TIME-FREQUENCY BASED ROBUST OFDM CHANNEL EQUALIZATION

Aydın Akan[1], *Erol Önen*[1], *and Luis F. Chaparro*[2]

[1] Department of Electrical and Electronics Eng., Istanbul University
Avcilar, 34320, Istanbul, TURKEY
phone: + (90) 212 473 7070, fax: + (90) 212 473 7180, email: {*akan,eeerol*}@*istanbul.edu.tr*
[2] Department of Electrical and Computer Eng., University of Pittsburgh
348 Benedum Hall, Pittsburgh, 15261 PA, USA, email: *chaparro@ee.pitt.edu*

ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) has become the standard method in the next generation mobile communication systems because of its advantages over single carrier modulation schemes on harsh channel environments. However, inter-carrier interference due to Doppler frequency shifts, and multi-path fading severely degrades the performance of OFDM systems. Estimation of channel parameters is required to use coherent receivers. In this paper, we present a computationally efficient, time-frequency (TF) based channel estimation method for OFDM channels whose frequency response may be changing over a single transmit symbol. The Discrete Evolutionary Transform is used to obtain a complete model of a multi-path, fading and frequency selective channel. A TF receiver is designed to detect the sent data by using estimated channel parameters. Performance of the proposed method is tested on different levels of channel noise, and Doppler frequency shifts.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is considered an effective method for broadband wireless communications because of its great immunity to fast fading channels and inter-symbol interference (ISI). It has been adopted in several wireless standards such as digital audio broadcasting (DAB), digital video broadcasting (DVB-T), the wireless local area network (W-LAN) standard; IEEE 802.11a, and the metropolitan area network (W-MAN) standard; IEEE 802.16a [1, 2]. OFDM partitions the entire bandwidth into parallel subchannels by dividing the transmit data bitstream into parallel, low bit rate data streams to modulate the subcarriers of those subchannels. However, inter-carrier interference (ICI) due to Doppler shifts, phase offset, local oscillator frequency shifts, and multi-path fading severely degrades the performance of multi-carrier communication systems [1]. For fast-varying channels, especially in mobile systems, large fluctuations of the channel parameters are expected. Estimation of the channel parameters is required to employ coherent receivers. Most of the channel estimation methods assume that the communication channel is either completely time–invariant or it remains time-invariant over an OFDM symbol, which is not valid for fast varying environments [3]. A complete time-varying model of the channel can be obtained by employing TF representation methods. A new time–varying channel estimation method based on the Discrete Evolutionary Transform (DET) is recently presented for OFDM systems [4]. The DET provides a TF representation of the received signal by means of which the time-varying frequency response of the multi-path, fading channel with Doppler shifts can be estimated. In this work, we improve above method and present a sub-sampled technique where the time-varying channel information for each OFDM symbol can be estimated by using channel response to known pilot symbols.

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2. WIRELESS CHANNEL MODEL

In wireless communications, the multi-path, fading channel with Doppler frequency shifts may be modeled as a linear time-varying system with the following discrete-time impulse response [5, 6]

$$h(m,\ell) = \sum_{i=0}^{L-1} \alpha_i \, e^{j\psi_i m} \, \delta(\ell - N_i) \tag{1}$$

where L is the total number of transmission paths, ψ_i represents the Doppler frequency, α_i is the relative attenuation, and N_i is the time delay caused by path i. The Doppler frequency shift ψ_i, on the carrier frequency ω_c, is caused by an object with radial velocity υ and can be approximated by $\psi_i \cong \frac{\upsilon}{c}\,\omega_c$ where c is the speed of light in the transmission medium [6]. In wireless mobile communication systems, with high carrier frequencies, Doppler shifts become significant and have to be taken into consideration. Time-varying channel parameters can be estimated in the TF domain by means of the so called spreading function which is related to the time-varying frequency response and the bi-frequency function of the channel. Time-varying transfer function of this linear channel is calculated by taking the discrete Fourier transform (DFT) of the impulse response with respect to ℓ, i.e.,

$$H(m,\omega_k) = \sum_{i=0}^{L-1} \alpha_i \, e^{j\psi_i m} \, e^{-j\omega_k N_i} \tag{2}$$

where $\omega_k = \frac{2\pi}{K}k, k = 0,1,\cdots,K-1$. The bi-frequency function of the channel $B(\Omega_s,\omega_k)$, is found by computing the DFT of $H(m,\omega_k)$ with respect to m.

$$B(\Omega_s,\omega_k) = \sum_{i=0}^{L-1} \alpha_i e^{-j\omega_k N_i} \delta(\Omega_s - \psi_i). \tag{3}$$

and $\Omega_s = \frac{2\pi}{K}s, s = 0,1,\cdots,K-1$. Furthermore, the spreading function of the channel is obtained by calculating the DFT of

$h(m,\ell)$ with respect to m, or by taking the inverse DFT of $B(\Omega_s,\omega_k)$ with respect to ω_k;

$$S(\Omega_s,\ell) = \sum_{i=0}^{L-1} \alpha_i \delta(\Omega_s - \psi_i)\delta(\ell - N_i) \tag{4}$$

which displays peaks located at the TF positions determined by the delays and the corresponding Doppler frequencies, and with α_i as their amplitudes [6]. If we extract this information from the received signal, we will be able to equalize for the channel and estimate the transmitted data symbol.

3. OFDM SYSTEM MODEL

In an OFDM communication system, the available bandwidth B_d is divided into K subchannels. The input data is also divided into K-bit parallel bit streams, and then mapped onto some transmit symbols $X_{n,k}$ drawn from an arbitrary constellation points where n is the time index, and $k = 0,1,\cdots,K-1$, denotes the frequency or subcarrier index. We then insert some pilot symbols, $p_{n,k}$ at some pilot positions (n',k'), known to the receiver: $(n',k') \in \mathscr{P} = \{(n',k')|n' \in \mathscr{X}, k' = id + (n' \text{mod}(d)), i \in [0,P-1]\}$ where P is the number of pilots, and the integer $d = K/P$ is the distance between adjacent pilots in an OFDM symbol [7].

The n^{th} OFDM symbol $s_n(m)$ is obtained by taking the inverse DFT and then adding a cyclic prefix (CP) of length L_{CP} where L_{CP} is chosen such that $T \le L_{CP}+1$, and T is the time-support of the channel impulse response. This is done to mitigate the effects of intersymbol interference (ISI) caused by the channel time spread [1, 2].

$$s_n(m) = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} X_{n,k} e^{j\omega_k m} \tag{5}$$

$m = -L_{CP}, -L_{CP}+1,\cdots,0,\cdots,K-1$ where again $\omega_k = \frac{2\pi}{K}k$, and each OFDM symbol has $N = K + L_{CP}$ length. The channel output suffers from multi-path propagation, fading and Doppler frequency shifts:

$$\begin{aligned} y_n(m) &= \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} X_{n,k} \sum_{i=0}^{L-1} \alpha_i\, e^{j\psi_i m}\, e^{j\omega_k(m-N_i)} \\ &= \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} H_n(m,\omega_k)\, e^{j\omega_k m} X_{n,k} \end{aligned} \tag{6}$$

The transmit signal is also corrupted by additive white Gaussian noise $\eta(m)$ over the channel. The received signal for the n^{th} frame can then be written as $r_n(m) = y_n(m) + \eta_n(m)$. At the receiver the CP is discarded and the signal is demodulated using a K-point DFT:

$$\begin{aligned} R_{n,k} &= \frac{1}{K} \sum_{s=0}^{K-1} X_{n,s} \sum_{i=0}^{L-1} \alpha_i\, e^{-j\omega_s N_i} \\ &\times \sum_{m=0}^{K-1} e^{j\psi_i m}\, e^{j(\omega_s-\omega_k)m} + Z_{n,k}. \end{aligned} \tag{7}$$

If the Doppler effects in all the channel paths are negligible, $\psi_i = 0,\ \forall i$, then the channel can be assumed time–invariant within one OFDM symbol. In that case, channel output above becomes,

$$\begin{aligned} R_{n,k} &= X_{n,k} \sum_{i=0}^{L-1} \alpha_i\, e^{-j\omega_k N_i} + Z_{n,k} \\ &= X_{n,k}\, H_{n,k} + Z_{n,k} \end{aligned} \tag{8}$$

where $H_{n,k}$ is the channel frequency response, and $Z_{n,k}$ is the Fourier transform of the channel noise for the n^{th} OFDM frame. By estimating the channel frequency response coefficients $H_{n,k}$, data symbols, $X_{n,k}$, can be recovered by a simple equalizer, $\hat{X}_{n,k} = R_{n,k}/H_{n,k}$. However, if there are large Doppler frequency shifts in the channel, then the above time–invariance assumption is no longer valid. Here we consider a completely time–varying channel that might change during an OFDM symbol and approach the problem from a TF point of view [6, 8].

4. TIME-VARYING CHANNEL ESTIMATION FOR OFDM SYSTEMS

In the following we briefly explain the Discrete Evolutionary Transform (DET) as a tool for the TF representation of wireless channel output.

4.1 Time-Frequency Analysis by DET

A non-stationary signal, $x(n), 0 \le n \le N-1$, may be represented in terms of a time-varying kernel $X(n,\omega_k)$ or its corresponding bi-frequency kernel $X(\Omega_s,\omega_k)$. The TF discrete evolutionary representation of $x(n)$ is given by [9],

$$x(n) = \sum_{k=0}^{K-1} X(n,\omega_k) e^{j\omega_k n}, \tag{9}$$

where $\omega_k = 2\pi k/K$, K is the number of frequency samples, and $X(n,\omega_k)$ is the evolutionary kernel. The discrete evolutionary transformation (DET) is obtained by expressing the kernel $X(n,\omega_k)$ in terms of the signal. This is done by using conventional signal representations [9]. Thus, for the representation in (9), the DET that provides the evolutionary kernel $X(n,\omega_k)$, $0 \le k \le K-1$, is given by

$$X(n,\omega_k) = \sum_{\ell=0}^{N-1} x(\ell) \mathrm{w}_k(n,\ell) e^{-j\omega_k \ell}, \tag{10}$$

where $\mathrm{w}_k(n,\ell)$ is, in general, a time and frequency dependent window. Details of how the windows can be obtained using signal decompositions such as Gabor and Malvar expansion are given in [9]. However, for the representation of multipath wireless channel output, signal-dependent windows that are adapted to the Doppler frequencies of the channel are proposed [4, 8].

4.2 OFDM Channel Estimation

We will now consider the computation of the spreading function by means of the evolutionary transformation of the received signal. The output of the channel, after discarding the cyclic prefix, for the n^{th} OFDM symbol is given in equation (6). Now calculating the DET of $y_n(m)$, we get

$$y_n(m) = \sum_{k=0}^{K-1} Y_n(m,\omega_k) e^{j\omega_k m}. \tag{11}$$

The above equation can also be given in a matrix form as, $\mathbf{y} = \mathbf{A}\mathbf{x}$ where

$$\begin{aligned} \mathbf{y} &= [y_n(0), y_n(1), \ldots, y_n(K-1)]^T, \\ \mathbf{x} &= [X_{n,0}, X_{n,1}, \ldots, X_{n,K-1}]^T, \\ \mathbf{A} &= \left[a_{m,k}\right]_{K\times K}, \quad a_{m,k} = \frac{H_n(m,\omega_k)\, e^{j\omega_k m}}{\sqrt{K}}. \end{aligned}$$

If the time-varying frequency response of the channel $H_n(m,\omega_k)$ is known, then $X_{n,k}$ may be estimated by

$$\hat{\mathbf{x}} = \mathbf{A}^{-1}\mathbf{y}. \quad (12)$$

A TF procedure to estimate $H_n(m,\omega_k)$ is explained in the following. Comparing the representations of $y_n(m)$ in (6) and (11), we require that the kernel is,

$$Y_n(m,\omega_k) = \frac{1}{\sqrt{K}} \sum_{i=0}^{L-1} \alpha_i\, e^{j\psi_i m}\, e^{-j\omega_k N_i} X_{n,k}. \quad (13)$$

Finally, the time-varying channel frequency response for n^{th} OFDM symbol can be obtained as,

$$H_n(m,\omega_k) = \frac{\sqrt{K}\, Y_n(m,\omega_k)}{X_{n,k}}. \quad (14)$$

Calculation of $Y_n(m,\omega_k)$ in such a way that it satisfies equation (13) is achieved by using signal-dependent windows that are adapted to the Doppler frequencies in [8]. According to (14), we need the input data symbols $X_{n,k}$ to estimate the channel frequency response. In [4], a two step solution is proposed: *i)* estimate the frequency response $H_{n,k}$ using a pilot aided channel estimation method [2], *ii)* use it to get a preliminary estimate of $X_{n,k}$. Then use the detected data for the estimation of $H(m,\omega_k)$, and the spreading function $S(\Omega_s,\ell)$ via (14).

Here we present an improved method by eliminating the above intermediate step where preliminary channel coefficients $H_{n,k}$ are estimated. Equation (14) can be given in a matrix form,

$$\mathbf{H} = \sqrt{K}\,\mathbf{Y}\,\mathbf{X}^{-1}, \quad (15)$$

where

$$\begin{aligned} \mathbf{H} &\triangleq \left[h_{m,k}\right]_{KxK}, \quad h_{m,k} = H_n(m,\omega_k) \\ \mathbf{Y} &\triangleq \left[y_{m,k}\right]_{KxK}, \quad y_{m,k} = Y_n(m,\omega_k) \\ \mathbf{X} &\triangleq \mathbf{I}\,\mathbf{x}, \end{aligned} \quad (16)$$

where $\mathbf{I}$ denotes a $K \times K$ identity matrix. Above relation is also valid at the preassigned pilot positions $k = k'$;

$$H'_n(m,\omega_p) = H_n(m,\omega_{k'}) = \frac{\sqrt{K}\, Y_n(m,\omega_{k'})}{X_{n,k'}} \quad (17)$$

where $p = 1, 2\cdots, P$ and $H'_n(m,\omega_p)$ is a decimated version of the $H_n(m,\omega_k)$. Note that P is again the number of pilots, and $d = K/P$ is the distance between adjacent pilots. Taking the inverse DFT of $H'_n(m,\omega_p)$ with respect to ω_p and DFT with respect to m, we obtain the sub-sampled spreading function $S'(\Omega_s,\ell)$,

$$S'(\Omega_s,\ell) = \frac{1}{d} \sum_{i=0}^{L-1} \alpha_i \delta(\Omega_s - \psi_i)\, \delta(\frac{\ell - N_i}{d}). \quad (18)$$

By comparing $S'(\Omega_s,\ell)$ and $S(\Omega_s,\ell)$, we observe that the channel parameters α_i, ψ_i, and N_i calculated from $S(\Omega_s,\ell)$ in [4], can also be estimated from the sub-sampled spreading function $S'(\Omega_s,\ell)$. Hence, the evolutionary kernel $Y_n(m,\omega_k)$ can be calculated directly from $y_n(m)$ and channel parameters can be obtained according to (17) and (18).

4.3 Time-Frequency Receiver

After estimating the spreading function and the corresponding frequency response $H_n(m,\omega_k)$ of the channel, data symbols $X_{n,k}$ can be detected using a TF receiver given in (12). In fact, the channel output in equation (7) can be rewritten as

$$\begin{aligned} R_{n,k} &= \frac{1}{K} \sum_{s=0}^{K-1} \left\{ \sum_{m=0}^{K-1} H_n(m,\omega_s) e^{j(\omega_s - \omega_k)m} \right\} X_{n,s} + Z_{n,k} \\ &= \frac{1}{K} \sum_{s=0}^{K-1} B_n(\omega_k - \omega_s, \omega_s) X_{n,s} + Z_{n,k}. \end{aligned} \quad (19)$$

where $B_n(\Omega_s,\omega_k)$ is the bi-frequency function of the channel during n^{th} OFDM symbol, and above equation indicates a circular convolution with the data symbols. It is possible to write the above equation in a matrix form as

$$\mathbf{r} = \mathbf{B}\mathbf{x} + \mathbf{z} \quad (20)$$

where $\mathbf{B} = \left[b_{k,s}\right]_{K\times K}, b_{k,s} = B_n(\omega_k - \omega_s, \omega_s)$ is a $K \times K$ matrix and, $\mathbf{r}$, $\mathbf{x}$ and $\mathbf{z}$ are $K \times 1$ vectors defined by $\mathbf{r} = [R_{n,1}, R_{n,2}, \ldots, R_{n,K}]^T$, $\mathbf{x} = [X_{n,1}, X_{n,2}, \ldots, X_{n,K}]^T$, and $\mathbf{z} = [Z_{n,1}, Z_{n,2}, \ldots, Z_{n,K}]^T$ respectively. Finally, data symbols $X_{n,k}$ can be estimated by using a simple TF equalizer,

$$\hat{\mathbf{x}} = \mathbf{B}^{-1}\,\mathbf{r}$$

which is an extension of the LTI channel equalizer to the time-varying channel model given in (1).

5. SIMULATIONS

First, we show the estimated spreading function of a 3-path channel shown in Fig. 1 by using 16 pilot symbols in an OFDM frame. Then we test the bit error rate (BER) performance of the proposed method. The wireless channel is simulated randomly, i.e, the number of paths, $1 \le L \le 5$, the delays, $0 \le N_i \le L_{CP} + 1$ and the Doppler frequency shift $0 \le \psi_i \le \psi_{\max}$, $i = 0, 1, \cdots, L-1$ of each path are picked randomly. Input data is 4-PSK coded and modulated onto $K = 128$ sub-carriers, 8 or 16 of which are assigned to the pilot symbols. The OFDM symbol duration is chosen to be $T = 200\mu$s, and $T_{CP} = 50\mu$s. Frequency spacing between the sub-carriers is $F = 5k$Hz. Signal-to-Noise Ratio (SNR) of the channel noise is changed between 0 and 15dB, and the BER is calculated by four approaches: 1) No Channel Estimation, 2) Proposed Approach with 8 pilots, 3) with 16 pilots, and 4) Known Channel parameters. Fig. 2 shows the

BER versus SNR for $\psi_{max} = 500$Hz. Notice that the proposed sub-sampled method is able to track the channel parameters even with only 8 pilots. To compare the results, we show in Fig. 3 the performance for the same OFDM system using the two step method presented in [4]. Notice that, the new method outperforms the PSA channel estimation and the previous two-step estimation.

6. CONCLUSIONS

In this work, we present a time-varying modeling of the multi-path, fading OFDM channels with Doppler frequency shifts by means of discrete evolutionary transform of the channel output. This approach allows us to obtain a representation of the time-dependent channel transfer function from the noisy channel output. At the same time, using the estimated channel parameters, a better detection of the input data can be achieved.

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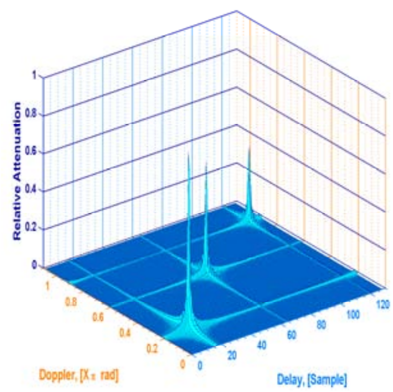

Figure 1: Spreading function estimated using 16 pilots.

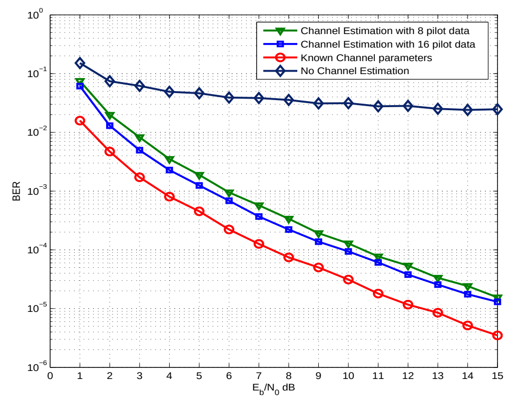

Figure 2: BER versus SNR for 4 different receivers.

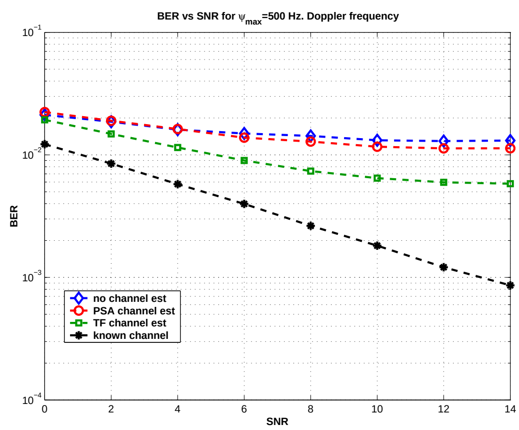

Figure 3: BER versus SNR for the two-step estimation in [4].