

# A COMBINED APPROACH FOR NLOS MITIGATION IN CELLULAR POSITIONING WITH TOA MEASUREMENTS

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## ABSTRACT

In this paper, we propose a method to mitigate the effects of non line of sight errors in range-based localization of mobile phones. The TOA/range measurement errors are dynamically tracked with a Kalman filter, which are then combined through a constrained weighted least squares (CWLS) algorithm to locate a mobile station with respect to three or more base stations. NLOS situations, which cause a large error in TOA readings, are initially detected and handled with a highly biased Kalman filter. The use of CWLS further mitigates NLOS errors. Simulation results clearly demonstrate the improvement offered by this combined approach.

## Introduction

Mobile station (MS) positioning has gained considerable attention with the evolution of new wireless technologies and services. The location of a user is a valuable information which can be used for mobile advertising, asset tracking, fleet management, location-based wireless access security and location sensitive billing, etc., [1]. In addition, in 1999, FCC mandated the wireless operators to provide the location information of the emergency calls in United States. According to FCC regulations, the accuracy requirement is 100 meters for at least 67 per cent of all calls, and 300 meters for at least 95 per cent of all calls[2].

For location estimation, various methods can be employed. These methods can be based on measurements of angle of arrival (AOA), time of arrival (TOA), time difference of arrival (TDOA), cell global identification (CGI), timing advance (TA), received signal strength (RSS) or a hybrid method that is composed of a combination of these measurements [3]. This work focuses on range-based positioning, which requires TOA measurements of a mobile station (MS) from at least three base stations (BS).

In this paper, a novel location estimation scheme for NLOS mitigation in TOA-based positioning is proposed, as summarized in figure 1. Often, TOA measurements deflect severely from the desired range value due to the the lack of a line-of-sight (LOS) propagation path between a BS and a MS. This NLOS condition is addressed here in two steps. In the first step, time series of range measurements for each BS to MS path is treated individually. At each time step, the current TOA reading is classified as LOS or NLOS based recent TOA values. Depending on the decision, the current range reading is corrected through a different Kalman filter on that time series. In the second step, the filtered time series of TOA for several BSs are combined through constrained weighted least squares to estimate the location of the MS.

The rest of the paper is organized as follows: In section 1 the LOS/NLOS identification technique will be explained; in

section 2 the Kalman filter and biasing Kalman filter will be described; in section 3 Constrained Weighted Least Squares (CWLS) algorithm will be mentioned, the simulation results and the simulation environment are shown in section 4.

## 1. LOS/NLOS IDENTIFICATION

At each time step  $k$  the current sample of the corresponding TOA time series  $y_m(k)$ , for a particular BS-MS combination, is identified as LOS or NLOS as follows:

Step 1: Average of recent TOA data is calculated as in eq. 1 with a sliding window with a length of 20 steps (4 seconds).

$$\bar{y}_m(k) = \frac{1}{M} \sum_{j=k-M+1}^k y_m(j) \quad (1)$$

Step 2: Standard deviation for range measurements is calculated as in eq. 2

$$\hat{\sigma}_m(k) = \sqrt{\frac{1}{M} \sum_{j=k-M+1}^k (y_m(j) - \bar{y}_m(k))^2} \quad (2)$$

where:

$M$  : is the window size.

$y_m(j)$  : is the range measurement at time sample  $j$  for  $m^{th}$  base station.

$\bar{y}_m(k)$  : is the mean of range measurements inside the window (from time sample  $k - M + 1$  to  $k$ ) which is calculated and used at time sample  $k$  (for  $m^{th}$  base station).

Step 3: Calculated standard deviation is compared to a threshold as follows

$$\begin{aligned} \text{if } \hat{\sigma}_m(k) \geq \gamma \sigma_m \text{ then decision is NLOS} \\ \text{if } \hat{\sigma}_m(k) < \gamma \sigma_m \text{ then decision is LOS} \end{aligned} \quad (3)$$

where:

$\sigma_m$  : is the standard deviation of the LOS measurements, which is known.

$\gamma$  : is a coefficient used to reduce false alarms due to small changes in variance at LOS condition.

This is based on the fact that under the LOS condition, typical measurement error standard deviation for a particular BS to MS TOA time series is known to correspond to  $150m$ .

The choice of  $\gamma$  parameter in this paper is based on figure 2. Figure 2 shows a Receiver Operating Characteristic(ROC) curve which assigns the probability of detection for each probability of false alarm. Operation at a particular probability of false alarm can be obtained by a choice of threshold

$\gamma$ . In the case of this paper,  $\gamma$  is chosen to be 1.35 since this allows a very high detection rate (97.88%), for a relatively low probability of false alarm (13.15%). It is judged that, correct identification of NLOS situation is more critical than incorrect classification of LOS as NLOS (ie. false alarm).

## 2. KALMAN FILTER AND BIASED KALMAN FILTER

The Kalman Filter, which is mostly used as a path tracking tool in discrete time signals, is used here in tracking merely range variations for a single BS-MS combination as in [4]. It uses the recent data and the range variation model to estimate the current range a priori, and then it corrects it according to the most recent measurement. The Kalman filter is known to work well even if the range variation model is not exactly known. [4]

### 2.1 Model Used in Kalman Filter

The mobile station motion model for this approach is defined as:

$$\begin{pmatrix} r_{k+1} \\ v_{k+1} \end{pmatrix} = \begin{pmatrix} 1 & \Delta t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} r_k \\ v_k \end{pmatrix} + \begin{pmatrix} 0 \\ \Delta t \end{pmatrix} w_k \quad (4)$$

$$\mathbf{x}_{k+1} = \Phi \mathbf{x}_k + \Gamma \mathbf{w}_k$$

where:

$r_k$  : is the range value at time sample  $k$ .

$v_k$  : is the speed of the range values at time sample  $k$ .

$\Delta t$  : is the sampling period.

$w_k$  : is the random change in speed from time sample  $k$  to  $k+1$ .

$w_k$  is used to compensate the effect of a random velocity change as well as the effect of a random route change. Here  $Q$  is defined as the covariance matrix of  $w_k$ .

And the measurement model is:

$$y_k = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} r_k \\ v_k \end{pmatrix} + u_k \quad (5)$$

$$y_k = \mathbf{M} \mathbf{x}_k + u_k$$

where:

$y_k$  : is the measured range value

$\mathbf{M}$  : is the observation matrix

$u_k$  : is the measurement error whose covariance matrix is  $R$ .

The iterative operations are as follows:

$$\hat{\mathbf{x}}_{k+1}^- = \Phi \hat{\mathbf{x}}_k \quad (6)$$

$$\mathbf{P}_{k+1}^- = \Phi \mathbf{P}_k \Phi^T + \Gamma \mathbf{Q} \Gamma^T \quad (7)$$

$$\mathbf{K}_{k+1} = \mathbf{P}_{k+1}^- \mathbf{M}^T (\mathbf{M} \mathbf{P}_{k+1}^- \mathbf{M}^T + R)^{-1} \quad (8)$$

$$\mathbf{P}_{k+1} = \mathbf{P}_{k+1}^- - \mathbf{K} \mathbf{M} \mathbf{P}_{k+1}^- \quad (9)$$

$$\hat{\mathbf{x}}_{k+1} = \hat{\mathbf{x}}_{k+1}^- + \mathbf{K} (y_{k+1} - \mathbf{M} \Phi \hat{\mathbf{x}}_{k+1}^-) \quad (10)$$

### 2.2 Biasing the Kalman Filter

The NLOS condition causes very high errors in range measurements. When the TOA value is identified as NLOS, it is directed to a highly biased Kalman filter. This filter is biased in the sense that range error variance in the covariance matrix  $Q$  is assumed to be very high compared to that of the LOS case. This, in turn decreases the dependence of the output to the NLOS measurement. Here the filter's output follows previous outputs more closely.

The error covariance matrix of the Kalman filter is multiplied by a so called bias parameter, which is chosen here experimentally based on the simulation whose results are shown in figure 3. The idea in this simulation is to minimize the RMSE of the TOA time series at the output of the filter by carefully selecting the best bias parameter.

### 3. CONSTRAINED WEIGHTED LEAST SQUARES

Given the current values of the Kalman filtered range sequence of three or more BS's, the current location of the MS can be estimated via a series of equations given in 11 through CWLS. The CWLS algorithm reorganizes the non-linear hyperbolic equations into a set of linear equations by introducing an intermediate variable,  $R^2$  [5]. Then the relation of  $R^2$  with the source position is imposed a constraint and the resulting constrained least square function is minimized by employing Lagrange multipliers.

The range measurements with a measurement error are modeled as [5]

$$r_i = d_i + n_i = [(x_1 - X_{i1})^2 + (x_2 - X_{i2})^2]^{1/2} + n_i \quad (11)$$

where  $i = 1, 2, 3, \dots, M$

$n_i$  : is the measurement error in the corresponding range measurement  $r_i$ .

$X_{i1}$  : is the x coordinate of the  $i^{th}$  base station.

$X_{i2}$  : is the y coordinate of the  $i^{th}$  base station.

$x_1$  : is the x coordinate of the estimated mobile station.

$x_2$  : is the y coordinate of the estimated mobile station.

$d_i$  : is the distance between the  $i^{th}$  base station and the estimated mobile station position.

If the measurement error is omitted, the following equation can be obtained by squaring both sides, as

$$r_i^2 = R^2 - 2x_1 X_{i1} - 2x_2 X_{i2} + (X_{i1}^2 + X_{i2}^2)$$

$$\implies x_1 X_{i1} + x_2 X_{i2} - 0.5R^2 = (1/2) \cdot [(X_{i1}^2 + X_{i2}^2) - r_i^2]$$

where  $R^2 = (x_1^2 + x_2^2)$  is the intermediate variable.

These equations can be represented in matrix notation as:

$$\mathbf{A} \Theta = \mathbf{b} \quad (12)$$

where:

$$\mathbf{A} = \begin{pmatrix} X_{11} & X_{12} & -0.5 \\ \vdots & \vdots & \vdots \\ X_{M1} & X_{M2} & -0.5 \end{pmatrix} \quad \Theta = \begin{pmatrix} x_1 \\ x_2 \\ R^2 \end{pmatrix}$$

$$\mathbf{b} = \frac{1}{2} \begin{pmatrix} X_{11}^2 + X_{12}^2 - r_1^2 \\ \vdots \\ X_{M1}^2 + X_{M2}^2 - r_M^2 \end{pmatrix}$$

This equation is solved indirectly by finding a least squares fit to  $\mathbf{A}\Theta - \mathbf{b}$  subject to the constraint  $R^2 = x_1^2 + x_2^2$ , which is expressed in terms of  $\Theta$ , as

$$\mathbf{q}^T \Theta + \Theta^T \mathbf{P} \Theta = 0 \quad (13)$$

where:

$$\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \mathbf{q} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

In the model of eq.s of 11, for small values of the measurement noise, its effects can be approximated as:

$$r_i^2 = (d_i + n_i)^2 \approx d_i^2 + 2d_i n_i \quad (14)$$

As a result the difference between the squares of the measured and the true distances, which corresponds to  $[\mathbf{A}\Theta - \mathbf{b}]_i$ , is approximated by

$$\varepsilon_i = r_i^2 - d_i^2 = 2d_i n_i \quad (15)$$

where  $\varepsilon_i$  value is defined as the disturbance.

Therefore the true values of the distance and the noise are not known, the covariance matrix of the disturbance  $\varepsilon_i$  can be approximated by

$$E [(\varepsilon_1 \varepsilon_2 \dots \varepsilon_M)(\varepsilon_1 \varepsilon_2 \dots \varepsilon_M)^T] = \mathbf{BQB} \quad (16)$$

where

$$\mathbf{B} = \begin{pmatrix} 2r_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 2r_M \end{pmatrix} \quad \text{and} \quad \mathbf{Q} = \begin{pmatrix} 2\sigma_1^2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 2\sigma_M^2 \end{pmatrix}$$

If each range error variance  $\sigma_i^2$  is assumed known, the disturbances can be weighted using a weighting matrix  $\mathbf{W} = (\mathbf{BQB})^{-1}$ . This turns the solution of eq. 12 into the following optimization problem:

$$\hat{\Theta} = \arg \min_{\Theta} (\mathbf{A}\Theta - \mathbf{b})^T \mathbf{W} (\mathbf{A}\Theta - \mathbf{b}) \quad (17)$$

subject to the constraint of eq. 13.

This turns to be a Lagrange multiplier problem whose solution requires finding the roots of a 5<sup>th</sup> order polynomial [5],[6]. The roots of this polynomial are found using the 'roots' function of MATLAB. The root which minimizes the objective function  $(\mathbf{A}\Theta - \mathbf{b})^T \mathbf{W} (\mathbf{A}\Theta - \mathbf{b})$ , gives the global solution of the CWLS.

Although in [5] CWLS is proposed for LOS cases, it is used here after the mitigation of NLOS error. CWLS provides us the ability to weight each range measurement according to its variance. In addition, here each range error variance is estimated before NLOS mitigation.

## 4. SIMULATION ENVIRONMENT AND RESULTS

### 4.1 Simulation Environment

In these simulations, the base stations are located at the points  $BS1 = [4\sqrt{3}, 12]$  km;  $BS2 = [4\sqrt{3}, 20]$  km;  $BS3 = [8\sqrt{3}, 16]$  km;  $BS4 = [8\sqrt{3}, 8]$  km;  $BS5 = [4\sqrt{3}, 4]$  km. The MS speed is 30 m/s. The TOA sampling period is 0.2 seconds.

In the first scenario, the MS is assumed to be moving along a straight line beginning from the point  $(x_1, x_2) = (1.7, 8)$  km. The MS is moving within the coverage area of one BS.

In the second scenario, the MS is again moving within the coverage area of one BS, though its motion is no longer linear but piecewise linear. The MS changes its direction randomly at undetermined times.

A LOS error, which is assumed to be additive white Gaussian noise with zero mean and  $\sigma_{BS_i}^2 = 150^2$ , is added to the true BS-MS ranges in both scenarios to obtain range measurements. An additional NLOS error, when it exists, is assumed to have uniform distribution over the interval (0 – 1000m) [4] and to persist for 250 m over each BS-MS range sequence. Several sub-scenarios, which assume a predetermined number of BS's at NLOS condition to the MS are considered for both scenario 1 & 2.

## 4.2 Simulation Results

The simulation results for first scenario is shown on table 1 and figure 5. It can be seen that the Kalman filter reduces the error significantly. Also the use of constrained weighted least squares provides a better performance than that of a simple least squares method. Table 1 shows the accuracy at 50 percent, 67 percent and 95 percent of all the position estimates for different LOS/NLOS conditions. The 1999 FCC requirements are achieved when there are at least two base stations in line of sight situation. Figure 5 shows the RMS errors for different algorithms at different LOS/NLOS conditions. It is obvious that the Kalman and CWLS algorithms decrease the error at each LOS/NLOS condition. In figure 4 an example of the filtered range measurements can be seen.

The simulation results for second scenario is shown on table 2 and figure 6. The result show that use of Kalman filter and constrained weighted least squares algorithms together improves the performance significantly. The error in the second scenario is greater than the first one, as expected. The reason is, as the mobile station changes direction, it takes the Kalman filter some time to catch up with the new situation and follow the track.

## 5. CONCLUSION

In the proposed method, a biased Kalman filter is used to mitigate NLOS error in range measurements for each BS-MS combination. The filtered range measurements are then used to obtain a location estimate with CWLS. The Kalman filter includes LOS/NLOS decisions, based on range variances, which are calculated from the recent range measurements with a sliding time window. Next a constrained weighted least squares algorithm is used to extract the location information from the range measurements. Each range measurement is weighted inversely proportional to its variance. Simulation results show that the proposed combined algorithm offers improved NLOS mitigation. In addition, in most of the simulations, the proposed algorithm satisfies the 1999 FCC requirements.

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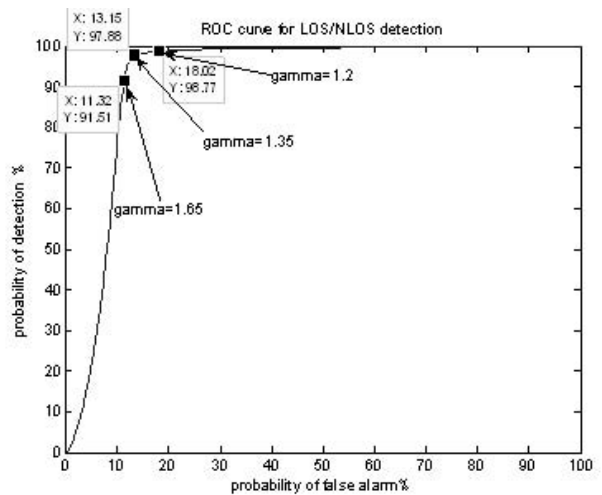


Figure 2: ROC curve for LOS/NLOS detection

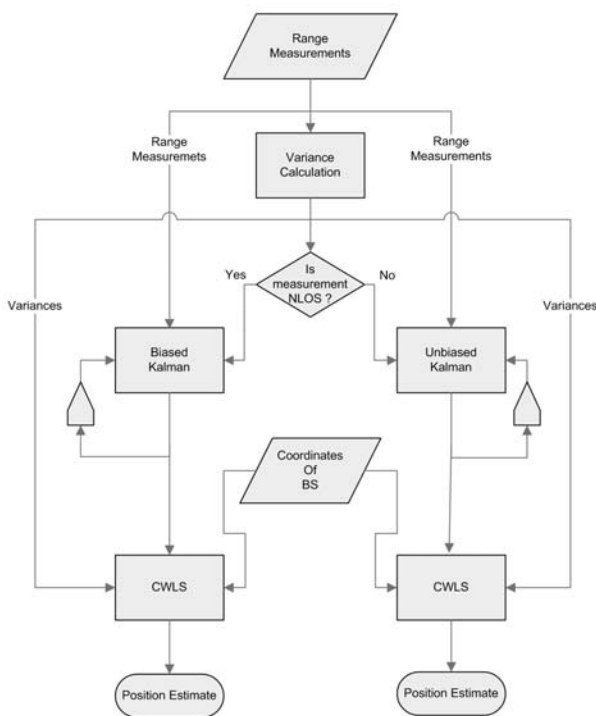


Figure 1: Block diagram of the Proposed Scheme

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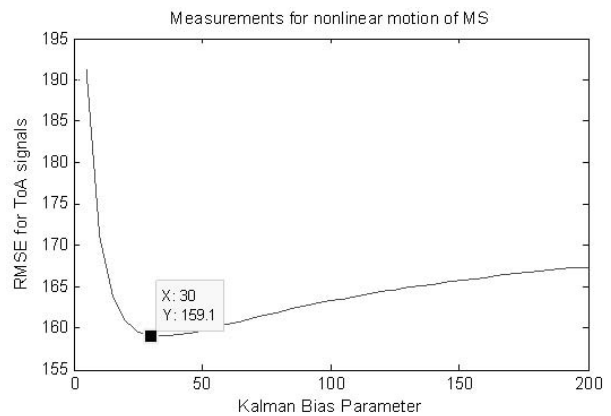


Figure 3: Kalman bias parameter for nonlinear motion of MS

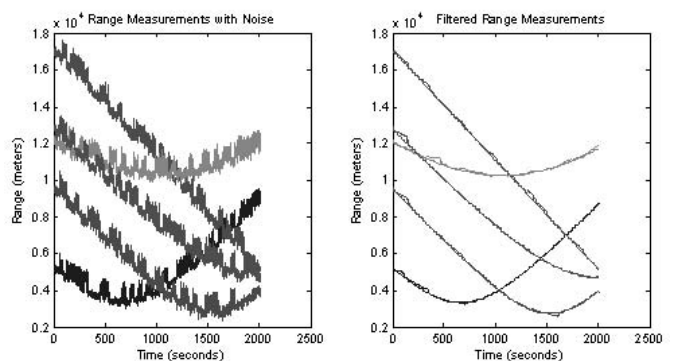


Figure 4: Filtered range measurements

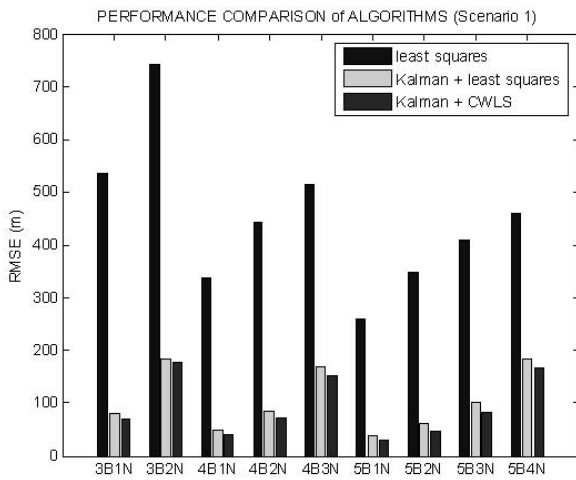


Table 1: Location Error for Proposed Algorithm( Scenario 1)

	95 % accuracy(m)	67 % accuracy(m)	50 % accuracy(m)
3BS 1NLOS	132.43	62.31	46.88
3BS 2NLOS	400.00	137.90	97.87
4BS 1NLOS	73.09	39.80	30.85
4BS 2NLOS	137.59	60.57	45.69
4BS 3NLOS	330.11	131.69	91.17
5BS 1NLOS	53.27	30.28	23.65
5BS 2NLOS	84.56	42.45	32.64
5BS 3NLOS	168.02	66.85	49.26
5BS 4NLOS	337.51	153.17	106.68

Figure 5: Performance comparison of algorithms for scenario 1

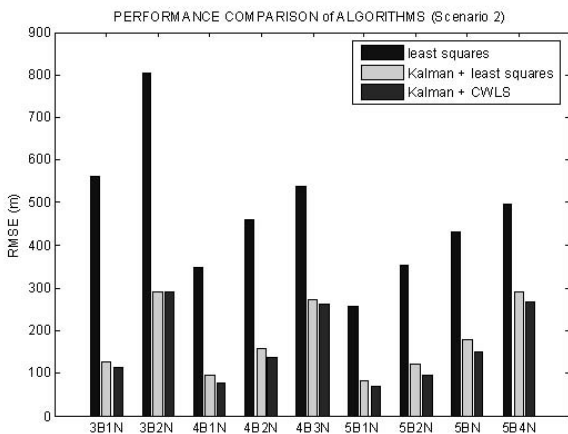


Table 2: Location Error for Proposed Algorithm( Scenario 2)

	95 % accuracy(m)	67 % accuracy(m)	50 % accuracy(m)
3BS 1NLOS	229.88	90.24	62.15
3BS 2NLOS	634.10	226.53	150.24
4BS 1NLOS	156.72	68.85	44.99
4BS 2NLOS	283.93	112.08	77.35
4BS 3NLOS	545.94	236.26	163.69
5BS 1NLOS	146.01	57.13	35.27
5BS 2NLOS	194.48	82.11	54.47
5BS 3NLOS	311.31	129.80	90.45
5BS 4NLOS	537.41	254.34	183.08

Figure 6: Performance comparison of algorithms for scenario 2