbf{w}$ and $\mathbf{b}$, respectively. Although two separate update equation could be written for these variables, the convergence of the algorithm would be determined by the slowest one. Therefore in the sequel, we define a composite variable $\mathbf{f} = [\mathbf{w}\ \mathbf{b}]^T$ for notational clarity.

**Remark:** Note that since this is a blind algorithm, we only have access to the signal at the channel output. We also assume that the data symbols are i.i.d. random variables known to be zero mean with unit variance and the noise also has zero mean. Apart from these assumption, there is no access to neither the CSI nor the noise variance, i.e. the operating SNR.

The update equation of the stochastic gradient descent algorithm can be written as

$$\mathbf{f}^{n+1} = \mathbf{f}^n - \frac{1}{2}\mu\nabla_{\mathbf{f}}J(n)$$

$$\nabla_{\mathbf{f}}J(n) = \begin{bmatrix}\nabla_{\mathbf{w}}J(n)\\ \nabla_{\mathbf{b}}J(n)\end{bmatrix}$$

$$\begin{aligned}\nabla_{\mathbf{w}}J(n) &= \nabla_{\mathbf{w}}\left(\sum_{l=0}^{n_c-1}\left(R_{yy}(l)-R_{bb}(l)\right)^2\right)\\ &= 2\sum_{l=0}^{n_c-1}\left(R_{yy}(l)-R_{bb}(l)\right)\nabla_{\mathbf{w}}R_{yy}(l)\end{aligned} \tag{12}$$

recall that $R_{yy}(l) = E\{y(n)y(n-l)\}$, then using the fact $y(n) = \mathbf{w}^T\mathbf{r}_n$

$$\begin{aligned}\nabla_{\mathbf{w}}J(n) = 2\sum_{l=0}^{n_c-1}&\left(E\{y(n)y(n-l)\}-R_{bb}(l)\right)\\ &\left(E\{\mathbf{r}_n y(n-l)\}+E\{y(n)\mathbf{r}_{n-l}\}\right]).\end{aligned}$$

Similarly,

$$\begin{aligned}\nabla_{\mathbf{b}}J(n) &= \nabla_{\mathbf{b}}\left(\sum_{l=0}^{n_c-1}\left(R_{yy}(l)-R_{bb}(l)\right)^2\right)\\ &= -2\sum_{l=0}^{n_c-1}\left(R_{yy}(l)-R_{bb}(l)\right)\left(\mathbf{b}_{up}(l)+\mathbf{b}_{dn}(l)\right)\end{aligned} \tag{13}$$

where $\mu$ is the step size, and $\nabla_{\mathbf{f}}$, $\nabla_{\mathbf{w}}$ and $\nabla_{\mathbf{b}}$ are the gradients of $J$ w.r.t $\mathbf{f}$, $\mathbf{w}$ and $\mathbf{b}$, respectively. The vectors $\mathbf{b}_{up}(l)$ and $\mathbf{b}_{dn}(l)$ are obtained from the gradient $\nabla_{\mathbf{b}}R_{bb}(l)$, and their structure are $\mathbf{b}_{up}(l) = [b_l \ \cdots \ b_{n_b-1} \ \mathbf{0}_{1\times l}]^T$ and $\mathbf{b}_{dn}(l) = [\mathbf{0}_{1\times l} \ b_0 \ \cdots \ b_{n_b-l-1}]^T$. Then, the update rule for $\mathbf{f}$ can be written as

$$\begin{aligned}\mathbf{f}^{n+1} = \mathbf{f}^n - \mu\sum_{l=0}^{n_c-1}&\left(R_{yy}(l)-R_{bb}(l)\right)\\ &\begin{bmatrix}\left(E\{\mathbf{r}_n y(n-l)\}+E\{y(n)\mathbf{r}_{n-l}\}\right])\\ -\left(\mathbf{b}_{up}(l)+\mathbf{b}_{dn}(l)\right)\end{bmatrix}\end{aligned} \tag{14}$$

where the first vector component of the right-most vector corresponds to the update of the equalizer coefficients whereas the second vector component corresponds to the update of the TIR.

To finalize the update rule, the expectation terms in (14) has to be calculated. Since direct access to the CSI is not possible, these expectations have to be calculated over signal samples. As stated in [8] there are several unbiased estimates for this calculation. In the sequel, we will investigate the moving average (MA) and autoregressive (AR) estimates.

### 4.1 Moving Average (MA)

For a fair comparison, we will follow the methodology in [8], and calculate the expectations in a block-by-block manner, where each block contains $N$ samples. Hence for the $k$-th block $R_{yy}(l)$ is

$$R_{yy}(l) = \frac{1}{N}\sum_{n=kN}^{(k+1)N-1} y(n)y(n-l) \tag{15}$$

Similarly, we have

$$E\{\mathbf{r}_n y(n-l)\} = \frac{1}{N}\sum_{n=kN}^{(k+1)N-1}\mathbf{r}_n y(n-l) \tag{16a}$$

$$E\{y(n)\mathbf{r}_{n-l}\} = \frac{1}{N}\sum_{n=kN}^{(k+1)N-1}\mathbf{r}_{n-l}y(n). \tag{16b}$$

Substituting (15) and (16b) into (14) the update rule with moving average estimates can be obtained.

### 4.2 Auto-regressive (AR)

Another method to calculate the expectations in (14) is to use autoregressive (AR) estimates. Again following the methodology in [8], in this case the calculations are carried out in a sample-by-sample manner. In order to achieve this, for each lag $l$, first define the following

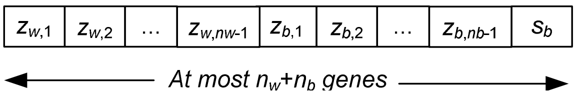


Figure 3: Structure of a chromosome. Genes are binary valued.

$$\mathbf{a}^{n+1}(l) = (1-\alpha)\,\mathbf{a}^{n}(l) + \alpha y(n)\begin{bmatrix} r(n-l) \\ r(n-l-1) \\ \vdots \\ r(n-l-(L_w+1)) \end{bmatrix}$$

$$\mathbf{b}^{n+1}(l) = (1-\alpha)\,\mathbf{b}^{n}(l) + \alpha y(n-l)\begin{bmatrix} r(n) \\ r(n-1) \\ \vdots \\ r(n-(L_w+1)) \end{bmatrix}$$

$$c^{n+1}(l) = \mathbf{w}^T \mathbf{b}^{n+1}(l)$$

where $\alpha \in (0,1)$ can be considered as a forgetting factor. Decreasing the value of $\alpha$ will increase the contribution of the past estimates to the AR estimates. By using these definitions, the auto/cross-correlation functions are obtained as $E\{y(n)\mathbf{r}_{n-l}\} = \mathbf{a}^{n+1}(l)$, $E\{\mathbf{r}_n y(n-l)\} = \mathbf{b}^{n+1}(l)$ and $R_{yy}(l) = c^{n+1}(l)$.

## 5. MATCHING EQUALIZER AND TIR PAIR

As discussed in Corollary 1, there are $2^{n_w+n_b}$ combinations of the $\mathbf{w}$ and $\mathbf{b}$ pairs which give the same minimum value for (11). Therefore minimizing the proposed cost function does not necessarily give a good solution. Indeed, in parallel with the discussion in [8], half of the minima do not even perform channel shortening. Moreover, the TIR may not match the channel shortened by the equalizer, both found at the end of the iterative algorithm. However, among these pairs, one of them performs the maximum shortening and provides the highest bit rate. If no a priori information related to the channel is present, it may not be possible to determine the best initialization point for obtaining this solution at the end of the descent algorithm.

Nevertheless, even if it is not optimum, a solution starting from an arbitrary point will constitute a basis for further optimization, since all minima are related according to the rule explained in Corollary 1. Therefore, the zeros of the equalizer $\mathbf{w}$ and the coefficients of the TIR $\mathbf{b}$ are extracted from this possibly non-optimum solution.

Since the length of the TIR is limited, the equalizer yielding the shortened channel same as the TIR will form the matching equalizer-TIR pair. One can employ the pilot tone present in the downstream of the MCM transmission to determine this pair. For example in ADSL the 64-th tone in the downstream range is fixed to symbol '1' as the pilot signal.

Let the Fourier transforms of the shortened channel and the TIR be represented by $C(k)$ and $B(k)$, respectively. Ultimately, the TIR $B(k)$ should be equal to the shortened channel $C(k)$, hence it can be used as the FEQ, i.e. $Q(k) = B^*(k)/|B(k)|^2$. Then after TEQ, FFT and FEQ operations, the signal observed at the output of the receiver is $R(k) = \left[C(k)B^*(k)/|B(k)|^2\right]X(k)$.where $X(k)$ is the symbol transmitted from the $k$-th bin. Also let the pilot symbol transmitted at the $l$-th bin be $X(l) = 1$. Then only if the shortened channel and the TIR matches, $R(l) = 1$. Hence by minimizing the criterion $|R(63)-1|^2$ among possible $\mathbf{w}$-$\mathbf{b}$ pairs, one can find the matching $\mathbf{w}$-$\mathbf{b}$ pair.

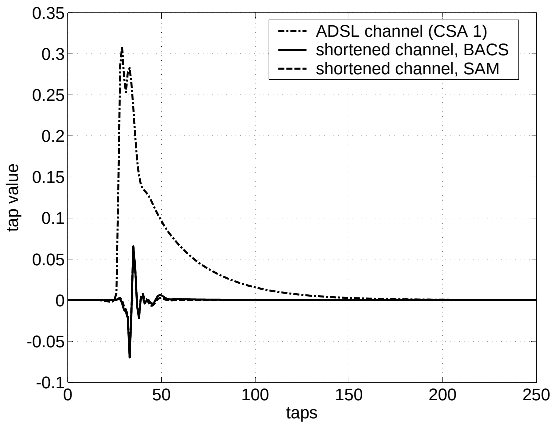


Figure 4: Shortened channel impulse response of the proposed BACS and the SAM algorithms compared to the CSA 1 ADSL channel model.

As the second phase of the optimization, to overcome the problem of finding this matching $\mathbf{w}$-$\mathbf{b}$ pair, we employ genetic algorithms. Genetic algorithms are global search techniques based on the principles of genetics and natural selection. The parameters to be optimized are represented by 'genes' and the combination of genes for a particular solution is called a 'chromosome' [10].

For searching the whole parameter space, one needs $n_w + n_b$ genes in a chromosome. However, in order to narrow down the search space, observing that the equalizer and TIR solutions are real values, hence, complex zeros of $\mathbf{w}$ and $\mathbf{b}$ will be in conjugate pairs, we can only alter the position of one of these zeros, and the other zero will automatically change position.

Hence, for the present problem, there are a variable number of binary valued genes in a chromosome as demonstrated in Figure 3. There are at most $n_w - 1$ genes ($z_{w,i}$) representing the position of the zeros of $\mathbf{w}$ with respect to the unit circle, e.g. a zero of the prospect solution remains inside the unit circle if the corresponding gene is set to 1 and it is transfered to outside if the gene is 0. Similarly; there are at most $n_w - 1$ genes ($z_{b,i}$) representing the position of the zeros of $\mathbf{b}$. The remaining gene ($s_b$) represent the relative sign of the searched equalizer and TIR solutions.

First, an initial population consisting of 10 chromosomes is generated randomly. Then at each iteration, two choromosomes are selected from the population to produce two new offsprings. Selection methods can be in several ways such as top to bottom, random, weighted random, etc [10]. After selecting two chromosomes, a crossover point is randomly selected within the $\mathbf{w}$ and $\mathbf{b}$ parts separately between the last and the first bits of the parents' choromosome and bits are swapped mutually at crossover point. Also, random mutations alter the bits of a selection of a percentage of the new population with a rate of 15% and the selection probability is 50%. Finally, the number of iterations depends on whether an acceptable solution or a set number of iterations exceeded. Here we use the maximum iteration number as 500.

## 6. SIMULATIONS AND RESULTS

solution.

For the simulations we consider an ADSL environment described in [8]. The cyclic prefix consists of 32 samples and the FFT size is 512. The TEQ has 16 taps, and the channel is chosen to be CSA test loop 1 [11]. The operating SNR is 40 dB. The initilization for the equalizer is all-zeros with a single one in the middle, whereas the TIR is initialized to all-zeros but the first entry is one. The step size is taken as $\mu = 10^{-4}$.

The first results provide the shortened channel impulse response

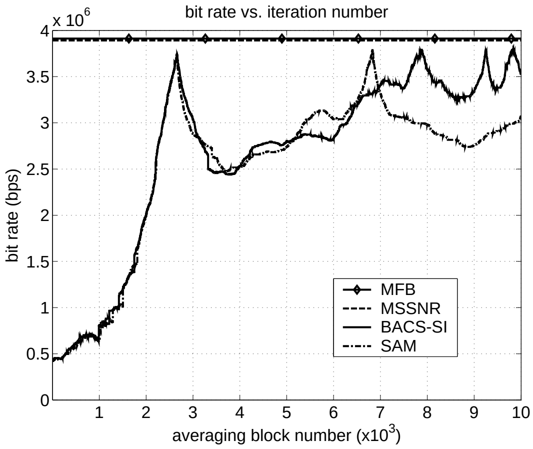


Figure 5: Bıt rate of the proposed and SAM algorithms compared to the matched filter bound.

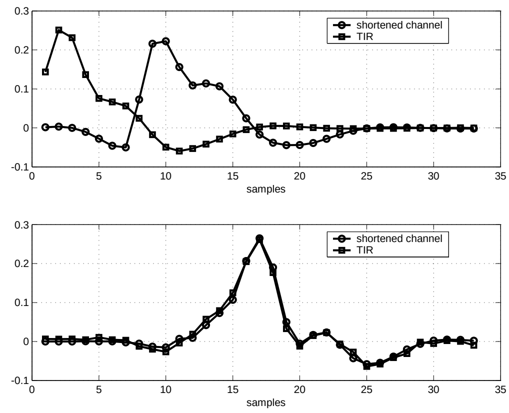


Figure 6: (a) Initial **c**-**b** pair, (b) **c**-**b** pair after the genetic algorithm.

as seen in Figure 4. It can be observed that both the proposed BACS-SI and SAM algorithm can successfully shorten the CSA 1 model for the relatively longer ADSL channel. Moreover, they converge to almost the same

Figure 5 demonstrates the bit rate comparison of the BACS and SAM algorithms as well as the matched filter bound. The price of blind channel shortening can be considered as the loss in the bit rate as compared to the matched filter bound. However, as seen from the figure, BACS-SI can considerably overcome this loss since the loss of SAM is around 25 % whereas for BACS-SI this loss is approximately 10%. Therefore it can be concluded that BACS-SI provides better bit rate efficiency compared to SAM.

In addition to the improvement in the bit rate, the main contribution of the present study is finding the shortened channel impulse response. Hence, Figure 6 demonstrates the improvement achieved by the genetic algorithm. The genetic algorithm also has the ability to find the optimum delay caused by the shortening equalizer. Note that, even if the outcome of the iterative algorithm may not have the ability to shorten the channel, the genetic algorithm can successfully find the shortening counterpart. The genetic algorithm is initialized with the outcome of the iterative algorithm. Working on a single ADSL frame, the result in Figure 6 is obtained after 500 iterations.

Therefore, although it is not possible to directly find the shortened channel impulse response in the SAM or SLAM algorithms (or even a successful shortening equalizer), the proposed BACS-SI algorithm provides this information as the TIR without any additional effort.

## 7. CONCLUSION

A blind, adaptive channel shortening algorithm (BACS-SI) is proposed which has two phases. In the first phase a stochastic gradient algorithm is employed to find some solution to the defined cost function. In the second phase this solution is fine tuned by a genetic algorithm to find the optimum equalizer-target impulse response pair which satisfies a criterion based on the pilot tone. The main advantage of this method is, in addition to performing channel shortening, it can provide the shortened channel information inherently due to the existence of the target impulse response in the algorithm. This is a major contribution when compared to similar algorithms SAM and SLAM where the shortened channel has to be obtained by further channel estimation. It is also demonstrated that the BACS-SI algorithm has better bit rate performance compared to the SAM algorithm.

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