DATA-DEPENDENT SUPERIMPOSED TRAINING BASED CHANNEL ESTIMATION AND SYMBOL DETECTION USING ZERO CORRELATION ZONE (ZCZ) SEQUENCES

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ABSTRACT

We address the problem of channel estimation and symbol detection using data-dependent superimposed training (DDST) in the presence of dc-offset for both single-input/single-output (SISO) and multiple-input/multiple-output (MIMO) systems using a set of Zero Correlation Zone (ZCZ) sequences. The use of ZCZ sequences in the proposed method not only results in dc-offset free channel estimation but also ensures that the equalisation output is completely independent of the dc-offset. The proposed approach is shown to perform significantly better than existing methods for any value of dc-offset and hence removes the computational burden of having to estimate the unknown dc-offset.

1. INTRODUCTION

Digital communication systems require an estimate of the channel prior to equalisation. Channel estimation using superimposed training (ST) [1], where the training sequence is added to the data symbols, has recently received considerable attention. But since the training and information are sent at the same time, from the channel estimation point of view, the information interferes with the training and effectively acts as unwanted noise. Later, in [2, 3], a modified ST known as data-dependent ST (DDST) was able to make the information sequence transparent to the training sequence, thus removing the “information noise” and hence significantly improving channel estimation. A disadvantage of these methods is that performance is affected by a possible dc-offset at the receiver, especially for a direct-conversion receiver [7], since the channel estimation is carried out using first-order statistics. If it is not taken into account, the algorithm’s performance could be seriously degraded. In [2, 3] the dc-offset was assumed to be zero. It was pointed out in [1] that the dc-offset can be directly removed if the sum of all the elements in the training sequence is equal to zero. In [4], a set of binary zero correlation zone (ZCZ) training sequences with balanced property was used for the ST method and the harmful dc-offset was completely removed without extra complexity for channel estimation. Although equalisation in the presence of dc-offset was not considered in [4], the dc-offset still poses a problem for equalisation and still needs to be estimated. In [8] channel estimation for ST in the presence of dc-offset was proposed, but it suffered from a high computational burden.

In this paper we extend the use of the ZCZ training sequences for the DDST method and show that the performance of the system is independent of the dc-offset in terms of MSE of the channel estimate as well as BER after equalisation for both SISO and MIMO systems.

Notation: Superscripts $^{-1}$, $\dagger$, $*$ and ^{T} represent inverse, pseudo-inverse, Hermitian and transpose operators respectively. $\mathbf{I}$ denotes the identity matrix. The trace is denoted by Tr$\{.\}$.

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2. DDST SCHEME

Consider a baseband equivalent digital communications system within the DDST scenario, where a periodic training sequence $c(k)$ of length N and period P is added to the information bearing symbols $b(k)$, along with a periodic (period P) data-dependent sequence given by the term $e(k) = -\frac{1}{N_P}\sum_{i=0}^{N_P-1} b(iP+k)$, $k = 0, 1, \ldots, P-1$ and $N_P = \frac{N}{P}$. For an FIR channel $\{h(k)\}_{k=0}^{M-1}$, a cyclic prefix of length $M-1$ is appended to each data block, and we require $P \geq M$ [1] . The transmitted signal (after passing through the channel) is then contaminated by additive, white, Gaussian noise, $n(k)$. So in general,

$$x(k) = \sum_{m=0}^{M-1} h(m)b(k-m) + \sum_{m=0}^{M-1} h(m)e(k-m) + \sum_{m=0}^{M-1} h(m)c(k-m) + n(k) + d \quad (1)$$

where $k = 0,\ 1,\ \ldots,\ N-1$. This can be written in matrix form as

$$\mathbf{x} = \mathbf{Sh} + \mathbf{n} + \mathbf{d} \quad (2)$$

where $\mathbf{S}$ is the $N \times M$ data matrix with $s(k) = b(k) + c(k) + e(k)$ and other matrices are of appropriate dimensions. We will assume that: all terms in (1) can be complex; $b(k)$ and $n(k)$ are from independent, identically distributed (i.i.d.) random, zero-mean processes, with powers σ_b^2 and σ_n^2 respectively; $c(k)$ is known with power $\sigma_c^2 = \frac{1}{P}\sum_{k=0}^{P-1}|c(k)|^2$; and d is an unknown dc-offset [1, 7]. Now as in [1] we can write

$$\hat{y}(j) = \frac{1}{N_P}\sum_{i=0}^{N_P-1} x(iP+j) \tag{3}$$

with $j = 0,\ 1,\ \ldots,\ P-1$, where $\hat{y}(j)$ is an estimate of the periodic (period P) cyclic mean $y(j) \equiv E\{x(iP+j)\}$. So from (2) and (3) we can easily show that

$$\begin{aligned}\hat{y}(j) = &\sum_{m=0}^{M-1} h(m)\tilde{b}(j-m) + \sum_{m=0}^{M-1} h(m)e(j-m) + \\ &+ \sum_{m=0}^{M-1} h(m)c(j-m) + \tilde{n}(j) + d\end{aligned} \tag{4}$$

with $j = 0,\ 1,\ \ldots,\ P-1$, where

$$\tilde{b}(k) = \frac{1}{N_P}\sum_{i=0}^{N_P-1} b(iP+k) \tag{5}$$

with $k = 1-P,\ 2-P,\ \ldots,\ P-1$, and

$$\tilde{n}(j) = \frac{1}{N_P}\sum_{i=0}^{N_P-1} n(iP+j) \tag{6}$$

with $j = 0,\ 1,\ \ldots,\ P-1$. So (4) can now be written as

$$(\mathbf{C} + \tilde{\mathbf{B}} + \mathbf{E})\mathbf{h} = \hat{\mathbf{y}} - \tilde{\mathbf{n}} - \mathbf{d} \tag{7}$$

where $\mathbf{C}$, $\tilde{\mathbf{B}}$ and $\mathbf{E}$ are the leading $(P \times M)$ submatrices of the $(P \times P)$ circulant matrices whose first columns are $[c(0)\ c(1)\ \ldots\ c(P-1)]^T$, $[\tilde{b}(0)\ \tilde{b}(1)\ \ldots\ \tilde{b}(P-1)]^T$ and $[e(0)\ e(1)\ \ldots\ e(P-1)]^T$ respectively. All other vectors in (7) are obvious from (4). It is not difficult to see that if we choose $e(k) = -\tilde{b}(k)_P$, with $k = 0,\ 1,\ \ldots,\ N-1$ and $(\cdot)_P$ implying arithmetic modulo-P, then $\mathbf{E} = -\tilde{\mathbf{B}}$ and so for DDST (7) becomes

$$\mathbf{C}\mathbf{h} = \hat{\mathbf{y}} - \tilde{\mathbf{n}} - \mathbf{d}. \tag{8}$$

3. CHANNEL ESTIMATION AND TRAINING DESIGN

The channel estimation can be carried out at the receiver as in the conventional ST using a time-domain estimator based on the synchronised averaging of the received signal as in [1]. Now the channel estimates can be computed using

$$\hat{\mathbf{h}} = \mathbf{C}^{\dagger}\hat{\mathbf{y}} = \mathbf{h} + \mathbf{C}^{\dagger}\tilde{\mathbf{n}} + \mathbf{C}^{\dagger}\mathbf{d}. \tag{9}$$

Further, if the training sequence has the property such that, the sum of all elements of any column of $\mathbf{C}$ is zero, then we get

$$\mathbf{C}^{\dagger}\mathbf{d} = \mathbf{0}. \tag{10}$$

Using (10) in (9) we obtain

$$\hat{\mathbf{h}} = \mathbf{h} + \mathbf{C}^{\dagger}\tilde{\mathbf{n}}. \tag{11}$$

So we can see from (11) that the channel estimate is completely independent of the dc-offset. Now the ideal training sequence should be balanced, i.e., the sum of all the elements within one period should be equal to zero (i.e., (10)) and should have zero auto-correlation and zero cross-correlation zones (for later use in MIMO systems). One such training sequence ($c_i(n)$), with period P, is called a ZCZ sequence ([4, 5, 6]), which is not only orthogonal to shifts within Z lags but also orthogonal to the other similar training sequences with shifts within Z lags. These kind of training sequences can identify a channel impulse response when the channel length M is less than or equal to $Z+1$. Their autocorrelation and cross-correlation functions have the following properties [4]:

$$\begin{aligned}\phi_{i,j}(m) &= \sum_{n=0}^{P-1} c_i(n)c_j(n+m) \\ &= \begin{cases} P, & \text{if } m = 0, i = j \\ 0, & \text{if } 0 < m \leq Z, i = j \\ 0, & \text{if } 0 < m \leq Z, i \neq j \end{cases}\end{aligned} \tag{12}$$

where $c_i(n)$ and $c_j(n)$ are any two ZCZ sequences. From (12), and for $\mathbf{C}$ defined as in (7), it is not difficult to show that

$$\mathbf{C}^*\mathbf{C} = P\sigma_c^2\mathbf{I}_M. \tag{13}$$

If we define MSE($\hat{\mathbf{h}}$) $:= E\Big\{\sum_{k=0}^{M-1}|\hat{h}(k) - h(k)|^2\Big\}$, then

$$\begin{aligned}\text{MSE}(\hat{\mathbf{h}}) &= E\Big\{\tilde{\mathbf{n}}^*[(\mathbf{C}^*\mathbf{C})^{-1}\mathbf{C}^*]^*(\mathbf{C}^*\mathbf{C})^{-1}\mathbf{C}^*\tilde{\mathbf{n}}\Big\} \\ &= \text{E}\Big\{\tilde{\mathbf{n}}^*\mathbf{C}(\mathbf{C}^*\mathbf{C})^{-2}\mathbf{C}^*\tilde{\mathbf{n}}\Big\} \\ &= \text{Tr}\Big\{\text{E}(\tilde{\mathbf{n}}\tilde{\mathbf{n}}^*)\mathbf{C}(\mathbf{C}^*\mathbf{C})^{-2}\mathbf{C}^*\Big\} \\ &= \frac{\sigma_n^2}{N_P}\text{Tr}\Big\{\mathbf{C}^*\mathbf{C}(\mathbf{C}^*\mathbf{C})^{-2}\Big\}.\end{aligned} \tag{14}$$

Now with (13) we get

$$\text{MSE}(\hat{\mathbf{h}}) = \frac{\sigma_n^2}{N_P}\frac{1}{P\sigma_c^2}\text{Tr}\Big\{\mathbf{I}_M\Big\} = \frac{M\sigma_n^2}{N\sigma_c^2}. \tag{15}$$

Therefore the above MSE shows that the channel estimation is completely independent of dc-offset. Some typical ZCZ sequences with zero correlation zone Z, are listed in Table 1. These are obtained from the construction presented in ([5, 6]). It should be noted that the third property of ZCZ sequences in (12) (i.e. , $\phi_{i,j}(m) = 0$, if $0 < m \leq Z, i \neq j$) can be used in MIMO systems, as will be shown shown later.

Table 1. Balanced binary ZCZ sequences

Zero correlation zone (Z)	Balanced ZCZ sequences
1	-1 -1 -1 -1 1 1 1 1 -1 1 -1 1 1 -1 1 -1
1	-1 -1 1 1 1 1 -1 -1 -1 1 1 -1 1 -1 -1 1 1
2	-1 1 1 -1 -1 1 -1 -1 -1 -1 1 -1 1 1 1 1
2	-1 -1 1 1 -1 -1 -1 1 -1 1 1 1 1 -1 1 -1

4. SYMBOL DETECTION

After the channel is estimated, we can remove the contribution of training and dc-offset by simply removing the cyclic mean from $\mathbf{x}$ in (2). This can be computed as in (3). Removing the cyclic mean has the effect of setting to zero the DFT of $\mathbf{x}$ at all the P pilot frequencies. Once the training and dc-offset have been removed the equalisation can be performed using an MMSE equaliser to obtain the estimate $\hat{\mathbf{x}}$. Since the data was corrupted at the transmitter by the addition of the data dependent sequence $e(n)$ in (1), a symbol-by-symbol detection algorithm to recover the cyclic mean of the data can be initiated by the hard detector of $\hat{\mathbf{x}}$ as in [2]. The next section extends the proposed method to multiple antenna systems.

5. EXTENSION TO MIMO SYSTEMS

In this section we extend the proposed approach to multiple antenna systems. Again we assume that a cyclic prefix is inserted between the data blocks. Let T and R respectively denote the number of transmit and receive antennas. Here we focus on MIMO flat fading channels but it should be noted that that the ZCZ training sequences can also be used to identify a frequency selective channel impulse response when the channel length M is less than or equal to $Z+1$. Let $h_{r,t}$ denote the channel between the tth transmit and rth receive antenna. The block received at the rth antenna after removing the cyclic prefix is given by

$$\mathbf{x}_r = \sum_{t=0}^{T-1} \mathbf{s}_t h_{r,t} + \tilde{\mathbf{n}}_r + \mathbf{d}_r, \;\; r = 1, 2, ..., R. \tag{16}$$

where $\mathbf{s}_t$ is a $N \times 1$ data vector with $s_t(k) = b_t(k) + c_t(k) + e_t(k)$. The $N \times 1$ vectors $\mathbf{d}_r$ and $\tilde{\mathbf{n}}_r$ represent the noise and dc-offset respectively. The estimate of the periodic cyclic mean at the rth receive antenna is given by

$$\hat{\mathbf{y}}_r = \sum_{t=0}^{T-1} (\mathbf{c}_t + \tilde{\mathbf{b}}_t + \mathbf{e}_t) h_{r,t} + \tilde{\mathbf{n}}_r + \mathbf{d}_r \tag{17}$$

where $\mathbf{c}_t$ is the $N \times 1$ vector formed using periodic training sequences with period P as given in Table 1. The $N \times 1$ vectors $\mathbf{b}_t$ and $\mathbf{e}_t$ are formed using $b_t(k)$ and $e_t(k)$ respectively. It should be noted that since we are considering a flat fading channel (i.e., $M = 1$ in (1)), then $\mathbf{c}_t, \tilde{\mathbf{b}}_t, \mathbf{e}_t$ are all vectors of dimension $P \times 1$. Therefore when we set $\mathbf{e}_t = -\tilde{\mathbf{b}}_t$ (to eliminate the interference due to the data) we get

$$\hat{\mathbf{y}}_r = \sum_{t=0}^{T-1} \mathbf{c}_t h_{r,t} + \tilde{\mathbf{n}}_r + \mathbf{d}_r. \tag{18}$$

Now the channel estimate can easily be computed using

$$\hat{h}_{r,t} = \mathbf{c}_t^\dagger \hat{\mathbf{y}}_r. \tag{19}$$

Since the ZCZ training sequence used in each antenna is not only orthogonal to itself within Z lags but also orthogonal to the training sequences in other antennas within Z lags, then from (18) and (19) we get

$$\hat{h}_{r,t} = h_{r,t} + \mathbf{c}_t^\dagger \tilde{\mathbf{n}}_r + \mathbf{c}_t^\dagger \mathbf{d}_r. \tag{20}$$

(It should be noted that another choice for training, but only for the flat fading case, would be Walsh sequences with balanced property. Note, as we have already said, ZCZ sequences can also be used for the frequency dispersive case.) Now again, since the sum of all elements of any column of $\mathbf{C}$ is zero, we get $\mathbf{c}_t^\dagger \mathbf{d}_r = 0$. Therefore the channel estimate is completely independent of the dc-offset and is given by

$$\hat{h}_{r,t} = h_{r,t} + \mathbf{c}_t^\dagger \tilde{\mathbf{n}}_r. \tag{21}$$

Now, if we define $\hat{\mathbf{H}}$ as

$$\hat{\mathbf{H}} = \begin{pmatrix} \hat{h}_{1,1} & \hat{h}_{1,2} & \dots & \hat{h}_{1,T} \\ \hat{h}_{2,1} & \hat{h}_{2,2} & \dots & \hat{h}_{2,T} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{h}_{R,1} & \hat{h}_{R,2} & \dots & \hat{h}_{R,T} \end{pmatrix} \tag{22}$$

then the MSE is given by

$$\text{MSE}(\hat{\mathbf{H}}) := E\Big\{||\hat{\mathbf{H}} - \mathbf{H}||_F\Big\} = \frac{TR\sigma_n^2}{N\sigma_c^2} \tag{23}$$

where $||.||_F$ is the Frobenius norm. After the channel is estimated, we can remove the contribution of training and dc-offset by simply removing the cyclic mean from $\mathbf{x}_r$ as in Section 4. Once the training and dc-offset have been removed the equalisation can be performed using a MMSE equaliser given by $\mathbf{G} = (\hat{\mathbf{H}}^*\hat{\mathbf{H}} + \sigma_n^2 \mathbf{I})^{-1}\hat{\mathbf{H}}^*$. A symbol-by-symbol detection algorithm (as in Section 4) is then used to recover the cyclic mean of the data in order to compensate for the data dependent term $\mathbf{e}_t$ which was added at the transmitter.

6. SIMULATION RESULTS

The results of the simulations are shown in Figures 1 and 2 for three-tap Rayleigh fading channels for SISO systems and in

Figures 3 and 4 for Rayleigh flat fading channels for a MIMO system. The channel coefficients were complex Gaussian, i.i.d. with unit variance. The average energy of the channel was set to unity. The data was a BPSK sequence, to which a ZCZ training sequence $[-111-1-11-1-1-1-11-11111]$ with balanced property satisfying (12) was added before transmission. The training to information power ratio $\left(\text{TIR} = \frac{\sigma_c^2}{\sigma_{b+e}^2}\right)$ was set to -6.9798 dB — we used this value in order to have a proper comparison with previous papers.

For the SISO system $P = 16$ and $N = 420$. Two dc offsets were used: $0.3162, 0.7071$, and a linear MMSE equaliser of length $Q = 11$ taps and optimum delay was used throughout. In order to make a fair comparison, we have included the results of channel estimation and BER using the optimum channel independent (OCI) sequences (as proposed in [1]) with period $P = 4$ and for similar dc offsets.

The Figures 3 and 4 shows the results for Rayleigh flat fading channels for a MIMO system. We illustrate the performance of the proposed system when applied to a two by two spatial multiplexing system, i.e., $T = R = 2$. The block length $N = 200$. Although only the flat fading case is considered here for simplicity, the proposed method can also be used for frequency selective fading. We have also included the results using the method in [3] with period $P = 2$ and two dc-offsets (0.3162 and 0.7071) for a good comparison.

It can be seen from Figures 1 and 3 that the MSE of the channel estimates for the DDST scheme using ZCZ sequences are the same for all the values of dc-offset for both SISO and MIMO systems respectively. They lie exactly on top of each other and are in very good agreement with their respective theoretical curves. On the other hand, it can be observed that the DDST scheme for SISO and MIMO systems using the OCI sequence and the method in [3], suffer heavily from the dc-offset and the performance degrades as the value of the dc-offset increases. The Figures 2 and 4 show the BER performance for the DDST scheme using ZCZ sequences for SISO and MIMO systems respectively along with the curve when the channel is exactly known at the receiver and full power is assigned to the data symbols. Again it can be seen that the BER curves for the DDST scheme using ZCZ sequences are indistinguishable for different values of dc-offsets.

7. CONCLUSION

In this paper, we have presented the data-dependent superimposed training (DDST) scheme using Zero Correlation Zone (ZCZ) sequences with balanced property in order to provide dc-offset free channel estimation and equalisation for both single input single output (SISO) and multiple input multiple output (MIMO) systems. The proposed scheme was found to be completely independent of the dc-offset and therefore proves to be computationally efficient in the presence of dc-offset, unlike the scheme to remove the dc-offset using HOS

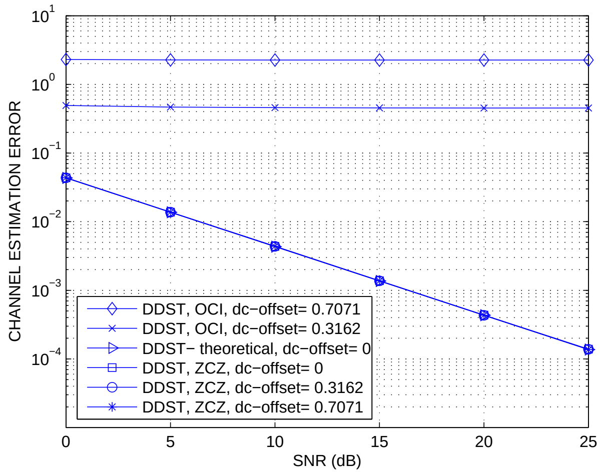

Fig. 1. MSE of channel estimates for different values of dc-offset using ZCZ sequences and OCI sequences (from[1]) for a SISO system. Note that the curves using ZCZ sequences lie on top of each other for any value of dc-offset and hence are indistinguishable from DDST with zero dc-offset.

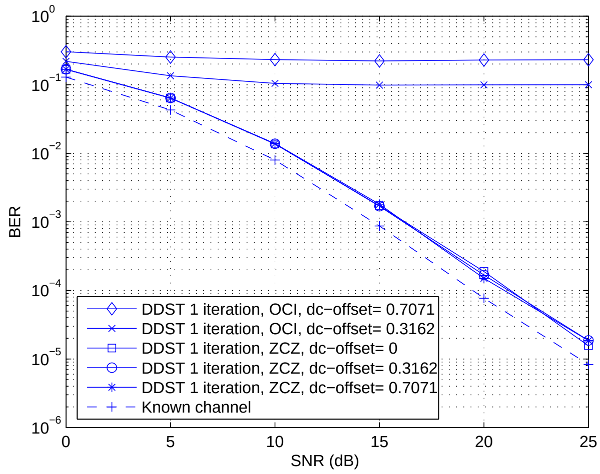

Fig. 2. BER for different values of dc-offset using ZCZ sequences and OCI sequences (from [1]) for a SISO system. Note that the curves using ZCZ sequences lie on top of each other for any value of dc-offset and hence are indistinguishable from DDST with zero dc-offset. (The “1 iteration” comment refers to using only one iteration of the non-linear symbol detection algorithm (described in Section 4) to compensate for the $e(n)$ term.)

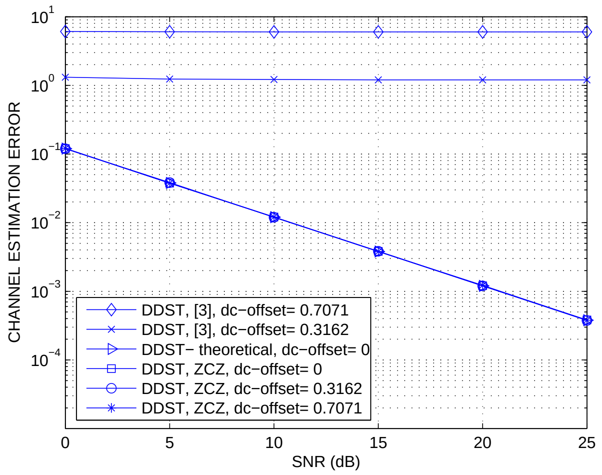

Fig. 3. MSE of channel estimates for different values of dc-offset using ZCZ sequences and training sequences in [3] for a 2×2 MIMO system. Note that the curves using ZCZ sequences lie on top of each other for any value of dc-offset and hence are indistinguishable from DDST with zero dc-offset.

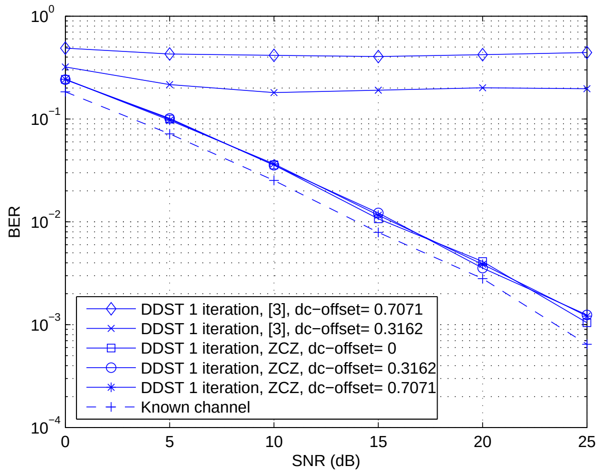

Fig. 4. BER for different values of dc-offset using ZCZ sequences and training sequences in [3] for a 2×2 MIMO system. Note that the curves using ZCZ sequences lie on top of each other for any value of dc-offset and hence are indistinguishable from DDST with zero dc-offset. (The "1 iteration" comment refers to using only one iteration of the non-linear symbol detection algorithm (described in Section 4) to compensate for the $e(n)$ term.)

proposed in [1]. Further, the BER for the proposed scheme is close to the ideal case where the channel estimates and dc-offset are exactly known. Therefore these ZCZ sequences with balanced property are shown to be good candidates for use with the new DDST algorithm.

8. REFERENCES

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