ITERATIVE PRECODING VS DECODING IN VIRTUAL MIMO SYSTEMS WITH CHANNEL MISMATCHING ERRORS

B. Béjar, S. Zazo and I. Raos

Dpto. Señales, Sistemas y Radiocomunicaciones-ETSIT. Universidad Politécnica de Madrid
Avd. Complutense s/n, ETSIT, 08040, Madrid, Spain
phone: + (34) 915495700, fax: + (34) 913367350, e-mail: min@gaps.ssr.upm.es

ABSTRACT

This contribution addresses a mathematical formulation and a comparison in performance of two realistic approaches of cooperation in wireless sensor networks. Tomlinson - Harashima precoding has been used as representative of transmitter cooperation while V-BLAST equalization technique was used as representative for receiver cooperation. The key point of this contribution is that non ideal conditions of carrier synchronization are considered. Recent theory suggests that cooperating at the transmitter results more beneficial than doing it at the receiver. In our hand we will show that, when carriers are not accurate synchronized, receiver cooperation appears to be a more robust selection.

1. INTRODUCTION

[1] Sensor networks and ad-hoc networks are receiving more and more attention from the research community. It has been shown that, under certain assumptions, WSN can be viewed as virtual Multiple-Input Multiple-Output (MIMO) systems [1, 2]. This point allows to relate developed theory and experience in MIMO systems with WSN. One important topic of research in WSN is cooperative communications, that is, how the elements of a WSN arrange the distribution of some common resource to improve the system performance. Basically, there are three possible configurations for cooperation depending on the available information: a) at the transmitter side a group of nodes interchange their codewords and also the channel coefficients, b) at the receiver side nodes share their received signals and channel coefficients and c) cooperation is performed jointly at both sides of the system. It is important to say that if cooperation were ideal, i.e. no restrictions on power or bandwidth and ideal cooperation channel, performance of a WSN can achieve that of a MIMO system. However this assumption can not be applied as cooperation is always by some means penalized. In [5] a very suitable model is provided that addresses penalizations in terms of power, bandwidth and even network topology and establishes a trade-off between them in order to achieve cooperation benefits. Conclusions in [5] reveal that cooperating at the transmitter is more suitable than doing it at the receiver side and even better than doing it at both sides because extra gain is negligible. However when considering some non ideal conditions in the communication channel this assessment may not hold any more.
Recent work [8] goes in this direction and presents a model that introduces two important aspects: First,some shift between the carrier frequencies of different sensors even among those belonging to the same cluster are considered. This aspect is very critical because nodes need to be very cheap (so inaccurate) and also because full time / frequency synchronization is unreachable even using very carefully PLL designs [6]. A second aspect is that it considers realistic strategies of cooperation. Our work starts from this point but introducing some new aspects that describe more accurate the system under non idealities of carrier synchronization. In this contribution deviations to the center carrier frequency are quantified in terms of the coherence bandwidth of the channel and the effects of phase rotation due to frequency mismatch is considered even among nodes of the same cluster. For our evaluation, Tomlinson - Harashima Precoding (THP) based on the V-BLAST idea (THP-VB) [7] is used as transmitter cooperation strategy while equalization based on V-BLAST technique has been used as representative for receiver cooperation. THP based on the V-BLAST idea and V-BLAST equalization are considered as two equivalent strategies. Both of them perform nearly the same in MIMO applications although there is a small power penalty in the THP case due to the modulo operator. To be more specific, the approach we have followed in both cases is based on the QR decomposition of the estimated matrix, performing sequential interference cancellation. Clearly, THP is a sub optimum implementation of Dirty Paper Coding (DPC) [3], while V-BLAST is a sub optimum Multiuser Detector (MUD). Joint cooperative approach is not considered here because conclusions from [8] show that this last one suffers a high degradation when transmitter and receiver have different or mismatching Channel State Information (CSI) and therefore makes this approach nearly useless.

Simulation results confirm what was said in [8] and give more insight into about the problem. Our conclusions show that deviations of only a fraction of the coherence bandwidth makes the system degrade significantly when cooperation is performed at the transmitter. However these deviations are too small to produce an appreciable degradation in performance when cooperating at the receiver and hence, performance of this cooperation strategy is only affected by the errors introduced through the cooperation channel.

The paper is organized as follows: In Section II the system model for a sensor network with imperfect carrier synchronization is presented. Section III provides some examples and simulations and some conclusions and future directions of research are given in Section IV. References are provided at the end of the paper.

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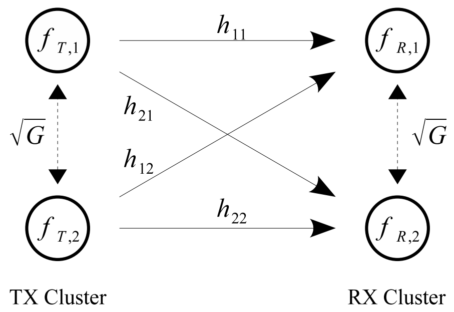

Figure 1: Sensor network consisting of two transmitters and two receivers.

2. SYSTEM DESCRIPTION

Consider a sensor network consisting on a cluster of N transmitters and a cluster of M receivers. Assuming the flat fading model, the channel coefficients from the j-th transmitter to the i-th receiver are denoted as h_{ij}, where h_{ij} are circularly symmetric complex-Gaussian random variables with zero mean and unit variance, $h_{ij} \sim \mathcal{N}_C(0,1),\ \forall ij$. Let $\mathbf{x} = [x_1, x_2, \ldots, x_N]^T$ be a vector of complex transmit signals and $\mathbf{y} = [y_1, y_2, \ldots, y_M]^T$ a complex vector containing the received signals. In matrix formulation, the system can be described as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{n} = [n_1, n_2, \ldots, n_M]^T$ is a vector of independent and identically distributed (iid) complex-Gaussian random variables with zero mean and unit variance, $n_i \sim \mathcal{N}_C(0,1), i = 1, \ldots, M$ and the matrix $\mathbf{H}$ contains the channel coefficients

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & & h_{1N} \\ h_{21} & h_{22} & \cdots & & h_{2N} \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ h_{M1} & h_{M2} & \cdots & & h_{MN} \end{bmatrix}. \tag{2}$$

Additional to the transmission channel and analog to the model in [5], there also exist two Additive White Gaussian Noise (AWGN) cooperation channels with channel gain $\sqrt{G}$ for the communication between elements of the same cluster, see Fig. 1. These channels are supposed to be orthogonal to the transmission channels by means of some frequency or time duplex technique. Denoting x as the complex transmit signal, the received signal through cooperation channel is given by

$$y = \sqrt{G}x + n\,, \tag{3}$$

where $n \sim \mathcal{N}_C(0,1)$. This means, transmitters / receivers interchange their information about the transmit / received symbols / signals and channel fading, resulting in a virtual BC / MAC MIMO channel. The information provided by the other elements is used in order to appropriately encode / equalize data.

According to this, the j-th transmitter will have access to

$$\mathbf{x}_j = \begin{bmatrix} \tilde{x}_1 \\ \vdots \\ \tilde{x}_{j-1} \\ x_j \\ x_{j+1} \\ \vdots \\ \tilde{x}_N \end{bmatrix} = \begin{bmatrix} x_1 \\ \vdots \\ x_{j-1} \\ x_j \\ x_{j+1} \\ \vdots \\ x_N \end{bmatrix} + \frac{1}{\sqrt{G}} \begin{bmatrix} n_{t,1} \\ \vdots \\ n_{t,j-1} \\ 0 \\ n_{t,j+1} \\ \vdots \\ n_{t,N} \end{bmatrix}, \tag{4}$$

where $n_{t,j} \sim \mathcal{N}_C(0,1)\ ,\ j = 1, \ldots, N$. In an analog way, i-th receiver will have information related to

$$\mathbf{y}_i = \begin{bmatrix} \tilde{y}_1 \\ \vdots \\ \tilde{y}_{i-1} \\ y_i \\ \tilde{y}_{i+1} \\ \vdots \\ \tilde{y}_M \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_{i-1} \\ y_j \\ y_{i+1} \\ \vdots \\ y_M \end{bmatrix} + \frac{1}{\sqrt{G}} \begin{bmatrix} n_{r,1} \\ \vdots \\ n_{r,i-1} \\ 0 \\ n_{r,i+1} \\ \vdots \\ n_{r,N} \end{bmatrix}, \tag{5}$$

where $n_{r,i} \sim \mathcal{N}_C(0,1)\ ,\ i = 1, \ldots, M$.

For the sake of simplicity, let assume that no errors occur when cooperating nodes interchange information related to the transmission channel $\mathbf{H}$, i.e. by appropriate coding of the coefficients. All channels are supposed to have the same available bandwidth B.

Consider now that each element on the network has its own operating frequency as it is illustrated in Fig. 1 and that these frequencies are given by

$$\begin{aligned} f_{T,j} &= f_0 + \Delta_{f_{T,j}}\ , \quad j = 1, \ldots, N \\ f_{R,i} &= f_0 + \Delta_{f_{R,i}}\ , \quad i = 1, \ldots, M \end{aligned} \tag{6}$$

where $\Delta_{f_{T,j}}, \Delta_{f_{R,i}}$ are iid uniform random variables.

Let denote $h_{ij}^{(f_0)}$ the ij-th channel coefficient when transmission takes place at a carrier frequency f_0. In the following, we will see how these imperfect synchronization affects the system.

2.1 Ideal case

When all elements operate at the same carrier frequency $f_{T,j} = f_{R,i} = f_0,\ \forall i,j$, transmissions in both directions of the link experience the same attenuation by the channel and hence, the channel matrix

$$\mathbf{H}^{(f_0)} = \begin{bmatrix} h_{11}^{(f_0)} & h_{12}^{(f_0)} & \cdots & & h_{1N}^{(f_0)} \\ h_{21}^{(f_0)} & h_{22}^{(f_0)} & \cdots & & h_{2N}^{(f_0)} \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ h_{M1}^{(f_0)} & h_{M2}^{(f_0)} & \cdots & & h_{MN}^{(f_0)} \end{bmatrix}$$

will be available at both transmitter and receiver sides. If gain $\sqrt{G}$ is large enough we can assume that no additional noise component is present due to transmission over cooperation channels and the system can be viewed as a virtual MIMO system. Under this situation, the different approaches of cooperation will achieve ideal performance.

2.2 Imperfect synchronization

Consider now the situation where each element has its own operating frequency as given in Eq. (6) and denote $\mathbf{H}_T$ and $\mathbf{H}_R$ as the channel matrices for transmitter to receiver link and receiver to transmitter link, respectively. It is important to note that transmitters will know $\mathbf{H}_R$, while $\mathbf{H}_T$ will be available at receiver side.
Because transmission follows the link transmitter to receiver, the received signal $\mathbf{y}$ is then given by

$$\mathbf{y} = \mathbf{H}_T\mathbf{x} + \mathbf{n} \ . \tag{7}$$

2.2.1 *Cooperation at the receiver*

Taking all the previous considerations into account, the transmission channel matrix $\mathbf{H}_T$ will be given by

$$\mathbf{H}_T = \begin{bmatrix} h_{11}^{(f_0+\Delta_{f_{T,1}})} e^{j\varphi_{11}} & \cdots & h_{1N}^{(f_0+\Delta_{f_{T,N}})} e^{j\varphi_{1N}} \\ \vdots & \ddots & \vdots \\ h_{M1}^{(f_0+\Delta_{f_{T,1}})} e^{j\varphi_{M1}} & \cdots & h_{MN}^{(f_0+\Delta_{f_{T,N}})} e^{j\varphi_{MN}} \end{bmatrix} \tag{8}$$

where φ_{ij} is the phase difference introduced between the i-th receiver and j-th transmitter given as

$$\varphi_{ij} = 2\pi\left(f_{R,i} - f_{T,j}\right) = 2\pi\left(\Delta_{f_{R,i}} - \Delta_{f_{T,j}}\right) \ , \quad \forall i,j \ . \tag{9}$$

As receivers always know $\mathbf{H}_T$, imperfections on the oscillators do not degrade Channel State Information (CSI) and therefore, equalization will not degrade due to these imperfections. However when interchanging information through the cooperation channel an additional noise term is present but also a phase shift will occur as elements within the receiver cluster have different carrier frequencies. Based on this, at the i-th receiver the information available about the received signals $\mathbf{y}'_i$ will be

$$\mathbf{y}'_i = \begin{bmatrix} (y_1 + n_{r,1}/\sqrt{G})e^{j\Delta_{R,i1}} \\ \vdots \\ (y_{i-1} + n_{r,i-1}/\sqrt{G})e^{j\Delta_{R,i(i-1)}} \\ y_i \\ (y_{i+1} + n_{r,i+1}/\sqrt{G})e^{j\Delta_{R,i(i+1)}} \\ \vdots \\ (y_M + n_{r,M}/\sqrt{G})e^{j\Delta_{R,iM}} \end{bmatrix} = \mathbf{\Sigma}_{R,i} \cdot \mathbf{y}_i \ , \tag{10}$$

where $\mathbf{y}_i$ is given in Eq. (5), $\Delta_{R,ik} = 2\pi(\Delta_{f_{R,i}} - \Delta_{f_{R,k}})$, $i,k = 1,\ldots,M$ and $\mathbf{\Sigma}_{R,i}$ is a diagonal matrix whose elements are $\{e^{j\Delta_{R,i1}},\ldots,e^{j\Delta_{R,i(i-1)}},1,e^{j\Delta_{R,i(i+1)}},\ldots,e^{j\Delta_{R,iM}}\}$. Receiver i will use $\mathbf{y}_i$ to perform equalization of the i-th symbol.

2.2.2 *Cooperation at the transmitter*

The information about the channel at the transmitter is assumed to be obtained from the receiver using pilot symbols transmitted via feedback channel. This means that at the transmitter the information about the channel will correspond to the matrix $\mathbf{H}_R$ which, analog to $\mathbf{H}_T$, is given by

$$\mathbf{H}_R = \begin{bmatrix} h_{11}^{(f_0+\Delta_{f_{R,1}})} e^{-j\varphi_{11}} & \cdots & h_{1N}^{(f_0+\Delta_{f_{R,N}})} e^{-j\varphi_{1N}} \\ \vdots & \ddots & \vdots \\ h_{M1}^{(f_0+\Delta_{f_{R,1}})} e^{-j\varphi_{M1}} & \cdots & h_{MN}^{(f_0+\Delta_{f_{R,N}})} e^{-j\varphi_{MN}} \end{bmatrix} . \tag{11}$$

The received signal $\mathbf{y}$ depends on $\mathbf{H}_T$ while transmitters have access to $\mathbf{H}_R$. The coefficients of these two matrices are different as they are the response of the channel to different frequencies and hence, reciprocity of the channel is lost. This lost of reciprocity will translate into an error on the precoding / pre-equalization procedure. Analog to the receiver cooperation case, we do not have perfect information about the transmit symbols from other users. A noise term is introduced by the cooperation channel and constellation points will experience a rotation due to phase errors between cooperating nodes. This will cause that some symbols may not be decoded correctly which will translate into an erroneous information about symbols to be transmitted. The information about the transmit signals at the j-th transmitter $\mathbf{x}'_j$ would be

$$\mathbf{x}'_j = \lceil \mathbf{\Sigma}_{T,j} \cdot \mathbf{x}_j \rfloor \ , \tag{12}$$

where $\lceil\cdot\rfloor$ represents the hard-decision procedure that takes place at the transmitters, $\mathbf{\Sigma}_{T,j}$ is a diagonal matrix with elements on the main diagonal equal to $\{e^{j\Delta_{T,j1}},\ldots,e^{j\Delta_{T,j(j-1)}},1,e^{j\Delta_{T,j(j+1)}},\ldots,e^{j\Delta_{T,jN}}\}$ and $\Delta_{T,jm} = 2\pi(\Delta_{f_{T,j}} - \Delta_{f_{T,m}})$, $j,m = 1,\ldots,N$, are the phase shifts introduced due to the offsets between transmitters. At the j-th transmitter, $\mathbf{x}'_j$ and $\mathbf{H_R}$ will be provided to the precoder instead of the correct information $\mathbf{x}$ and $\mathbf{H}_T$.

3. SIMULATIONS

Apart from the noise component of variance $1/\sqrt{G}$ introduced by the cooperation channel, frequency offsets between carriers of different users introduce additional distortion that affects the system performance. In the case of receiver cooperation, this effect produces a rotation on the received symbols from other users while still keeping the correct information about the channel. On the other hand, when cooperation is performed at the transmitters, reciprocity of the channel is lost and the information available about the channel will be erroneous. This mismatch will depend on how the channel is correlated. If it is highly uncorrelated, only a small deviation will cause a strong degradation. Additional to this, phase errors will rotate the transmit symbols received from other users. These two effects are expected to severe degrade performance.

For the simulations we will consider a scenario where only two transmitters and two receivers operate, $N = M = 2$. For simplicity, we will assume that a total transmit power of P_{coop} is available for the cooperation channel and that P_{Tx} corresponds to the transmission one, so no optimization is carried out regarding the total transmit power constraint $P = P_{coop} + P_{Tx}$. It becomes clear that system performance will depend on how the channel is correlated when cooperating at the transmitters. Due to this reason, we will use the Coherence Bandwidth B_C to characterize frequency offsets. That is, we will assume $\Delta_{f_{T,j}}, \Delta_{f_{R,i}} \sim \mathscr{U}(-\alpha\frac{B_C}{2}, \alpha\frac{B_C}{2})$,$\forall i,j$, where $0 \leq \alpha$ is a parameter to quantify frequency deviations in terms of B_C.

The channel will consists on two rays with a number of zeros between them in the form

$$h(n) = h_0\delta(n) + h_K\delta(n-K), \tag{13}$$

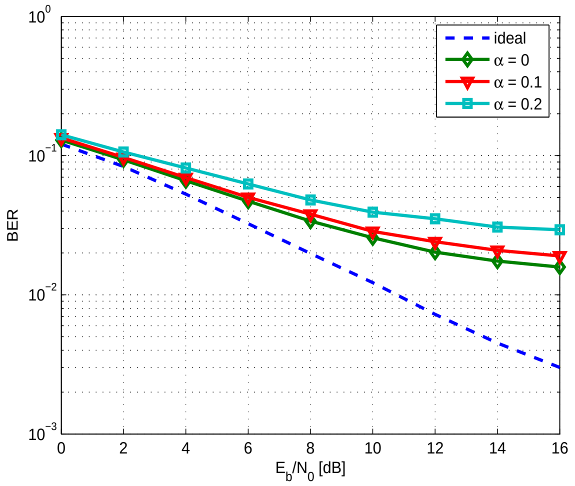

Figure 2: V-BLAST degradation for different values of the parameter α and $G = 10$ dB.

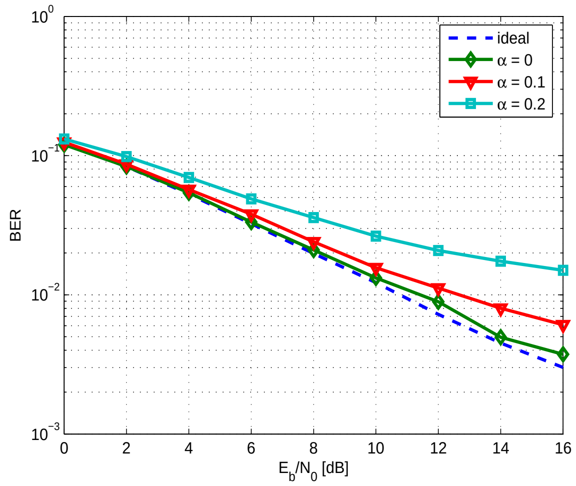

Figure 3: V-BLAST degradation for different values of the parameter α and $G = 20$ dB.

where $h_0, h_K \sim \mathcal{N}_C(0, 1/2)$. The corresponding Power Delay Profile $P_h(n)$ of $h(n)$ is given by

$$P_h(n) = \frac{1}{2}\left(\delta(n) + \delta(n-K)\right). \tag{14}$$

Time dispersion T_h of the channel can be computed as

$$T_h = \sqrt{\frac{\sum_n (n-D)^2 P_h(n)}{\sum_n P_h(n)}}, \quad D = \frac{\sum_n n P_h(n)}{\sum_n P_h(n)}. \tag{15}$$

And the Coherence Bandwidth B_C can be then obtained as the inverse of the time dispersion such that

$$B_C \approx \frac{1}{T_h}. \tag{16}$$

For our particular channel of two rays delayed in time, B_C takes the value

$$B_C = \frac{2}{K}. \tag{17}$$

3.1 Simulation results

We will consider the case where $K = 4$ and will provide the simulations in terms of BER and depending on α and G. For all simulations QPSK have been employed for modulating the data. Two different techniques to mitigate the impact of the channel have been considered. V-BLAST is used as equalization technique for receiver cooperation and Tomlinson-Harashima precoding based on the V-BLAST idea has been selected as representative strategy when cooperating at the transmitter. In all simulations the ideal curves $(\alpha = 0, G \to \infty)$, labeled as "ideal", have been included for the ease of comparison.

3.1.1 Receiver cooperation

Simulation results for the receiver cooperation case are displayed in Figures 2 and 3 for values of $G = 10$ dB and $G = 20$ dB, respectively. We can observe that for $G = 10$ dB, noise introduced by the cooperative transmission causes a significant performance degradation. This is due to the fact that no decision can be made when cooperating between receiver nodes and hence, an additional noise component is present. On the other hand, error phases cause a rotation of the received signals which produces an increase in BER. It is important to note that degradation will be higher for modulation schemes of higher order as constellation points are closer.

3.1.2 Transmitter cooperation

For the transmitter cooperation case, simulation results depending on α are presented in Figures 4 and 5 for the cases where $G = 10$ dB and $G = 20$ dB, respectively. It can be observed that these two simulations coincide although they are realizations to different values of the cooperation channel's gain G. This means that the decision procedure that takes place at the transmitter provides a very few errors on the cooperation channel for these values of G and small values of αB_C. Phase errors also cause a rotation on the received symbols through cooperation channel but if these values of αB_C are small, this rotation is negligible and has no effect on the decision of the received symbols from other transmitters. However, having a look at Figures 5 and 4, we can see that for phase offsets uniformly distributed in $[-0.05B_C, 0.05B_C]$, the system degrades significantly. This means that mismatch between $\mathbf{H_R}$ and $\mathbf{H}_T$ dominates BER performance as symbols are precoded using erroneous CSI.

4. CONCLUSIONS

A performance evaluation of two realistic cooperation strategies under non-ideal conditions of carrier synchronization has been studied. V-BLAST equalization has been considered for receiver cooperation while Tomlinson-Harashima based on the V-BLAST idea has been selected as representative for transmitter cooperation. Simulation results show that the effect of the cooperation channel gain G has a greater impact when cooperation is performed at the receiver than cooperating at the transmitter. The reason for that is that a decision procedure can be performed at the transmitter and

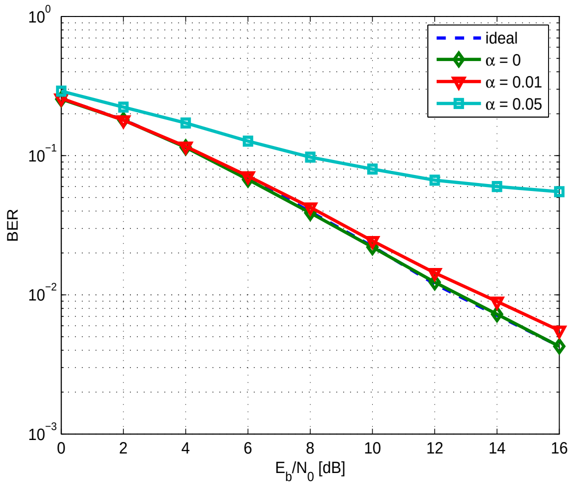

Figure 4: THP-VB degradation for different values of the parameter α and $G = 10$ dB.

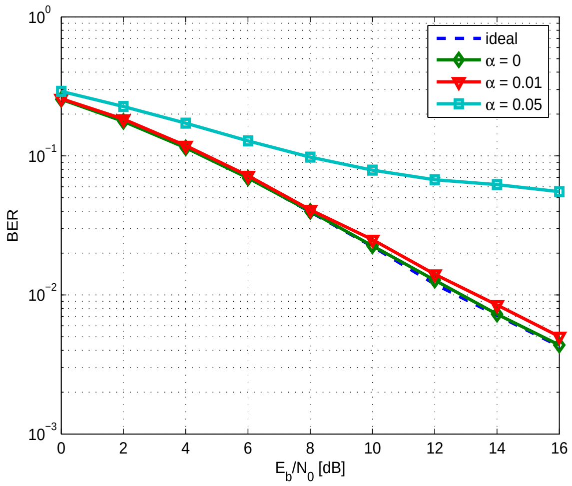

Figure 5: THP-VB degradation for different values of the parameter α and $G = 20$ dB.

very few errors occur through the cooperation channel. As gain G is expected to be large, it will not have a significant impact on performance.
The effect of carrier mismatch has, however, much more impact on performance when cooperating at the transmitter than at the receiver. Frequency offsets of only a small fraction of B_C at the transmitter cause a severe degradation in performance. The reason is that at the receiver we only have the effect of a phase shift due to frequency offsets between nodes of the cluster while in the transmitter case, an additional error is present as we are precoding with channel coefficients that correspond to a different frequency response.

Based on the results, we can say that receiver cooperation appears to be a more robust approach under non-ideal conditions of synchronization than transmitter cooperation. Future work on the topic could include some constraints and optimizations related to the total available bandwidth and / or power in order to get a more robust model and be able to establish a criterion when selecting the appropriate cooperating strategy.

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