

# TIME AND FREQUENCY EQUALIZATION FOR UWB SYSTEMS: A COMPARISON STUDY

G. Ferré, J. P. Cancès and V. Meghdadi

Université de Limoges - Ecole Nationale Supérieure d'Ingénieurs de Limoges (ENSIL)

XLIM/C2S2/ESTE - CNRS UMR 6172

16, Rue Atlantis - Parc ESTER - BP 6804 - 87068 Limoges cedex, France

Email: {ferre,cances,meghdadi}@ensil.unilim.fr

## ABSTRACT

*In this paper we compare the performance of time and frequency equalization for high data rate ultra-wideband communications. For time-domain equalization, we propose a joint Rake and minimum mean square error equalizer receiver. The proposed receiver combats inter-symbol interference by taking advantage of the Rake and equalizer structure. We focus our attention on the effects of the number of Rake fingers and equalizer taps on the error performance. We show that, for a MMSE equalizer operating at low to medium SNR's, the number of Rake fingers is the dominant factor to improve system performance, while, at high SNR's the number of equalizer taps plays a more significant role in reducing error rates. The performances of this structure are compared with those of a frequency-domain equalizer operating in single carrier mode. We show that the frequency domain equalizer outperforms the combined Rake-MMSE equalizer structure in all the range of studied SNR's.*

## 1. INTRODUCTION

Ultra-wideband (UWB) has recently evoked great interest and its potential strength lies in its use of extremely wide transmission bandwidth. Furthermore, UWB is emerging as a solution for the IEEE 802.15a (TG3a) standard which is to provide a low complexity, low cost, low power consumption and high data-rate among Wireless Personal Area Network (WPAN) devices. An aspect of UWB transmission is to combat multipath propagation effects. Rake receivers can be employed since they are able to provide multipath diversity [1-3]. Another aspect is to eliminate or combat the inter-symbol interference (ISI) which distorts the transmitted signal and causes bit errors at the receiver, especially when the transmission data rate is very high as well as for systems which are not well synchronized. ISI can be suppressed by employing an equalizer at the receiver that requires periodic transmission of a training sequence [4]. Equalization for UWB has been addressed in a number of recent publications. Different types of RAKE receivers are studied by e.g. Cassioli et al [5] and their performance is evaluated for pulse-position modulation. Mielczarek et al [6] analyze fractionally-spaced RAKE receivers for DS-UWB systems employing Gaussian monocycles. Rajeswaran et al [7] investigate the RAKE performance and Eslami and Dong [8] discuss the performance of decision-feedback and linear equalization techniques for carrierless pulse-based UWB transmission. Ishiyama and Ohtsuki [9] consider frequency-domain equalization for modulation with Gaussian monocycles and a cyclic prefix.

Different from the above mentioned literature, in this paper, we explicitly consider BPSK modulation with root raised cosine (RRC) pulse shaping and carrier modulation as specified in [10], instead of modulation with Gaussian monocycles that do not require a carrier. We propose at first to study time equalization with combined Rake MMSE equalizer structure. We show that, for a MMSE equalizer operating at low to medium SNR's, the number of Rake fingers is the dominant factor to improve system performance, while, at high SNR's the number of equalizer taps plays a more significant role in reducing error rates. We show that for high frequency selective

channels such as the CM4 one, a linear equalizer structure is not sufficient and must be replaced by a decision feedback equalizer (DFE) structure. Furthermore, we propose a simple recursive gradient based algorithm to implement the equalizer structures.

Then, we propose to operate channel estimation and equalization in the frequency domain. This yields to a new receiver structure whose efficiency is compared to those of time-domain equalization with linear or DFE equalizer filters.

The rest of the paper is organized as follows. In section 2 we study the system model and UWB channel modelling. Section 3 is devoted to the time-domain equalization with linear or DFE equalizers. In section 4, we study frequency-domain equalization. Simulation results with comparison of time and frequency domain equalization are given in section 5. Section 6 eventually concludes the paper.

## 2. SYSTEM MODEL

For a single user system, the continuous transmitted data stream is written as

$$s(t) = \sum_{k=-\infty}^{+\infty} d(k) \cdot p(t - kT_s) \quad (1)$$

Where  $d(k)$  are stationary uncorrelated BPSK data and  $T_s$  is the symbol duration. Throughout this paper we consider the application of a root raised cosine (RRC) transmit filter  $p(t)$  with roll-off factor  $\alpha = 0.3$ .

The channel models used in this paper are the model proposed by IEEE 802.15.3a Study Group [11]. In the normalized models provided by IEEE 802.15.3a Study Group, different channel characteristics are put together under four channel model scenarios having rms delay spreads ranging from 5 to 26 nsec. For this paper two kinds of channel models, derived from the IEEE 802.15 channel modelling working group, are considered and named CM3 and CM4 channels. The first one CM3 corresponds to a non-line of sight communication with range 4-10 meters. The second corresponds to a strong dispersion channel with delay spread of 26 nsec. The impulse response can be written as

$$h(t) = \sum_{p=0}^M h_p \cdot \delta(t - \tau_p) \quad (2)$$

Parameter  $M$  is the total number of paths in the channel.

## 3. TIME DOMAIN EQUALIZATION

### 3.1 Receiver structure

The receiver structure is illustrated in Fig. 1 and consists in a Rake receiver followed by a linear equalizer (LE) or a DFE. As we will see later on, a DFE Rake structure gives better performances over UWB channels when the number of equalizer taps is sufficiently large. The received signal first passes through the receiver filter matched to the transmitted pulse and is given by

$$\begin{aligned} r(t) &= s(t) * h(t) * p(-t) + n(t) * p(-t) \\ &= \sum_{k=-\infty}^{+\infty} d(k) \sum_i h_i \cdot m(t - kT_s - \tau_i) + \hat{n}(t) \end{aligned} \quad (3)$$

where  $p(-t)$  represents the receiver matched filter, “\*” stands for convolution operation and  $n(t)$  is the additive white Gaussian noise (AWGN) with zero mean and variance  $N_0/2$ . Also,  $m(t) = p(t) * p(-t)$  and  $\hat{n}(t) = n(t) * p(-t)$ .

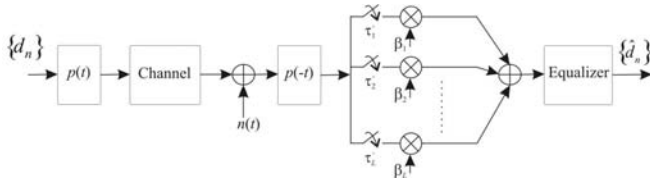


Figure 1 - UWB Rake-MMSE-equalizer structure

Combining the channel impulse response (CIR) with the transmitter pulse shape and the matched filter, we have

$$\tilde{h}(t) = p(t) * h(t) * p(-t) = \sum_{i=0}^M h_i m(t - \tau_i) \quad (4)$$

The output of the receiver filter is sampled at each Rake finger. The minimum Rake finger separation is  $T_m = T_s / N_u$ , where  $N_u$  is chosen as the largest integer value that would result in  $T_m$  spaced uncorrelated noise samples at the Rake fingers. In a first approach, complete channel state information (CSI) is assumed to be available at the receiver. For general selection combining, the Rake fingers ( $\beta_l$ 's) are selected as the largest  $L$  ( $L \leq N_u$ ) sampled signal at the matched filter output within one symbol time period at time instants  $\tau'_l$ ,  $l = 1, 2, \dots, L$ . In fact, since a UWB signal has a very wide bandwidth, a Rake receiver combining all the paths of the incoming signal is practically unfeasible. This kind of Rake receiver is usually named a ARake receiver. A feasible implementation of multipath diversity combining can be obtained by a selective-Rake (SRake) receiver, which combines the  $L$  best, out of  $N_u$ , multipath components. Those  $L$  best components are determined by a finger selection algorithm. For a maximal ratio combining (MRC) Rake receiver, the paths with highest signal-to-noise ratios (SNRs) are selected, which is an optimal scheme in the absence of interfering users and intersymbol interference (ISI). For a minimum mean square error (MMSE) Rake receiver, the “conventional” finger selection algorithm is to choose the paths with highest signal-to-interference-plus-noise ratios (SINRs) [12]. Our case doesn't deal with multiuser UWB communication but we study channels with high delay dispersion, so the first criterion ( $L$  highest SNR's) can be chosen. The noiseless received signal sampled at the  $l^{\text{th}}$  Rake finger in the  $n^{\text{th}}$  data symbol interval is given by

$$v(n.T_s + \tau'_l + t_0) = \sum_{k=-\infty}^{+\infty} \tilde{h}((n-k).T_s + \tau'_l + t_0).d(k) \quad (5)$$

where  $\tau'_l$  is the delay time corresponding to the  $l^{\text{th}}$  Rake finger and is an integer multiple of  $T_m$ . Parameter  $t_0$  corresponds to a time offset and is used to obtain the best sampling time. Without loss of generality,  $t_0$  will be set to zero in the following analysis. The Rake combiner output at time  $t = n.T_s$  is

$$y[n] = \sum_{l=1}^L \beta_l v(n.T_s + \tau'_l) + \sum_{l=1}^L \beta_l \hat{n}(n.T_s + \tau'_l) \quad (6)$$

Choosing the correct Rake finger placement leads to the reduction of ISI and the performance can be dramatically improved when using an equalizer to combat the remaining ISI. Considering the necessary trade off between complexity and performance, a sub-optimum classical criterion for updating the equalizer taps is the MMSE criterion. In the next section, we derive the MMSE-based equalizer tap coefficients.

### 3.2 Performance analysis

In this part, due to the lack of place we will only discuss the matrix block computation of linear equalizers. Furthermore, we suppose perfect channel state information (CSI). Assuming that the  $n^{\text{th}}$  data bit is being detected, the MMSE criterion consists in minimizing

$$E \left[ |d(n) - \hat{d}(n)|^2 \right] \quad (7)$$

where  $\hat{d}(n)$  is the equalizer output. Rewriting the Rake output signal, one can distinguish the desired signal, the undesired ISI and the noise as

$$y(n) = \left[ \sum_{l=1}^L \beta_l \tilde{h}(\tau'_l) \right] .d(n) + \sum_{k \neq n} \sum_{l=1}^L \beta_l \tilde{h}((n-k).T_s + \tau'_l).d(k) + \sum_{l=1}^L \beta_l \hat{n}(n.T_s + \tau'_l) \quad (8)$$

where the first term is the desired output. The noise samples at different fingers,  $\hat{n}(n.T_s + \tau'_l)$ ,  $l = 1, \dots, L$ , are uncorrelated and therefore independent, since the samples are taken at approximately the multiples of the inverse of the matched filter bandwidth. It is assumed that the channel has a length of  $(n_1 + n_2 + 1).T_s$ . That is, there is pre-cursor ISI from the subsequent  $n_1$  symbols and post-cursor ISI from the previous  $n_2$  symbols, and  $n_1$  and  $n_2$  are chosen large enough to include the majority of the ISI effect. Using (8), the Rake output can be expressed now in a simple form as

$$y(n) = \alpha_0 .d(n) + \sum_{\substack{k=-n_1 \\ k \neq 0}}^{n_2} \alpha_k .d(n-k) + \tilde{n}(n) \\ = \Phi^T d[n] + \tilde{n}(n) \quad (9)$$

where coefficients  $\alpha_k$ 's are obtained by matching (8) and (9).

$\Phi = [\alpha_{-n_1} \dots \alpha_0 \dots \alpha_{n_2}]^T$  and  $d[n] = [d(n+n_1) \dots d(n) \dots d(n-n_2)]^T$ . The superscript  $T$  denotes the transpose operation. The noise at the Rake output is  $\tilde{n}(n) = \sum_{l=1}^L \beta_l \hat{n}(n.T_s + \tau'_l)$ . The output of the linear equalizer is obtained as

$$\hat{d}(n) = \sum_{r=-K_1}^{K_2} c_r .y(n-r) = \mathbf{c}^T \boldsymbol{\gamma}(n) + \mathbf{c}^T \boldsymbol{\eta}(n) \quad (10)$$

where  $\mathbf{c} = [c_{-K_1} \dots c_0 \dots c_{K_2}]^T$  contains the equalizer taps. Also

$$\boldsymbol{\gamma}[n] = [\Phi^T d[n+K_1] \dots \Phi^T d[n] \dots \Phi^T d[n-K_2]]^T \\ \boldsymbol{\eta}[n] = [\tilde{n}(n+K_1) \dots \tilde{n}(n) \dots \tilde{n}(n-K_2)]^T \quad (11)$$

The mean square error (MSE) of the equalizer,

$$E \left[ |d(n) - \mathbf{c}^T \boldsymbol{\gamma}[n] - \mathbf{c}^T \boldsymbol{\eta}[n]|^2 \right] \quad (12)$$

which is a quadratic function of the vector  $\mathbf{c}$ , has a unique minimum solution. Here, the expectation is taken with respect to the data symbols and the noise. Defining matrices  $\mathbf{R}$ ,  $\mathbf{p}$  and  $\mathbf{N}$  as

$$\mathbf{R} = E[\boldsymbol{\gamma}[n].\boldsymbol{\gamma}^T[n]] \quad (13)$$

$$\mathbf{p} = E[d(n).\boldsymbol{\gamma}[n]] \quad (14)$$

$$\mathbf{N} = E[\boldsymbol{\eta}[n].\boldsymbol{\eta}^T[n]] \quad (15)$$

The equalizer taps are given by

$$\mathbf{c} = (\mathbf{R} + \mathbf{N})^{-1} \cdot \mathbf{p} \quad (16)$$

and the MMSE is

$$\mathbf{J}_{\min} = \sigma_d^2 - \mathbf{p}^T (\mathbf{R} + \mathbf{N})^{-1} \cdot \mathbf{p} \quad (17)$$

where  $\sigma_d^2 = E[|d(n)|^2]$ . Evaluating the expectation over  $\mathbf{R}$  and  $\mathbf{p}$  with respect to the data and the noise, we have

$$\mathbf{p} = [\alpha_{K_1} \dots \alpha_0 \dots \alpha_{-K_2}]^T \quad \mathbf{R} = [r_{i,j}]_{K_1+K_2+1, K_1+K_2+1} \quad (18)$$

where:  $r_{i,j} = \Phi^T F_{ij} \Phi$  and  $F_{i,j} = [f_{ik}]_{n_1+n_2+1, n_1+n_2+1}$

$$f_{ik} = \begin{cases} 1, & l-k = j-i \\ 0, & l-k \neq j-i \end{cases} \quad (19)$$

$$\text{Also: } \mathbf{N} = E[\boldsymbol{\eta}[n] \boldsymbol{\eta}^T[n]] = \frac{N_0}{2} \cdot \left( \sum_{l=1}^L \beta_l^2 \right) \cdot \mathbf{I}_{K_1+K_2+1} \quad (20)$$

where  $\mathbf{I}$  is the identity matrix. This Rake-equalizer receiver will eliminate ISI as far as the number of equalizer's taps gives the degree of freedom required. In general, the equalizer output can be expressed as

$$\hat{d}(n) = q_0 \cdot d(n) + \sum_{i \neq 0} q_i \cdot d(n-i) + w(n) \quad (21)$$

with:  $q_n = \alpha_n \cdot c_n$  and  $w(n) = \tilde{n}(n) \cdot c_n$ . The variance of  $w(n)$  is

$$\sigma_{w(n)}^2 = \left( \sum_{i=-K_1}^{K_2} c_i^2 \right) \left( \sum_{l=1}^L \beta_l^2 \right) \cdot E_p \cdot N_0 / 2 \quad (22)$$

where  $E_p$  is the pulse energy.

### 3.3 Implementation Issue

Matrix blocks based implementation of equalizer implies computation of inverse matrix (16) and this may constitute a heavy task. Moreover, for practical implementation, channel state information is unknown at the receiver. To cope with these problems, adaptive iterative algorithms such as LMS with training sequences or blind algorithms such as the Constant Modulus Algorithm (CMA) can be employed. Their drawback consists in the required number of iterations to obtain the desired MMSE level. To describe the proposed algorithms we use the structure depicted in Fig. 2 for the linear adaptive equalizer. The channel time delay for the correlation fingers of the Rake is assumed statistically uniformly distributed. The timing of the first arriving desired signal path is taken at the time reference with zero delay, i.e.  $\tau_1 = 0$  and the timing of the  $i^{\text{th}}$  path is given by the delay,  $\tau_i$ .

In the proposed receiver, the received signal  $r(t)$  firstly passes through a  $L = N_u$  tapped-delay-line and performs cross correlations with the reference pulse  $p(t)$  at uniform time delays  $\tau_i = (i-1) \cdot \tau, 1 \leq i \leq L$  ( $\tau = T_s / L$ ).

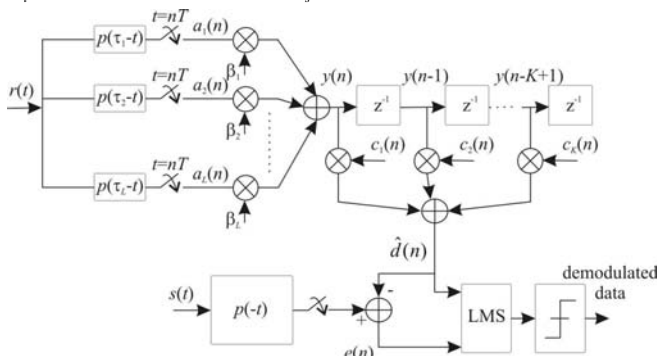


Figure 2 - Receiver structure with adaptive combined Rake equalizer structure

The correlation output is sampled at time  $t = nT$ , and then input into the combined Rake-equalizer structure, where  $T$  represents the sampling time interval. Let the weighting coefficients of the Rake fingers be

$\{\beta_i(n), i = 1, \dots, L\}$  and the tap coefficients of the equalizer be

$\{c_i(n), i = 1, \dots, K\}$ . Also let define the corresponding vectors

$\boldsymbol{\beta}[n] = [\beta_1(n), \beta_2(n), \dots, \beta_L(n)]^T$  and  $\mathbf{c}[n] = [c_1(n), c_2(n), \dots, c_K(n)]^T$ .

The vector of the bank of correlator outputs is denoted as

$\mathbf{a}[n] = [a_1(n), \dots, a_L(n)]^T$  with

$$a_i(n) = \int_0^t r(u) \cdot p(u - (i-1)\tau) \cdot du \Big|_{t=nT} \cdot \quad \text{We then define}$$

$\mathbf{y}[n] = [y(n), y(n-1), \dots, y(n-K+1)]^T$  as the channel samples at the input of the equalizer. We have the relationship

$$y(n) = \boldsymbol{\beta}^T[n] \bullet \mathbf{a}[n] \quad (23)$$

where  $\bullet$  represents the vector inner product. For the linear structure, a  $K$ -tap transversal FIR filter is employed. The proposed algorithm proceeds as follows. For the training based Rake combining equalizer, a training sequence  $\mathbf{d}(n)$  is employed. LMS algorithm is then used recursively to adjust the Rake and equalizer

tap weights to minimize the mean square error  $\text{MSE } E\{e(n)^2\}$  using the following three steps

- 1) Filtering:  $\hat{d}(n) = d_{\text{new}}(n) = \mathbf{c}^T[n] \bullet \mathbf{y}[n]$  (24)

- 2) Error estimation:  $e(n) = d(n) - d_{\text{new}}(n)$  (25)

- 3) Rake and tap weight vector adaptation:  $\boldsymbol{\beta}[n+1] = \boldsymbol{\beta}[n] + \mu \cdot e(n) \cdot \mathbf{a}[n]$  (26)

$$\mathbf{c}[n+1] = \mathbf{c}[n] + \mu \cdot e(n) \cdot \mathbf{y}[n] \quad (27)$$

$\mu$  represents a small positive convergence parameter and  $e(n)$  is the

prediction error obtained from the LMS algorithm.  $\hat{d}(n) = d_{\text{new}}(n)$  represents the output from the Rake combining equalizer.  $d(n)$  represents the training sequence which is obtained by convolving the transmitted pulse train with the pulse  $p(t)$ :

$$d(n) = \int_0^t s(u) \cdot p(u) \cdot du \Big|_{t=nT} \cdot \quad \text{Once the algorithm converges and}$$

taps are fixed, the output of the equalizer  $\hat{d}(n)$  is then passed through the decision making scheme to determine whether the transmitted bit is "1" or "0" by comparing it with the zero threshold. This simple gradient based adaptive algorithm can be easily generalized to DFE equalizer structure.

## 4. FREQUENCY DOMAIN EQUALIZATION

### 4.1 Frequency Equalization

We consider a cyclic-prefixed single carrier frequency domain equalizer (SC-FDE) transmission over UWB channels as illustrated on Fig. 3. A block of signals  $d(k), (0 \leq k \leq N-1)$  is transmitted with length  $N$ . A cyclic prefix (CP) is inserted between blocks to mitigate interblock interference (IBI). As long as the duration of CP is longer than that of the CIR, IBI effects can be ignored. For simplicity reason, we will assume here in our mathematical derivations that  $p(t) = \delta(t)$ . Assuming perfect time synchronization and supposing that the equivalent  $T$ -spaced CIR is of order  $L$  with taps  $\mathbf{h} = [h(0), h(1), \dots, h(M)]^T$  as in (2), the block received signal can be expressed in a matrix form as

$$\mathbf{y} = \tilde{\mathbf{H}} \cdot \mathbf{d} + \mathbf{n} \quad (28)$$

where  $\mathbf{d} = [d(0), d(1), \dots, d(N-1)]^T$ , and  $\tilde{\mathbf{H}}$  is the circulant Toeplitz matrix with the first column being  $\mathbf{h}$  zero-padded to length  $N$  yielding to vector  $\tilde{\mathbf{h}}$ .  $\mathbf{n}$  is a  $N \times 1$  vector of white Gaussian noise samples with variance  $\sigma_n^2 = N_0/2$ . The frequency domain received signal  $\mathbf{Y}$  can be expressed as

$$\mathbf{Y} = \mathbf{F} \cdot \mathbf{y} = \mathbf{A} \mathbf{d} + \mathbf{F} \mathbf{n} \quad (29)$$

where  $\mathbf{F}$  is the discrete Fourier transform (DFT) matrix and  $F_{l,k} = (1/\sqrt{N}) \cdot \exp(-j \cdot (2\pi/N) \cdot l \cdot k)$ ,  $0 \leq l, k \leq N-1$ . Matrix  $\mathbf{A}$  is a diagonal matrix, with its  $(k,k)$ th entry denoted as  $H_k$ , where  $H_k$  is the  $k$ th coefficient of channel frequency response and

$$H_k = \sum_{l=0}^M h(l) \cdot \exp(-j \cdot \frac{2\pi}{N} \cdot k \cdot l) \quad (30)$$

This is equivalent to:  $\mathbf{H} = \sqrt{N} \cdot \mathbf{F} \tilde{\mathbf{h}}$  (31)

where  $\mathbf{H} = [H_0, H_1, \dots, H_{N-1}]^T$ . The purpose of frequency-domain equalization (FDE) is to eliminate intersymbol interference (ISI) within individual transmission blocks. The frequency domain equalizer taps are given by

$$C_k = \frac{H_k^*}{|H_k|^2 + \sigma_n^2 / \sigma_d^2} \quad (32)$$

In order to perform frequency-domain channel equalization, the estimation of channel coefficients  $H_k$ ,  $k = 0, 1, \dots, N-1$ , are required. After FDE and IDFT, the received signal  $\mathbf{z}$  becomes

$$\begin{aligned} \mathbf{z} &= \mathbf{F}^H \cdot \mathbf{C} \mathbf{Y} \\ &= \mathbf{F}^H \cdot \mathbf{C} \mathbf{A} \mathbf{d} + \mathbf{F}^H \cdot \mathbf{C} \mathbf{F} \mathbf{n} \end{aligned} \quad (33)$$

where  $\mathbf{C}$  is an  $N \times N$  diagonal matrix with its  $k$ th diagonal element as the frequency-domain equalizer taps. Signal detection is then performed in the time domain.

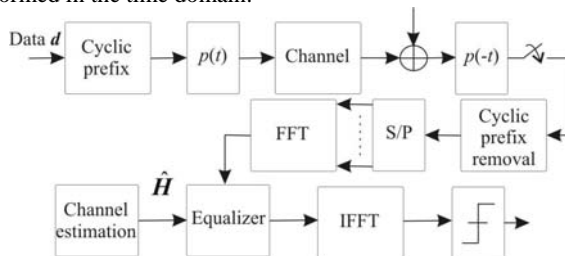


Figure 3 - SC-FDE system

#### 4.2 Channel Estimation

If we use a block of pilot symbols denoted as  $\mathbf{p} = [p_0, p_1, \dots, p_{N-1}]^T$ , it is straightforward to rewrite (29) as

$$\mathbf{Y} = \mathbf{P} \mathbf{H} + \mathbf{N} \quad (34)$$

where  $\mathbf{P}$  is a diagonal matrix with its  $k$ th diagonal element  $P_k$  as the  $k$ th coefficient of the frequency-domain spectrum of the pilot sequences  $\mathbf{p}$ , and  $\mathbf{N} = \mathbf{F} \mathbf{n}$ . It is known by Wiener filtering technique that the LMMSE estimator can be estimated as

$$\hat{\mathbf{H}}_{MMSE} = \mathbf{R}_{HY} \cdot \mathbf{R}_{YY}^{-1} \cdot \mathbf{Y} \quad (35)$$

where  $\mathbf{R}_{HY} = E[\mathbf{H} \mathbf{Y}^H] = \mathbf{R}_{HH} \cdot \mathbf{P}^H$  (36)

and  $\mathbf{R}_{YY} = E[\mathbf{Y} \mathbf{Y}^H] = \mathbf{P} \cdot \mathbf{R}_{HH} \cdot \mathbf{P}^H + \sigma_n^2 \cdot \mathbf{I}_N$  (37)

$\mathbf{R}_{HY}$  and  $\mathbf{R}_{YY}$  are the cross-correlation matrix between  $\mathbf{H}$  and  $\mathbf{Y}$  and the autocorrelation matrix of  $\mathbf{Y}$ , respectively.  $\mathbf{R}_{HH}$  is the autocorrelation matrix of channel frequency-domain response  $\mathbf{H}$ ,

and is supposed to be known at the receiver. The LMMSE channel estimator in (54) can be obtained as

$$\hat{\mathbf{H}}_{MMSE} = \mathbf{R}_{HH} (\mathbf{R}_{HH} + \sigma_n^2 (\mathbf{P}^H \mathbf{P})^{-1})^{-1} \cdot \hat{\mathbf{H}}_{LS} = \mathbf{Q} \cdot \hat{\mathbf{H}}_{LS} \quad (38)$$

with  $\hat{\mathbf{H}}_{LS} = \mathbf{P}^{-1} \mathbf{Y}$  and  $\mathbf{Q} = \mathbf{R}_{HH} \cdot (\mathbf{R}_{HH} + \sigma_n^2 (\mathbf{P}^H \mathbf{P})^{-1})^{-1}$ . In the case of severe channels such as CM4 one it is possible to increase the accuracy of the estimation by using several block of pilot symbols, the derivation of the frequency estimator remains similar. A good estimator should minimize the variance of the estimated error. Therefore, to evaluate the performance of the LMMSE estimators, we calculate the average MSE

$$\text{MSE} = \frac{1}{N} \cdot \text{Tr} \left\{ E[(\mathbf{H} - \hat{\mathbf{H}}_{MMSE}) \cdot (\mathbf{H} - \hat{\mathbf{H}}_{MMSE})^H] \right\} \quad (39)$$

### 5. SIMULATION RESULTS

As we mentioned it before, we study the case of UWB channels CM3 and CM4. For the root raised cosine (RRC) pulse, we use an oversampling factor of eight. According to this sampling rate, time channel spread is chosen equal to 100 for CM4 and 70 for CM3, this corresponds to respectively  $12 = 100/8$  and  $9 = 70/8$  transmitted symbols. This choice enables to gather 99% of the channel energy. The data rate is chosen to be 400 Mbps, one of the optional data rates proposed for IEEE standard. The size of the transmitted packets is equal to 2560 BPSK symbols including a training sequence of length 512. CIR remains constant over the time duration of a packet. The root raised cosine (RRC) pulse with rolloff factor  $\beta = 0.5$  is employed as the pulse-shaping filter.

In the case of time domain equalization, we have at first to optimize the number of Rake fingers  $L$  and the number of equalizer taps. The Rake fingers are regularly positioned according to time channel spread and the number of fingers. For example, in the case of CM4 channel, with  $L = 10$ , the time distance between two consecutive fingers is equal to 10 samples. Fig. 4 shows the effect of the number of equalizer taps and Rake fingers using Monte-Carlo simulation runs. For LE structure, at high SNR's, a 20 tap equalizer with 1 Rake fingers outperforms a 3 tap equalizer with 20 Rake fingers.

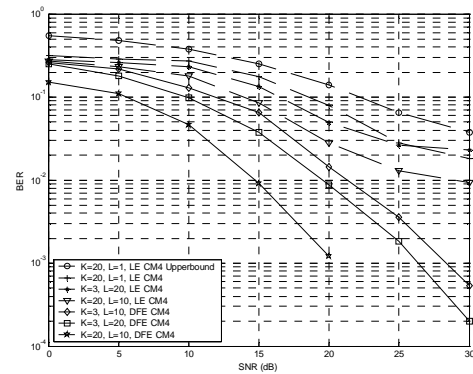


Figure 4 - Performance of UWB Rake-MMSE-receiver for different number of equalizer taps and Rake fingers

At low to medium SNR's, however, the receiver with more Rake fingers outperforms the one that has more equalizer taps but fewer Rake fingers. This result can be explained by considering the fact that at high SNR's it is mainly the ISI that affects the system performance, whereas at low SNR's the system noise is also a major contribution in system degradation (more signal energy capture is required). The performance dramatically improves when the number of Rake fingers and the equalizer taps are increased simultaneously, i.e.  $K = 20, L = 10$ .

One can observe a BER floor at high SNR's due to the difficulty for a linear equalizer to cope with the presence of zeros outside the unit circle. This can be circumvented by the use of DFE structure. For DFE structure, we use the same number of feedforward and

backward coefficients  $K$ . DFE performances are computed by Monte-Carlo computer simulations, using a training sequence with length 500 and  $\mu = 0.01$  (see (26-27)) and the BER at each SNR is averaged over 100 channel trials. We obtain the same conclusions as for LE structure, and we found that  $K = 20$  and  $L = 10$  yields to the best performances as illustrated on Fig. 4. Fig 5 illustrates the MSE of (7) averaged over sliding window blocks of 40 symbols during the training phase and for a particular channel trial at SNR = 20 dB.

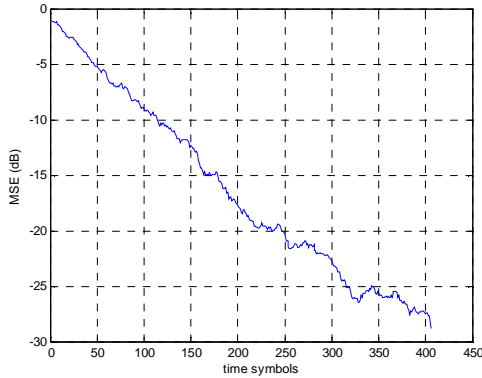


Figure 5 - MSE for the training phase (DFE), SNR = 20 dB,  $K = 20$ ,  $L = 10$

In the following, and particularly in order to compare time and frequency equalization, we will use a optimized RAKE-DFE structure for CM4 with  $K = 20$ ,  $L = 10$ .

In the case of frequency equalization, we take data blocks of length 256. This means, in order to have a fair comparison with time domain equalization (training sequence length of 512 symbols), that two blocks are used as pilots. Simulation results not shown here demonstrate that using two blocks yields to an improvement of 1 dB over the CM4 channel and 0.7 dB over the CM3 one. Higher size only brings marginal improvement.

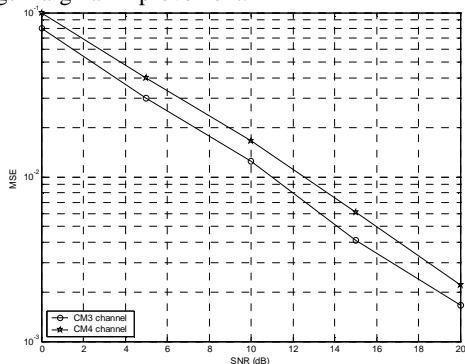


Figure 6 - Average MSE for CIR estimation (CM3 and CM4 channel)

A CP with length 20 is employed to prevent IBI. Fig. 6 illustrates the MSE obtained by averaging (48) over 100 different CM3-CM4 channel realizations. One can see that results are slightly better for CM3, which does not constitute a surprising fact due to the higher selectivity of CM4.

Finally, we compare time and frequency domain equalization in terms of BER. We do insist on the fact that the compared systems use the same symbol training length (512 symbols). The results are illustrated on Fig. 7. One can see that FDE structure always outperforms the combined Rake-DFE receiver. For example, the gain is equal to 1 dB at  $BER = 10^{-3}$  for the CM4 channel and 0.8 dB for the CM3 in the same conditions.

## 6. CONCLUSION

In this paper, we have proposed two approaches to equalize UWB channels. The first one is in the time domain and uses a combined

Rake-equalizer structure. We focus our attention on the effects of the number of Rake fingers and equalizer taps on the error performance to optimize this time-domain structure. The second one operates in the frequency domain and in a single carrier mode at the receiver. Simulation results clearly illustrate the superiority of the frequency domain equalizer which exhibits for example a gain of 1 dB at  $BER = 10^{-3}$  for the CM4 channel.

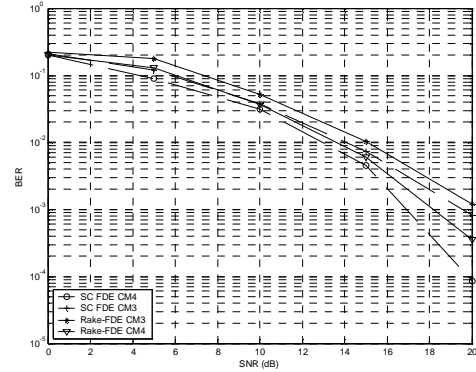


Figure 7 - BER performances for Rake-DFE and SC FDE

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