CORRELATION VS. DECORRELATION OF COLOR COMPONENTS IN IMAGE COMPRESSION - WHICH IS PREFERRED?

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ABSTRACT

Most image compression techniques are based on de-correlating the 3 color primaries before applying monochrome encoding to each of the resultant 3 components. Recently, however, a new approach to color image compression based on exploiting the correlations between the color primaries has been introduced. This method, in several cases, outperforms the common de-correlation approach. In this work we analyze and compare the two methods in their optimized cases using a Rate-Distortion framework for subband transform coders. Our goal is to provide theoretical tools to predict which method yields the best performance for color image coding. Our results show that both methods outperform JPEG. However, theoretically and perceptually, the correlation based technique should be preferred. Our conclusions are instrumental in the decision of which color compression method to select in various applications of limited bandwidth communication.

1. INTRODUCTION

The De-correlation Based Approach (DBA) to image coding is widely used for color images. It consists of applying a color components transform (CCT) to the RGB color components to reduce their high inter-color correlations [1], [2], [3], [4] and then coding each color component separately. Algorithms employing this approach are, for example, JPEG [5] and JPEG2000 [6]. This approach has also been presented in [7] in a general framework that allows the optimization of the CCT and quantization stages based on a Rate-Distortion theory for subband transform coders. Another approach, called Correlation Based Approach (CBA), has been presented in a general optimization framework in [8]. This approach utilizes the inter-color correlation of the color components to encode two of the components (called the dependent colors) as a polynomial approximation of the third in each frequency band of the subband transform used. Earlier versions of the approach apply a similar approximation of the dependent colors either in each block [9] or in each region [1] of the image. Considering the two approaches, the question that rises is which method is preferred for color image coding? More specifically, what should be preferred: to search for a color components transform that de-correlates the color primaries and apply a DBA scheme, or on the contrary, attempt to use their inter-color correlations by applying an algorithm based on CBA? In this work we compare the performance of the two approaches to answer these questions.

2. THE DBA AND CBA METHODS

2.1 DBA: De-correlation Based Approach

The DBA approach considers a general color subband transform coder where the image data is pre-processed, subband transformed and quantized and finally post-processed losslessly [10]. These stages are detailed next.

Pre-processing

A CCT is applied to the RGB color components of the image to reduce their inter-color correlations and concentrate most of the image energy in one or two of the components. We denote the RGB components in vector form as $\mathbf{x} = [R\ G\ B]^T$ and the new color components as $\widetilde{\mathbf{x}} = [C_1\ C_2\ C_3]^T$. Also the 3×3 size CCT matrix is denoted by $\mathbf{M}$. Then this stage means

$$\widetilde{\mathbf{x}} = \mathbf{M}\mathbf{x}. \tag{1}$$

Subband transforming and quantizing

A subband transform, e.g. DCT (Discrete Cosine Transform [11]) or DWT (Discrete Wavelet Transform), is applied to each color component. Then the transform coefficients of each color component are quantized using an independent uniform scalar quantizer for each subband.

Post-processing

The quantized coefficients are losslessly coded to reduce the bits budget they require. Techniques such as run-length coding, zero trees and entropy coding can be used here.

The Rate-Distortion model of this algorithm is [7]:

$$d\left(\{R_{bi}\},\mathbf{M}\right) = \frac{1}{3}\sum_{i=1}^{3}\sum_{b=0}^{B-1}\eta_b G_b \widetilde{\sigma}_{bi}^2 \varepsilon_i^2 e^{-aR_{bi}}\left((\mathbf{M}\mathbf{M}^T)^{-1}\right)_{ii}. \tag{2}$$

Here d is the average MSE distortion of the image in the RGB domain, $\widetilde{\sigma}_{bi}^2$ is the variance of subband b ($b \in \{0,1,...,B-1\}$) of color component C_i, G_b is its energy gain [12] and R_{bi} is the rate allocated to it. Also η_b is its sample rate, i.e., the relative part of the number of coefficients in it from the total number of samples in the signal. Furthermore ε_i^2 is a constant dependent upon the distribution of color component i and a is a constant equal to $2ln2$.

Optimal rates allocation for the subbands can be found by minimizing the expression of (2) under the rate constraint:

$$\sum_{i=1}^{3}\alpha_i\sum_{b=0}^{B-1}\eta_b R_{bi} = R \tag{3}$$

for a given total image rate R. Here down-sampling factors α_i are used. α_i denotes the factor by which the number of samples of color component i has been reduced due to down-sampling. E.g., if the down-sampling is by a factor of 2 horizontally and vertically then:

$$\alpha_i = \begin{cases} 1 & \text{full component} \\ 0.25 & \text{down-sampled component.} \end{cases}$$

The optimal rates under the rate constraint of (3) as well as non-negativity constraints are [13]:

$$R_{bi} = \frac{R}{\sum_{j=1}^{3} \alpha_j \xi_j} + \frac{1}{a} ln \left(\frac{\frac{\varepsilon_i^2 G_b \widetilde{\sigma}_{bi}^2 \left((\mathbf{MM}^T)^{-1}\right)_{ii}}{\alpha_i}}{\prod_{k=1}^{3} \left(\frac{\left((\mathbf{MM}^T)^{-1}\right)_{kk} \varepsilon_k^2 GM_k^{Act}}{\alpha_k} \right)^{\frac{\alpha_k \xi_k}{\sum_{j=1}^{3} \alpha_j \xi_j}}} \right) \quad (4)$$

for $b \in Act_i$. Act_i denotes here the set of non zero (or active) rates in the color component i, i.e.,

$$Act_i \triangleq \{ b \in [0, B-1] \mid R_{bi} > 0 \}. \quad (5)$$

Also

$$\xi_i \triangleq \sum_{b \in Act_i} \eta_b, \;\; GM_i^{Act} \triangleq \prod_{b \in Act_i} (G_b \widetilde{\sigma}_{bi}^2)^{\frac{\eta_b}{\xi_i}}. \quad (6)$$

Algorithms for finding the active subbands sets Act_i and deriving the optimal quantization steps can be found in [7]. Substituting (4) into (2) yields the minimal MSE value for DBA. For simplicity, we neglect the influence of the non active subbands and assume that the ξ_i terms are constant. Then this value can be shown to depend only on the term $\prod_{k=1}^{3} \left(\frac{\varepsilon_k^2 \left((\mathbf{MM}^T)^{-1}\right)_{kk}}{\alpha_k} \right)^{\alpha_k \xi_k} \prod_{b \in Act_i} \left(G_b \prod_{k=1}^{3} (\widetilde{\sigma}_{bk}^2)^{\alpha_k} \right)^{\eta_b}$. Thus, for a constant CCT matrix, the smaller the product

$$f(\{\widetilde{\sigma}_{bk}^2\}) = \prod_{k=1}^{3} (\widetilde{\sigma}_{bk}^2)^{\alpha_k} \quad (7)$$

in each subband, the smaller the MSE.

2.2 CBA: Correlation Based Approach

In the CBA, a general subband transform coder is also considered. The algorithm can be divided into stages of pre-processing, coding and post-processing, however, the coding stage is different [8]. A description of the algorithm follows.

- In the pre-processing stage a CCT is applied to the color primaries with the goal of providing better performance in the successive coding stage. Since in the coding stage (discussed next) high inter-color correlations are preferred, it may be argued that the CCT should raise these correlations. However, raising the correlations is usually associated with increasing the condition number of the CCT (it becomes closer to singular) resulting in greater amplification of the coding errors when the inverse CCT is applied. Thus the optimal CCT has to find a compromise between these two properties. Of course, this stage can be described mathematically using (1) as for DBA.
- In the coding stage one of the new color components (C_1, C_2 or C_3) is chosen to be the base color and the other two (the dependent colors) are approximated as a first order polynomial function of the base. Prior to the approximation, a subband transform is applied to each of the colors so that the polynomial coefficients estimation is performed in each subband separately. Mathematically, if we assume that we have B subbands and denote the subband b ($b \in \{0, 1, ..., B-1\}$) coefficients of C_i by $y_b^{C_i}$, the approximation in each subband is according to:

$$\begin{aligned} \hat{y}_b^{C_2} &= \tau_{b1} \cdot y_b^{C_1} + \tau_{b0} \\ \hat{y}_b^{C_3} &= \beta_{b1} \cdot y_b^{C_1} + \beta_{b0}. \end{aligned} \quad (8)$$

Here we assume without loss of generality that C1 is the base and C2 and C3 are the dependent colors. $\hat{y}_b^{C_2}$ denotes the approximated coefficients of subband b of C2, and similarly for $\hat{y}_b^{C_3}$. The terms τ_{b1}, τ_{b0} and β_{b1}, β_{b0} are the expansion coefficients for C2 and C3, respectively. They are calculated according to the least squares (LS) method, i.e., so that the expression

$$E\left[\left(y_b^{C_2} - \hat{y}_b^{C_2}\right)^2\right] = E\left[\left(y_b^{C_2} - \tau_{b1} \cdot y_b^{C_1} - \tau_{b0}\right)^2\right] \quad (9)$$

is minimized for C2. Note that $E()$ denotes here statistic mean. A similar expression is minimized for C3, with $y_b^{C_3}$ replacing $y_b^{C_2}$ and β replacing τ. In this work we consider a CBA version in which only the following first order coefficients τ_{b1} and β_{b1} are sent:

$$\tau_{b1} = \frac{cov\left(y_b^{C_1}, y_b^{C_2}\right)}{var(y_b^{C_1})} \quad \text{and} \quad \beta_{b1} = \frac{cov\left(y_b^{C_1}, y_b^{C_3}\right)}{var(y_b^{C_1})}. \quad (10)$$

Prior to sending the coefficients, they are quantized. After calculating the expansion coefficients (quantized and reconstructed) the approximation errors are calculated and coded. In [8] the base color was not coded. However, it can be coded using any monochromatic compression technique. In this work we extend the CBA algorithm to perform full image coding as described in Subsection 2.3. Optimal rates are determined for the three color components and are used to derive the optimal quantization steps, passed to the quantizers. An independent uniform quantizer is employed in each subband.
- The post-processing stage once again employs lossless techniques to reduce the bit budget required for the quantized subband coefficients by utilizing the intra-subband and inter-subband correlations.

2.3 Extension of CBA to full image coding

The CBA algorithm presented in [8] can be extended to full image coding (including the base) by applying a monochromatic subband transform coder to the base color. Thus, the base color and the approximation errors of the subbands of the dependent colors are compressed together and the total bit budget has to be split between them. We use the Rate-Distortion model of (2) to find the optimal subband rates. This is done by substituting the variances of the base color and the approximation errors in each subband for $\widetilde{\sigma}_{bi}^2$ in (4).

Note that the reconstructed base color should be used for the approximation of the dependent colors to achieve best performance. However, to derive the optimal rates we need the variances of the approximation errors prior to coding the base color. Thus the calculation of the expansion coefficients should be done both prior to the calculation of the optimal rates and after the coding and reconstruction of the base color component. In practice, the performance does not change significantly if the second calculation stage is not carried out. Once the optimal rates are determined, the corresponding quantization steps are derived using the iterative algorithm in [7]. Using (10), the approximation error variances of C_i ($i \in \{2,3\}$) in subband b can be shown to be $\left(\sigma^e_{bi}\right)^2 = \widetilde{\sigma}^2_{bi}(1-\rho^2_{b1i})$, where $\widetilde{\sigma}^2_{bi}$ stands for the variance of C_i and ρ_{b1i} is its correlation with the base color C_1 (in subband b). Clearly, $\left(\sigma^e_{bi}\right)^2 < \widetilde{\sigma}^2_{bi}$ and referring to (7), $f\left(\widetilde{\sigma}^2_{b1}, \left(\sigma^e_{b2}\right)^2, \left(\sigma^e_{b3}\right)^2\right) < f\left(\widetilde{\sigma}^2_{b1}, \widetilde{\sigma}^2_{b2}, \widetilde{\sigma}^2_{b3}\right)$ in each subband. Thus for the same CCT, the CBA is expected to achieve a smaller MSE than the DBA.

3. COMPARISON OF THE TWO METHODS

In this section we compare the DBA and the CBA methods as well as the common JPEG algorithm (see below). In order to do so we use the extended CBA algorithm that includes compression of all the color components according to Subsection 2.3. For simplicity, we consider the CBA algorithm in the same color space as DBA - that of the DCT [14]. When no down-sampling is used in either of the algorithms, the results for a group of images are shown in Table 1. We use the Laplacian probability model of the DCT subband transform coefficients [15] in all the algorithms for the estimation of subband rates [13]. The images are those presented in Fig. 1. The PSPNR (Peak Signal to Perceptual Noise Ratio) used is as defined in [7] and [8]. As can be seen from the table, the performance of the CBA algorithm is better than that of the DBA both in terms of PSNR and PSPNR. The performance gain is 0.21dB for both measures. However, as we will see below, the CBA has higher complexity than DBA and sometimes it may be more efficient in terms of run-time to use DBA despite this performance loss.

In Tables 2 and 3 we compare the DBA, the CBA and JPEG with down-sampling for the same images of Fig. 1. When comparing the results here to Table 1, the performance gains due to down-sampling of the two algorithms can be observed.

Image	PSNR [dB]		PSPNR [dB]		Rate [bpp]
	DBA	CBA	DBA	CBA	
Peppers	29.65	29.94	38.24	38.41	0.912
Lena	29.56	30.00	38.15	38.74	0.620
Goldhill	29.86	29.97	40.54	40.62	1.839
Jelly Beans	29.66	29.87	37.31	37.49	0.415
Tree	29.71	30.00	39.44	39.75	1.526
WaterFall	29.75	29.79	39.53	39.52	1.613
Fruit	30.04	30.12	39.04	39.19	0.562
Mean	**29.75**	**29.96**	**38.89**	**39.10**	

Table 1: PSNR (Peak Signal to Noise Ratio) and PSPNR results for the DBA and the CBA algorithms at the same compression rate. Both algorithms do not use down-sampling.

Figure 1: The test images.

The PSNR performance of the DBA algorithm increases by 0.53dB and the PSNR of the CBA by 0.45dB while the PSPNR increases by 0.69dB and 0.58dB, respectively. It is interesting that the performance of the DBA algorithm becomes closer to that of the CBA although the CBA is still slightly better on average. Also JPEG is outperformed by the other approaches by 1.9-2.0dB PSNR and 1.8-1.9dB PSPNR.

Image	PSNR [dB]			Rate [bpp]
	DBA DS	CBA DS	JPEG DS	
Peppers	29.98	30.26	28.15	0.912
Lena	30.09	30.53	29.03	0.620
Goldhill	30.20	30.22	28.05	1.839
Jelly Beans	30.68	30.69	29.22	0.415
Tree	29.87	30.15	28.64	1.526
WaterFall	30.29	30.26	25.99	1.613
Fruit	30.83	30.77	29.46	0.562
Mean	**30.28**	**30.41**	**28.36**	

Table 2: PSNR results for the DBA and the CBA algorithms and JPEG at the same compression rate. All the algorithms use down-sampling (DS).

3.1 Visual Results

A visual comparison of the CBA and the DBA algorithms with down-sampling and JPEG is given in Figs. 2 and 3 for two images: Tree and Peppers. As can be seen for the Tree image the CBA algorithm is superior to DBA. All the algorithms introduce color artifacts, for example, in the regions marked with a frame, but the CBA result is more pleasing to the eyes. A similar result is achieved for the Peppers image - again the CBA has best performance, especially in the marked region.

3.2 Run-time comparison

The average run times of the CBA and the DBA algorithms as well as JPEG (in Matlab 7.0 environment) are given in Table 4 for images of size 128×128, 256×256 and 512×512. All the algorithms considered here use down-sampling. It can be concluded from the table that while JPEG has the shortest run-times for all the image sizes considered and the

Image	PSPNR [dB]			Rate [bpp]
	DBA DS	CBA DS	JPEG DS	
Peppers	38.30	38.54	36.32	0.912
Lena	38.83	39.28	36.92	0.620
Goldhill	41.14	41.14	40.07	1.839
Jelly Beans	38.43	38.46	36.78	0.415
Tree	39.43	39.82	37.86	1.526
WaterFall	40.20	40.10	38.87	1.613
Fruit	40.75	40.42	37.54	0.562
Mean	**39.58**	**39.68**	**37.77**	

Table 3: PSPNR results for the DBA and the CBA algorithms and JPEG at the same compression rate. All the algorithms use down-sampling (DS).

	DBA DS	CBA DS	JPEG DS
128×128	0.60	1.20	0.50
256×256	2.05	4.69	1.83
512×512	7.39	17.52	6.93

Table 4: Average run times in sec. for the DBA and the CBA algorithms as well as JPEG for images of different sizes.

DBA is not far behind (10%-30% longer run-time relative to JPEG), the CBA algorithm turns out to be the slowest (about 2.5 times the run-time of JPEG).

4. SUMMARY

In this work we have analyzed the two general approaches to color image coding using subband transforms. The difference between the two methods is in the way the high inter-color correlations of the RGB color components are utilized. The De-correlation Based Approach uses the high correlations to achieve energy compactness in another de-correlated color space, such as YCbCr. A potential alternative to this method is the Correlation Based Approach, which exploits the high correlations by approximating two of the color components as functions of the third. For improved performance it also transforms the RGB primaries into another color space, where the approximation is performed.

Our analysis considers theoretical comparison, quantitative performance of the algorithms, visual quality of the compressed images and run-time as a measure of complexity. According to the Rate-Distortion theory used, the CBA is expected to be superior to the DBA for the same CCT. We have shown that without down-sampling this is indeed so with respect to image quality. Furthermore, when down-sampling is employed, the situation remains similar, although the performance gap narrows. Both algorithms have been shown to be superior to JPEG. As for run-time or complexity, it turns out that the CBA has about twice longer run-times than the DBA which is rather close in performance to JPEG.

Our conclusion that in terms of performance it may be more efficient to exploit the high inter-color correlations of the color primaries (as in the CBA) instead of attempting to de-correlate them as done in the DBA. The main drawback of the CBA method is its higher complexity and run-time. Thus, only when the algorithm complexity is of highest priority, the DBA algorithm may be preferred.

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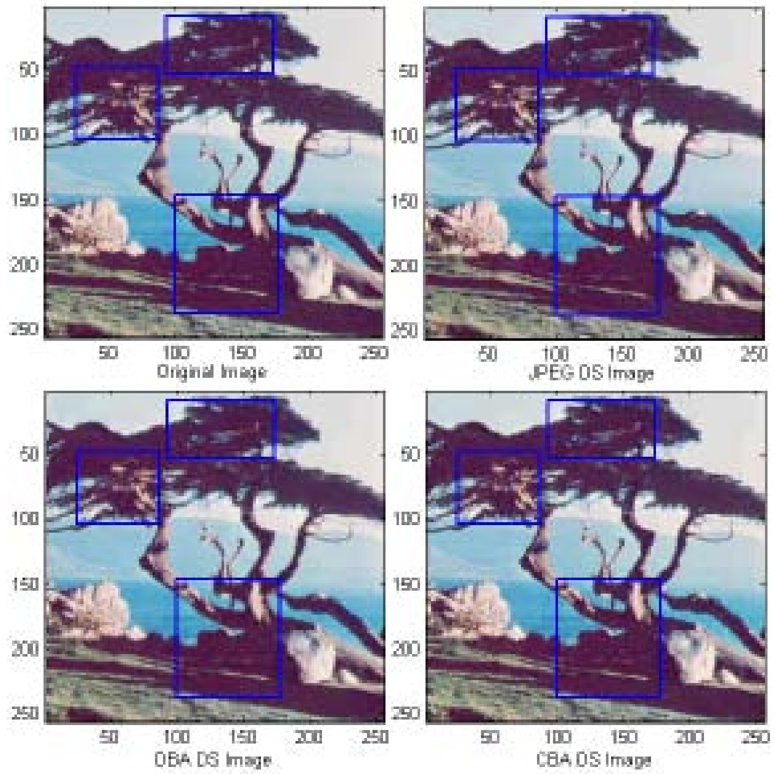

Figure 2: Tree at 0.839 bpp: original (top left), compressed by JPEG with DS (top right, PSNR=26.65dB), compressed by DBA with DS (bottom left, PSNR=27.50dB) and compressed by CBA with DS (bottom right, PSNR=27.71dB). PSPNR = 35.21dB (JPEG), 36.76dB (DBA), 37.19dB (CBA).

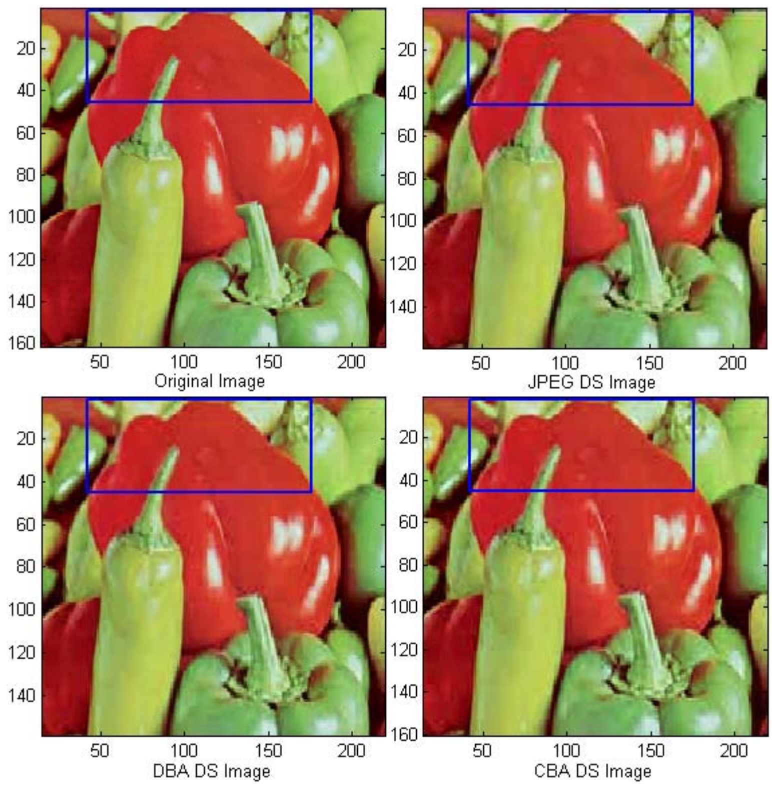

Figure 3: Peppers (zoomed in) at 1.021 bpp: original (top left), compressed by JPEG with DS (top right, PSNR=28.52dB), compressed by DBA with DS (bottom left, PSNR=30.51dB) and compressed by CBA with DS (bottom right, PSNR=30.76dB). PSPNR = 36.92dB (JPEG), 38.89dB (DBA) and 39.12dB (CBA).