ADAPTIVE DECISION FEEDBACK EQUALIZER FOR SUI CHANNEL MODELS

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ABSTRACT

This paper investigates the performance of the Non-Stationary Recursive Least Squares (NSRLS) algorithm in the adaptive equalization of realistic wireless channels. As in general the wireless channels are assumed to vary in time according to a Markov model, the NSRLS algorithm may represent a favorite candidate since it is designed to track Markovian time varying channels. The Stanford University Interim (SUI) channels are considered in this paper. To obey the constraints of the realistic transmission context, we propose in this paper a generalized version of the NSRLS algorithm. The performances of the Decision Feedback Equalizer (DFE) updated by the proposed NSRLS algorithm are compared with those of the conventional RLS-DFE through simulations. The reported results demonstrate the efficiency of the generalized NSRLS algorithm to capture the time variations of the SUI-1 and SUI-2 channels. Indeed, the Bit Error Rate (BER) is significantly reduced with the NSRLS-DFE. Moreover, it is shown that a high order Markov model is required to well represent the non-stationarity of the SUI channels.

1. INTRODUCTION

In wireless transmission, the channel characteristics are unknown and time varying, so it is difficult to design the optimal transmitter/receiver. An equalizer is needed to eliminate Inter-Symbol Interference (ISI) introduced by the time varying channel [1] [2]. Furthermore, the Decision Feedback Equalizers (DFEs) are preferred to transverse equalizers in the case of severe multipath time varying channels [2]. In this paper, we are interested in the SUI wireless channels [3] [4]. In particular, only the Line-Of Sight (LOS) models are considered here.

Basing on the idea that the wireless channels vary according to a Markov model [5] [6] [7], we propose to adapt the DFE that minimize the ISI introduced by an SUI channel by a specific algorithm: the NSRLS algorithm. In fact, this latter one is dedicated to track time varying Markovian channels' impulse responses. Based on a prior knowledge of the non-stationarity Markov structure, the NSRLS performs an adaptive identification of the unknown Markovian parameters followed by an adaptive estimation of the channel impulse response. The approach of the NSRLS algorithm is different from Kalman algorithm. Moreover, contrary to Kalman, the knowledge of the non-stationarity and the observation noises statistics are not required with NSRLS [1].

The NSRLS algorithm was proposed in [8] for the adaptive identification of Markovian time varying channels. In [9] it was applied to adapt the parameters of a DFE dedicated to the equalization of a first order Markov channels. However, in [8] and [9], only a real value transmission context and a synthetic first order Markov channel were considered. Therefore, in this paper, a generalized NSRLS algorithm is designed for a DFE performing in a realistic transmission context: the transmitted input and the channel impulse response are complex. Moreover, the equalized channel is assumed to vary according to a Markov model of order P that can be higher than one.

Furthermore, the proposed equalization approach allows not only the reduction of the BER but also the characterization of the Markovian non-stationarity of the considered SUI channels.

The remainder of the paper is organized as follows. In Section 2, the non-stationary equalization context as well as the test SUI channels models are presented. In Section 3, the generalized NSRLS algorithm is developed. Section 4 presents several simulation results to illustrate the efficiency of the NSRLS-DFE in the presence of the SUI channels.

2. ADAPTIVE EQUALIZATION FOR SUI CHANNELS

2.1 Equalization problem

For transmission over a wireless channels, the underlying signal space is one-dimensional and the equalizer has complex taps. The structure of such equalizer is shown in Figure 1.

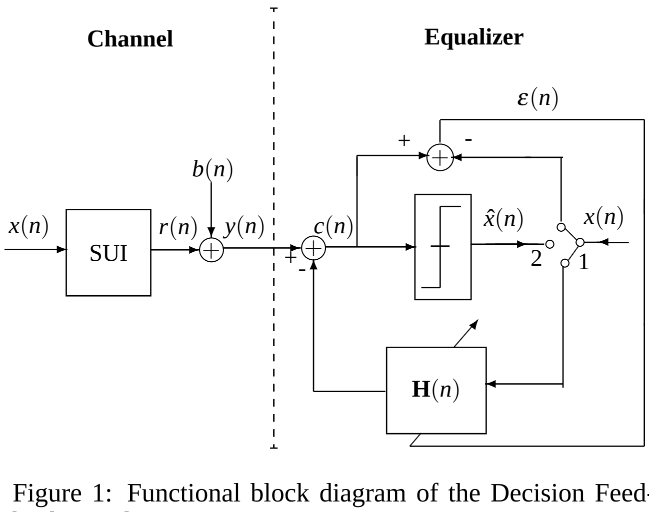

Figure 1: Functional block diagram of the Decision Feedback Equalizer

$\mathbf{X}(n) = \left[x(n-1),...,x(n-N)\right]^T$ is the input data vector. The signal $y(n)$ is the received symbols, it is a noisy version of the channel output $r(n)$. The observation noise $b(n)$ is assumed to be an i.i.d., zero mean value and independent of $x(n)$.

The equalizer modeled by a Feedback filter is characterized by its finite complex impulse response $\mathbf{H}(n)$ of length N. The input of the decision device is

$$c(n) = y(n) - \mathbf{H}^H(n)\hat{\mathbf{X}}(n), \tag{1}$$

where $\hat{\mathbf{X}}(n) = \left[\hat{x}(n-1),...,\hat{x}(n-N)\right]^T$ is the feedback filter input vector, and $\hat{x}(n)$ is an estimation of the transmitted symbol $x(n)$.

The time evolution of the adaptive filter $\mathbf{H}(n)$ is controlled by the error $\varepsilon(n)$. The adaptive equalizer operates in two modes. In the training mode, (1 in Figure 1), a known sequence of symbols is transmitted and a synchronized version is locally generated in the receiver. The true transmitted symbols are then used to compute the error:

$$\varepsilon(n) = \big(y(n) - \mathbf{H}^H(n)\mathbf{X}(n)\big) - x(n). \tag{2}$$

The tap coefficients are adjusted in order to obtain the desired response. Once the tap coefficients converge, the equalizer is switched to a decision directed mode (2 in Figure 1). The error signal derived from the estimated symbols is given by:

$$\varepsilon(n) = \big(y(n) - \mathbf{H}^H(n)\hat{\mathbf{X}}(n)\big) - \hat{x}(n). \tag{3}$$

As shown by Figure 1, only the Feedback filter of a conventional DFE is used. Two reasons explain this choice. Firstly, in the more general structure, it is usually possible to separate the adaptation of the FIR section from the Feedback one. Secondly, the aim of this paper is to investigate the behavior of the generalized NSRLS algorithm facing to realistic SUI-1 and SUI-2 channels, that are assumed to vary in time according to a Markov model.

2.2 SUI channel models

The SUI channel models are proposed in [3] to simulate, design, develop, and test a system suitable for fixed broadband wireless applications. In general, for SUI channels, it is noted that the shape of the used Doppler power spectral density is not similar to Jake's spectrum. It is given by equation (4) where $f_0 = \frac{f}{f_m}$ and f_m is the Doppler frequency. The SUI characterization is inspired by real measurements done in urban and suburban multipath conditions.

$$S(f) = \begin{cases} 1 - 1.72 f_0^2 + 0.785 f_0^4 & |f_0| \leq 1 \\ 0 & |f_0| > 1 \end{cases} \tag{4}$$

There are six types of SUI channels with different parameters corresponding to typical terrain types of the continental US. The SUI-1, SUI-2 and the SUI-6 models correspond to channel with Line Of Sight (LOS). However, SUI-3, SUI-4, and SUI-5 are models for Non Line Of Sight (NLOS) channels [3]. In this paper the SUI-1 and SUI-2 LOS channels are considered for the evaluation of the generalized NSRLS algorithm.

3. DESIGN OF THE GENERALIZED NSRLS ALGORITHM

The generalized NSRLS algorithm is designed to outperform the traditional RLS algorithm, when tracking a $P-$order Markov time varying channel. Indeed, the filter representing such channel is assumed to vary in time, n, as following:

$$\mathbf{F}(n) = \sum_{i=1}^{P} \beta_i \mathbf{F}(n-i) + \Omega(n), \tag{5}$$

where the vector $\mathbf{F}(n) = \left[f(1),...,f(N)\right]^T$ represents the time varying channel filter and $0 < (\beta_i)_{i=1..P} \leq 1$ are the Markovian parameters. The vector $\Omega(n)$ is the non-stationarity noise. The components of $\Omega(n)$ are assumed to be Gaussian white processes.

The algorithm is designed in such way to take into account the prior knowledge of the Markovian channel model structure [8]. Hence, the classical RLS with forgetting factor is modified as follows:

$$\mathbf{H}(n+1) = \sum_{i=1}^{P} \hat{\beta}_i(n)\mathbf{H}(n+1-i) + \varepsilon^*(n)\mathbf{K}(n), \tag{6}$$

where the N-by-1 vector $\mathbf{K}(n)$ is referred to the gain vector,

$$\mathbf{K}(n) = \frac{\mathbf{P}(n)\hat{\mathbf{X}}(n)}{\lambda + \hat{\mathbf{X}}^H(n)\mathbf{P}(n)\hat{\mathbf{X}}(n)}, \tag{7}$$

and the N-by-N matrix $\mathbf{P}(n)$ is referred to the inverse correlation matrix,

$$\mathbf{P}(n+1) = \lambda^{-1}\Big(\mathbf{P}(n) - \mathbf{K}(n)\hat{\mathbf{X}}^H(n)\mathbf{P}(n)\Big). \tag{8}$$

λ is the forgetting factor close to, but less than, 1. The parameters $\big(\hat{\beta}_i(n)\big)_{i=1..P}$ are the adaptive estimates of the unknown Markovian parameters $\big(\beta_i\big)_{i=1..P}$. At time n, the estimation of β_i is based on the minimization of the cost function $J_n\big(\hat{\beta}_i\big) = \sum_{k=1}^{n} |\varepsilon(k)|^2$. The principal computing details leading the adaptive estimation of the Markovian parameters $\big(\hat{\beta}_i(n)\big)_{i=1..P}$ are presented in the appendix. Therefore, the generalized NSRLS algorithm is described by:

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1. $\mathbf{H}(n+1) = \sum_{i=1}^{P} \hat{\beta}_i(n)\mathbf{H}(n+1-i) + \varepsilon^*(n)\mathbf{K}(n)$,
2. $\mathbf{K}(n) = \frac{\mathbf{P}(n)\hat{\mathbf{X}}(n)}{\lambda + \hat{\mathbf{X}}^H(n)\mathbf{P}(n)\hat{\mathbf{X}}(n)}$,
3. $\mathbf{P}(n+1) = \lambda^{-1}\Big(\mathbf{P}(n) - \mathbf{K}(n)\hat{\mathbf{X}}^H(n)\mathbf{P}(n)\Big)$,
4. $\hat{\beta}_i(n) = \frac{num_i(n)}{den_i(n)}$,
5. $num_i(n) = num_i(n-1) + Re\Big[\big(y(n) - \hat{x}(n)$ $-\varepsilon(n-1)\mathbf{K}^H(n-1)\hat{\mathbf{X}}(n)$ $-\sum_{j=1, j\neq i}^{P} \hat{\beta}_j \mathbf{H}^H(n-j)\hat{\mathbf{X}}(n)\big)\mathbf{H}^T(n-i)\hat{\mathbf{X}}^*(n)\Big]$,
6. $den_i(n) = den_i(n-1) + |\mathbf{H}^H(n-i)\hat{\mathbf{X}}(n)|^2$.

Note that, in the training mode the symbol $\hat{x}(n)$ is replaced by $x(n)$ its actual value.

4. SIMULATION RESULTS

The presented simulation results aim to investigate the performance of the NSRLS equalizer facing to SUI channels and also to characterize the assumed Markovian non-stationarity of the considered channels. The proposed equalizer performances were tested by several experiments based on SUI-1 and SUI-2 omni antenna channels. The reported results are obtained for the following considerations:

- The noise level added to the channel output was fixed by the Signal to Noise Ratio (SNR $[dB] = 10log10\left(\frac{P_r}{P_b}\right)$), where $P_r = E[r^2(n)]$ is the power of the output of the channel and $P_b = E[b^2(n)]$ is the noise power.
- An independent and identically distributed input with a Quaternary Phase-Shift Keying (QPSK) constellation was used.
- The DFE operates in training mode ($\approx$ 1000 symbols), after which it switches to a decision directed mode.
- The length of the DFE filter is fixed to $N = 4$. In fact, it is equal to the paths number of the considered SUI channels.

4.1 SUI channel characteristics

Table 1 and Table 2 show the time domain attribute of the SUI-1 and SUI-2 channels respectively. These tables show that the Doppler frequencies values are higher in the case of SUI-1 than in the case of SUI-2. Therefore, one can deduce that the SUI-1 non-stationarity is more severe than the one of SUI-2.

SUI-1				
	Tap1	Tap2	Tap3	Units
Delay	0	0.4	0.9	μs
Power (omni ant.)	0	-15	-20	dB
90% K-fact. (omni)	4	0	0	
Power (30° ant.)	0	-21	-32	dB
90% K-fact. (30°)	16	0	0	
Doppler	0.4	0.3	0.5	Hz

Table 1: SUI-1 channel model

SUI-2				
	Tap1	Tap2	Tap3	Units
Delay	0	0.4	1.1	μs
Power (omni ant.)	0	-12	-15	dB
90% K-fact. (omni)	2	0	0	
Power (30° ant.)	0	-18	-27	dB
90% K-fact. (30°)	8	0	0	
Doppler	0.2	0.15	0.25	Hz

Table 2: SUI-2 channel model

4.2 Tracking ability of the generalized NSRLS

Here, we analyze the tracking ability of the generalized NSRLS algorithm. The variations of the Mean Square Error (MSE $= E[\varepsilon^2(n)]$) versus the forgetting factor λ are presented in Figure 2, for the two test channels, SUI-1 and SUI-2. For these results, the first order NSRLS algorithm ($P = 1$) is used to update the filter equalizer. The SNR is set to 30 dB.

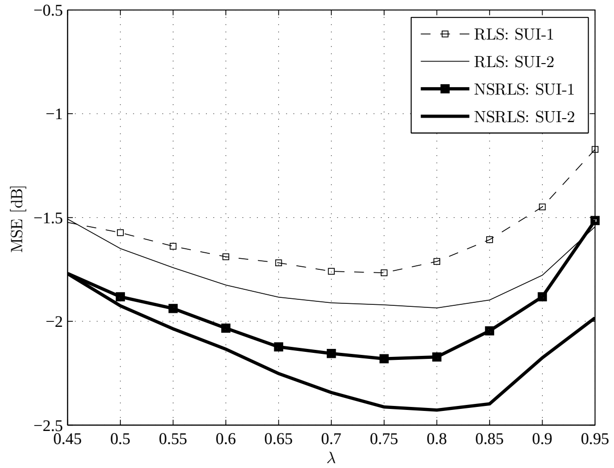

Figure 2: MSE versus the forgetting factor λ for SUI-1 and SUI-2 channels ($P = 1$, SNR $= 30$ dB)

Figure 2 shows that, for the two SUI channels, the NSRLS algorithm presents better tracking ability than the conventional RLS algorithm (gain $\simeq$ 0.7 dB). The optimum forgetting factor values (λ_{opt}) corresponding to the minimum mean square error are: $\lambda_{opt} = 0.75$ for SUI-1 channel and $\lambda_{opt} = 0.8$ for SUI-2 channel.

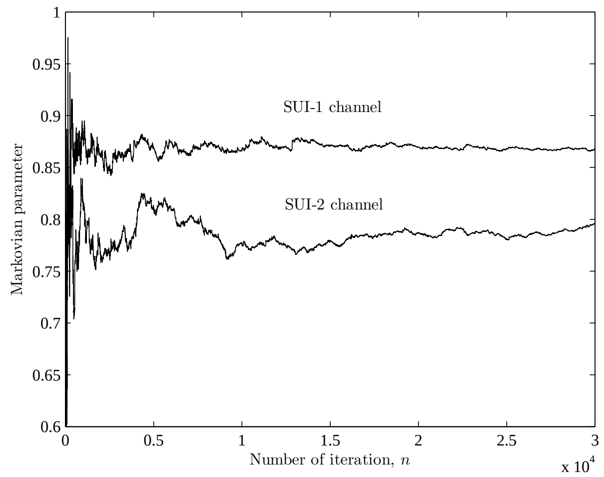

Figure 3: Convergence of the Markovian parameter $\hat{\beta}_1(n)$ for SUI-1 and SUI-2 channel ($P = 1$)

Figure 3 illustrates the time variations of the adaptive Markovian parameter $\hat{\beta}_1(n)$ corresponding to the two channels test. The forgetting factor is fixed to its optimal value. This figure shows that the Markovian parameters $\hat{\beta}_1(n)$ converge, in almost 500 simples, to an average value close to 0.86 for SUI-1 channel and 0.8 for SUI-2 channel. Therefore, the first order Markovian model can represent the time variation of SUI-1 and SUI-2 channels.

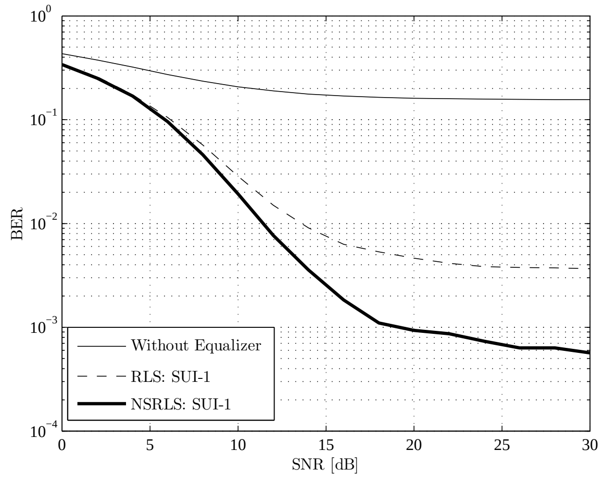

Figure 4: Superiority of the NSRLS-DFE over the RLS-DFE in term of BER (SUI-1 channel, $P = 1$)

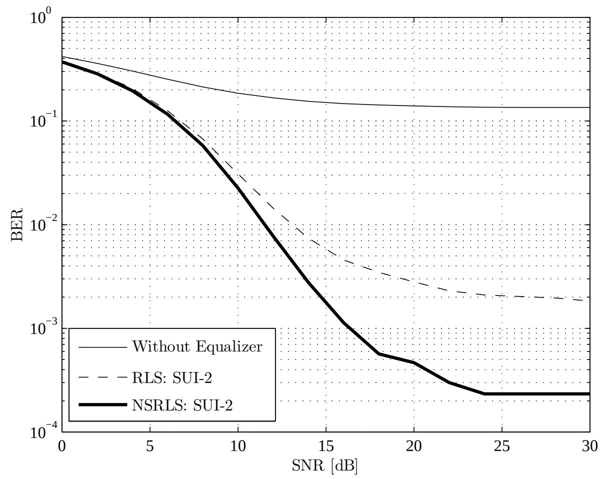

Figure 5: Superiority of the NSRLS-DFE over the RLS-DFE in term of BER (SUI-2 channel, $P = 1$)

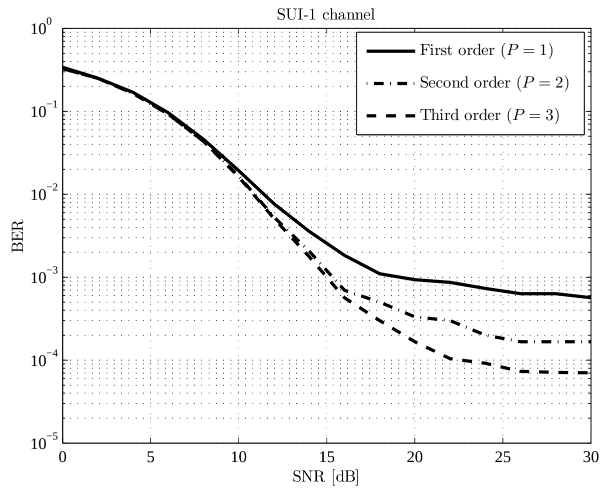

Figure 6: Influence of the Markov order P on the BER (SUI-1 channel case)

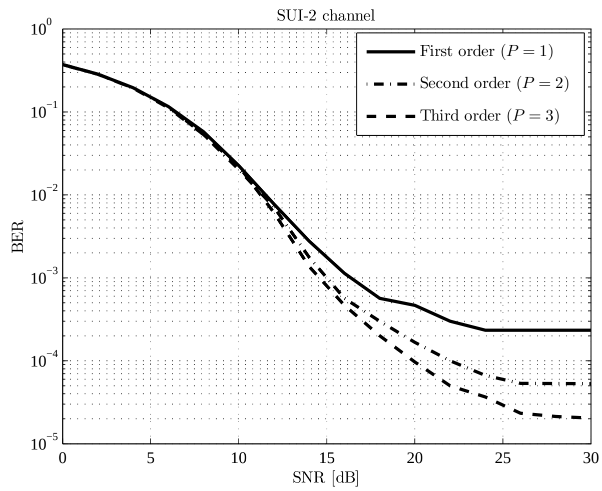

Figure 7: Influence of the Markov order P on the BER (SUI-2 channel case)

4.3 Equalization performances

In order to evaluate the equalization performances, the BER variations are analyzed versus the SNR values for different orders of the Markov model. Note that, for the following results, the forgetting factor is set to its optimal value for each value of P and of the SNR.

At a first stage, a first order NSRLS algorithm is used. In Figures 4 and 5 a comparison in term of BER of the RLS-DFE and the NSRLS-DFE is presented. These figures present the variations of the BER versus the SNR for the considered channels SUI-1 and SUI-2. It is shown that the BER given by NSRLS-DFE is lower than the one performed by the RLS-DFE. Moreover, the BER reduction is more significant with SUI-2 channel than with SUI-1. Such result was expected as the SUI-2 represents a moderate time variations, however, the SUI-1 corresponds to a relatively fast varying channel. In all studied cases, the gain in SNR realized is very important. For example, by setting the BER at 4 10^{-3}, the gain in SNR is equal to 10 dB for SUI-1 channel and 5 dB for SUI-2 channel. Although, in view of the Markovian parameters mean values computed in Subsection 4.2, we expect to enhance the performance of the equalizer by increasing the order P of the Markov model.

At a second stage, the non-stationarity of the channels is modeled by a high order Markov model ($P > 1$). Therefore, a second and a third order NSRLS algorithm were used. Figure 6 and Figure 7 display the BER performance of the equalizer for $P = 1$, $P = 2$ and $P = 3$. These results show that the BER is remarkably reduced by the increase of the Markov order P. Also, these results confirm the ability of the NSRLS algorithm to identify the Markovian parameters. Moreover, they can be helpful for the characterization of the considered SUI channels non-stationarity. Indeed, in view of the obtained results, one can deduce that a Markov model of order higher than three ($P \geq 3$) is suitable for SUI channels.

5. CONCLUSION

In this paper, the generalized NSRLS algorithm designed for a general Markovian time varying channel is proposed and used for the equalization of realistic SUI channel models. It was shown that a NSRLS-DFE exhibits better performance than a conventional RLS-DFE. A significant BER reduction is obtained. Moreover, the BER is all the most reduced as the Markov model order increases. The reported results confirm the efficiency of Markov model to represent the time variation of these channels. In particular, a Markov model of order higher than three is more suitable.

APPENDIX

An estimate of the Markovian parameter is the solution of:

$$\frac{\partial}{\partial \hat{\beta}_i} J_n(\hat{\beta}_i) = 2\sum_{k=1}^{n} \varepsilon_R(k)\frac{\partial \varepsilon_R(k)}{\partial \hat{\beta}_i} + 2\sum_{k=1}^{n} \varepsilon_I(k)\frac{\partial \varepsilon_I(k)}{\partial \hat{\beta}_i}, \\ = 0, \quad (9)$$

where $\varepsilon_R(k) = Re[\varepsilon(k)]$ and $\varepsilon_I(k) = Im[\varepsilon(k)]$. In a blind mode, $\varepsilon(k)$ is described by (3) and then the partial derivation of $\varepsilon_R(k)$ with respect to $\hat{\beta}_i$ is given by:

$$\begin{aligned}\frac{\partial}{\partial \hat{\beta}_i}\varepsilon_R(k) &= \frac{\partial}{\partial \hat{\beta}_i}\left(\frac{\varepsilon(k)+\varepsilon^*(k)}{2}\right), \\ &= \frac{1}{2}\frac{\partial}{\partial \hat{\beta}_i}\Big\{ y(k) - \mathbf{H}^H(k)\hat{\mathbf{X}}(k) - \hat{x}(k) + y^*(k) \\ &\quad - \mathbf{H}^T(k)\hat{\mathbf{X}}^*(k) - \hat{x}^*(k)\Big\}, \\ &= -Re\big[\mathbf{H}^H(k-i)\hat{\mathbf{X}}(k)\big] - \hat{\beta}_i \frac{\partial}{\partial \hat{\beta}_i} Re\big[\mathbf{H}^H(k-i)\hat{\mathbf{X}}(k)\big] \\ &\quad - \frac{\partial}{\partial \hat{\beta}_i} Re\big[\varepsilon^*(k-1)\mathbf{K}^T(k-1)\hat{\mathbf{X}}^*(k)\big] \\ &\quad - \frac{\partial}{\partial \hat{\beta}_i}\sum_{j=1, j\neq i}^{P} Re\big[\mathbf{H}^H(k-j)\hat{\mathbf{X}}(k)\big]. \end{aligned} \quad (10)$$

Based on classical approximation which aim to simplify the gradient, we can write the following:

$$\frac{\partial}{\partial \hat{\beta}_i}\varepsilon_R(k) = -Re\big[\mathbf{H}^H(k-i)\hat{\mathbf{X}}(k)\big]. \quad (11)$$

In the same manner, one can obtain the following:

$$\frac{\partial}{\partial \hat{\beta}_i}\varepsilon_I(k) = -Im\big[\mathbf{H}^H(k-i)\hat{\mathbf{X}}(k)\big]. \quad (12)$$

Finally, in view of (11) and (12) and by replacing $\varepsilon_R(k)$ and $\varepsilon_I(k)$ by their expressions in (9), one obtains,

$$\begin{aligned}\frac{\partial}{\partial \hat{\beta}_i} J_n(\hat{\beta}_i)\Big|_{\hat{\beta}_i = \hat{\beta}_i(n)} &= -2\sum_{k=1}^{n} Re\big[\varepsilon(k)\mathbf{H}^T(k-i)\hat{\mathbf{X}}^*(k)\big], \\ &= -2\sum_{k=1}^{n} Re\Big[\big(y(k) - \hat{x}(k) \\ &\quad - \varepsilon(k-1)\mathbf{K}^H(k-1)\hat{\mathbf{X}}(k) \\ &\quad - \sum_{j=1, j\neq i}^{P} \hat{\beta}_j \mathbf{H}^H(k-j)\hat{\mathbf{X}}(k)\big)\mathbf{H}^T(k-i)\hat{\mathbf{X}}^*(k)\Big] \\ &\quad + 2\hat{\beta}_i \sum_{k=1}^{n} |\mathbf{H}^H(k-i)\hat{\mathbf{X}}(k)|^2. \end{aligned} \quad (13)$$

Thus, it follows from (9) and (13) that

$$\hat{\beta}_i(n) = \frac{num_i(n)}{den_i(n)}, \quad (14)$$

where

$$\begin{aligned} num_i(n) &= num_i(n-1) + Re\Big[\big(y(n) - \hat{x}(n) \\ &\quad - \varepsilon(n-1)\mathbf{K}^H(n-1)\hat{\mathbf{X}}(n) \\ &\quad - \sum_{j=1, j\neq i}^{P} \hat{\beta}_j \mathbf{H}^H(n-j)\hat{\mathbf{X}}(n)\big)\mathbf{H}^T(n-i)\hat{\mathbf{X}}^*(n)\Big], \end{aligned} \quad (15)$$

and

$$den_i(n) = den_i(n-1) + |\mathbf{H}^H(n-i)\hat{\mathbf{X}}(n)|^2. \quad (16)$$

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