

## CLASSIFICATION OF QAM SIGNALS FOR MULTICARRIER SYSTEMS

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### ABSTRACT

*In this paper we highlight the benefit of automatic modulation classification (AMC) for quadrature amplitude modulation (QAM) constellations in the context of multicarrier transmission systems. Increasing computation power of multicarrier systems transceivers enables runtime adaptation of transmission parameters to cope with sudden changes of the noise conditions. AMC is a promising technique to verify that runtime adaptation was successful, and it can also be used in novel adaptation techniques. Using AMC, the modulation order for each subcarrier of the multicarrier system can be blindly detected, which reduces communication overhead during adaptation. In this work, we give an overview of existing AMC techniques, develop several novel low-complexity approaches and evaluate their performance.*

### 1. INTRODUCTION

The target application for our methods is process data communications (e.g. telecontrol and telemetry) using multicarrier communication systems. Here, we focus on wireline systems (discrete multitone (DMT)), since signal to noise ratios (SNR) for wireline systems are usually very good and channel parameters such as subcarrier gain and noise power can be easily estimated with great accuracy [1]. In process data communications, data rates are usually rather small. However great robustness and reliability of the systems is required for potentially very long transmission lines.

To guarantee reliable communication, transmitter and receiver must use the same transmission parameters (number of bits and transmit power per subcarrier). To this end, a training phase takes place before the actual data transmission starts. During this phase, the channel state is estimated and transmission parameters are optimized to reach some specific transmission constraints. The optimized parameters are then exchanged between receiver and transmitter.

If channel conditions change (usually due to an increase in noise power), it might be necessary to update transmission parameters. One way to restore reliable communication is to close the connection, retrain the system and establish communication a new connection with adapted transmission parameters. However, such a retraining is time-consuming and interrupts the data flow.

Another option is to use dynamic runtime adaptation such as proposed by ADSL (asymmetric digital subscriber line) standards. For example, some bits might be removed from subcarriers with poor SNR and re-assigned to subcarriers with better SNR. AMC can be used to verify that this redistribution took place as intended.

When noise changes render reliable communication impossible, runtime adaptation as described above is difficult.

The transceivers might then autonomously reduce the number of bits transmitted on some or all subcarriers and AMC can be used to detect such changes and update transmission parameters accordingly.

The main problem of using AMC in multicarrier systems is the possibly large number of subcarriers. Each subcarrier may use a different constellation and therefore, AMC must be performed for every subcarrier. Fast and low-complexity AMC methods are therefore mandatory.

### 1.1 Previous Work and New Contributions

AMC has been an active research area for many years. Maximum likelihood (ML) methods [2] have been shown to achieve optimum classification results. These methods, however, are computationally very expensive. Various techniques have been proposed to reduce the computational complexity while achieving close-to-optimum performance. Approaches include the use of higher order statistics (HOS) such as cumulants [3] and moments [4], feature extraction and artificial neural network (ANN) classification [5], fuzzy logic [6], and statistical sampling methods (e.g. with Monte Carlo Markov chain (MCMC) techniques [7] or Gibbs sampling [8]).

It is important to note that many previous publications only considered signal constellations that are relatively distinct from each other (either in terms of average power or constellation shape). Many of the above methods are not applicable when the signal constellations to be classified are very similar.

In DMT and OFDM systems, it is common to use square QAM constellations. In the remainder of this paper, we propose several low-complexity methods to distinguish between QAM constellations transmitting 2, 4, and 6 bits, respectively. By combining two methods, results can be further improved.

### 2. SYSTEM MODEL AND SIMULATION

The signal alphabet of a QAM constellation is represented as a set of points in the complex plane. In this paper, we use square equispaced QAM constellations, such that the number of bits that are transmitted with a constellation is a multiple of two and all signal points have the same minimum distance  $d_{\min}$  from each other.

The three possible constellations we want to classify are 4-QAM, 16-QAM, and 64-QAM. We refer to these as the constellation candidates  $\omega_k$ , with  $k \in \{1, 2, 3\}$ . Each candidate constellation transmits  $b_k$  bits with  $b_1 = 2$ ,  $b_2 = 4$ , and  $b_3 = 6$ , and has  $N_k$  constellation points with  $N_1 = 4$ ,  $N_2 = 16$ , and  $N_3 = 64$ .

Without loss of generality we assume that all constellations are normalized to have unit variance (average energy of one). Then,  $d_{\min}^k$  can be given by

$$d_{\min}^k = \sqrt{\frac{3}{2(N_k - 1)}}. \quad (1)$$

Each noise-free sample at instant  $n$  of a constellation point  $s^k(n) \in \omega_k$  can be given as

$$s^k(n) = \frac{\alpha_n^k + j\beta_n^k}{d_{\min}^k/2} \quad (2)$$

where

$$\alpha_n^k, \beta_n^k \in \{-\sqrt{N_k} + 1, -\sqrt{N_k} + 3, \dots, \sqrt{N_k} - 1\}. \quad (3)$$

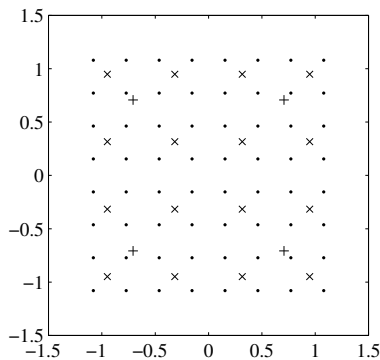


Figure 1: The three constellations, 4-QAM (+), 16-QAM (x), and 64-QAM (·).

Fig. 1 shows the three constellations plotted over each other.

Since the modulation of the transmit signal by the Fast Fourier transform (FFT) effectively separates all subcarriers from each other, we can perform AMC independently for each subcarrier. Accordingly, for each subcarrier we can consider a circular complex additive white Gaussian noise process  $g(n)$ . Its influence on noisy receive samples  $r(n)$  is given by

$$r(n) = x_n + jy_n = s^k(n) + g(n). \quad (4)$$

We assume the variance,  $\sigma^2$  of the two-dimensional noise process to be known. In practice, an estimate for  $\sigma^2$  is readily available, either by inspecting pure noise samples or by using pilot samples or decision feedback methods [9] during data transmission. Furthermore, we assume perfect symbol and carrier synchronization as well as zero phase offset, which is a reasonable assumption for DMT systems.

The task at hand is to estimate the correct constellation  $\omega_k$  from a vector  $\mathbf{r} = (r(1), r(2), \dots, r(N))$  of  $N$  received samples, all from the same constellation. Simulations are performed using 1000 iterations for each combination of  $\omega_k$ ,  $N$ , and SNR values.

### 3. SIGNAL SPACE CONCEPTS

In this section, we partition the complex signal plane into two sections. We consider an inner square section, centered at the origin, with a side length of  $d_s$  and the rest of the plane, outside the square. Depending on the SNR and the constellation size, we try to identify a value  $d_s$  that provides the best separation of the three constellation candidates in terms of the percentage of received signal points inside the square. We denote the probability of a signal point being received inside the square by  $p_s$ .

Results for the three constellation candidates for SNR of 10, and 20 dB are shown in Figs. 2 and 3.

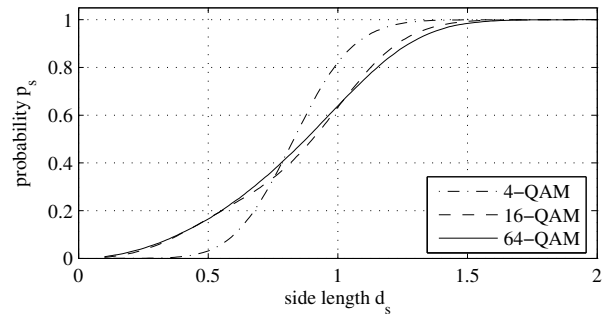


Figure 2: SNR = 10 dB.

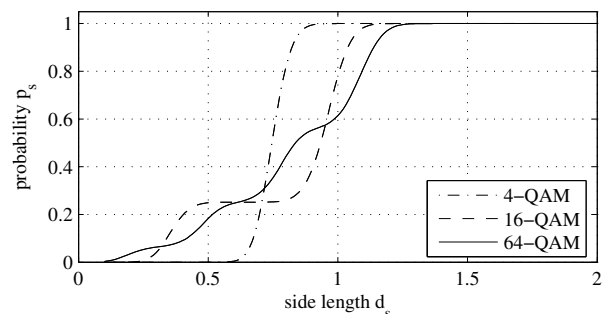


Figure 3: SNR = 20 dB.

For 0 dB SNR, a clear distinction is practically impossible. For 10 dB SNR, at least 4-QAM is easily identified. For 20 dB SNR, all three constellations can easily be distinguished. The best separation is achieved for side lengths of approximately  $d_s = 0.45$  and  $d_s = 0.8$ . Similarly, optimal values  $d_s$  and their corresponding  $p_s$  can be empirically derived for all SNR under consideration. For classification, actual values of  $p_s$  for all constellation candidates are compared with the empirical ones at values  $d_s$  providing optimal separation and the constellation candidate that shows the least deviation from empirical values is chosen.

In the following, we will refer to classification based on this method as SQUARE classification.

### 4. HOS METHODS

Higher order statistics (HOS) can be used to extract features for classification that are relatively robust to model mismatches [10].

Table 1: Noise-free HOS for QAM

Constellation	$C_{20}$	$C_{21}$	$C_{40}$	$C_{41}$	$C_{42}$
4-QAM	0	1	-1	0	-1
16-QAM	0	1	-0.6800	0	-0.6800
64-QAM	0	1	-0.6190	0	-0.6190

Second order moments for a complex-valued stationary random process  $y(n)$ , where  $y(n)$  represent the received QAM symbols, can be given as

$$C_{20} = E[y^2(n)] \text{ and } C_{21} = E[|y(n)|^2], \quad (5)$$

where  $E[\cdot]$  denotes expectation.

The value  $C_{21}$  is the average energy of the constellation (equal to one as explained in the previous section).

Sample estimates of second and fourth-order cumulants are given by [10]

$$\hat{C}_{20} = \frac{1}{N} \sum_{n=1}^N y^2(n) \text{ and } \hat{C}_{21} = \frac{1}{N} \sum_{n=1}^N |y(n)|^2$$

and

$$\begin{aligned} \hat{C}_{40} &= \frac{1}{N} \sum_{n=1}^N y^4(n) - 3\hat{C}_{20}^2 \\ \hat{C}_{41} &= \frac{1}{N} \sum_{n=1}^N y^3(n)y^*(n) - 3\hat{C}_{20}\hat{C}_{21} \\ \hat{C}_{42} &= \frac{1}{N} \sum_{n=1}^N |y(n)|^4 - |\hat{C}_{20}|^2 - 2\hat{C}_{21}^2 \end{aligned} \quad (6)$$

with complex conjugation operator  $\{\cdot\}^*$ .

The theoretical noise-free values for our three constellation candidates are given in table 1. For the modulation types considered in [10] (e.g. phase shift keying (PSK) and pulse amplitude modulation (PAM)) all values can be used for classification. The only useful parameters for our classification problem, though, are  $C_{40}$  and  $C_{42}$ . For noisy samples, reliable classification by means of these parameters is still difficult, because 16-QAM and 64-QAM can not be clearly separated.

We can apply a trick, however, and postulate that the transceivers pick one of the constellations and always rotate it by 45 degrees before transmission. For a constellation rotated in this way, the noise-free values of  $C_{40}$  and  $C_{42}$  have equal absolute value but different sign:  $C_{40}$  is positive. By inspecting Fig. 2, we can conclude that it is much more difficult to distinguish 16-QAM from 64-QAM than it is to distinguish both of them from 4-QAM. Accordingly, the largest performance improvement is realized when the 16-QAM constellation is rotated. The noise-free values for the rotated 16-QAM are then  $C_{40} = 0.6800$  and  $C_{42} = -0.6800$ .

For classification with noisy samples, it is sufficient to compare  $C_{40}$  and  $C_{42}$  and check for a sign difference without considering the specific values. With this method, the 16-QAM can be reliably classified even in poor noise conditions.

Fig. 4 shows the probability of correct classification  $P_{cc}$  of this method for various SNR values and sample sizes  $N$ .

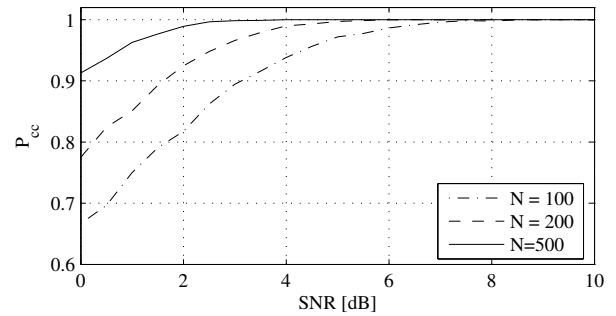


Figure 4: Performance of 16-QAM classifier.

Perfect recognition of 16-QAM is achieved for SNR values above 8 dB even with only  $N=100$  samples. Simulations have shown that when 4-QAM and 64-QAM are transmitted, 16-QAM is never erroneously classified. Thus, the estimator is very robust.

## 5. OPTIMAL CLASSIFICATION WITH ML CRITERION

The optimal classification rule is given by the maximum likelihood criterion.

We assume that all constellation points are chosen from the same constellation which is one of the three candidate constellations. As suggested in the previous section, we rotate the 16-QAM constellation by 45 degrees.

Each constellation candidate has a characteristic probability density function (PDF)  $p(x, y)$ , that can be given for points  $r(n) = x_n + jy_n$  as

$$p(x_n, y_n) = \frac{1}{N_k} \sum_{m=1}^{N_k} \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(x_n - x_m)^2 + (y_n - y_m)^2}{2\sigma^2}\right) \quad (7)$$

where  $\sigma^2$  is the one-dimensional noise variance of the additive Gaussian noise process  $g(n)$  and  $x_m$  and  $y_m$  are the coordinates of all  $N_k$  noise-free constellation points  $s^k$  from (2).

Given a vector  $\mathbf{r} = (r(1), \dots, r(N))$  with  $N$  received samples from the same constellation, the ML classifier can be described as follows [11]:

$$\text{Assign } \mathbf{r} \text{ to } \omega_i \text{ if } l(\mathbf{r}|\omega_i) \geq l(\mathbf{r}|\omega_j), \forall j \in \{1, 2, 3\} \quad (8)$$

where

$$l(\mathbf{r}|\omega_j) = \sum_{n=1}^N \ln(p(x_n, y_n)). \quad (9)$$

Note that generally, the implementation of the ML classifier is not practical since it requires extensive computations. Even if lookup tables are provided, the memory overhead for storing them is large.

Fig. 5 shows the simulation results for the combined classification performance over all constellation candidates and sample sizes of  $N=100$ ,  $N=200$ , and  $N=500$ .

Even for  $N=100$ , perfect classification is reached for SNR above 8 dB.

Fig. 6 shows the result when not rotating the 16-QAM constellation. It is evident that rotating does indeed greatly improve the overall classification performance.

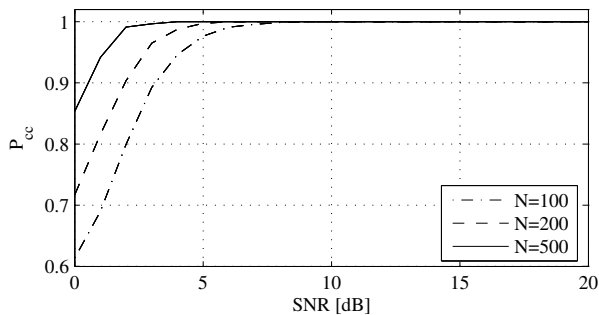


Figure 5: Averaged performance of ML classifier for various sample sizes.

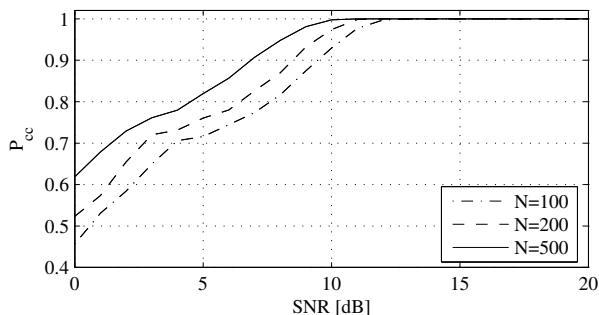


Figure 6: Averaged performance without rotation.

### 6. ML BASED PARTITIONING OF SIGNAL PLANE

We can make use of the above results for ML classification and incorporate them into a partitioning of the signal plane that assigns each point in the plane to the corresponding constellation candidate according to the ML criterion.

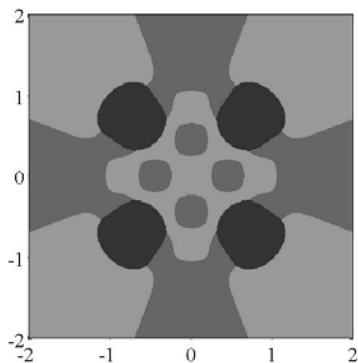


Figure 7: Signal plane partitioning for SNR = 10 dB.

In Fig. 7, an example is given for the three constellations and an SNR of 10 dB. Each constellation candidate dominates certain parts of the plane. Darker gray indicates a smaller constellation. Fig. 8 shows the partitioning for an SNR of 15 dB and shows how the partitioning changes with SNR.

All received points are assigned to the constellation can-

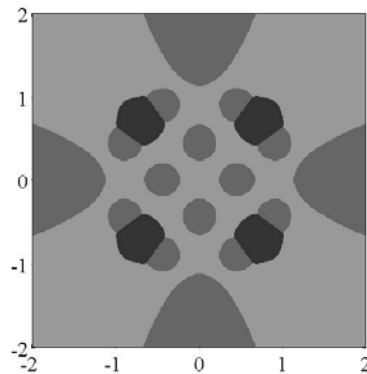


Figure 8: Signal plane partitioning for SNR = 15 dB.

didate in whose region they fall. A majority vote then leads to the final classification.

Formally, we divide the signal plane into three regions  $R_k$  and observe the number of samples that fall into each of those regions. The classification rule for the received vector  $\mathbf{r}$  is then

$$\text{Assign } \mathbf{r} \text{ to } \omega_i \text{ if } c(\mathbf{r}|\omega_i) \geq c(\mathbf{r}|\omega_j), \forall j \in \{1, 2, 3\} \quad (10)$$

with

$$c(\mathbf{r}|\omega_j) = |\{r(n)|r(n) \in R_j\}|, \quad (11)$$

the number of samples that falls into region  $R_j$ . In the following, this method is referred to as COUNT classification.

Of course, the proposed signal space partitioning yields worse results than ML classification since decisions are made on a point-by-point basis. However, for good SNR (above 13 dB), performance is virtually identical to that of ML classification at a much lower complexity as shown by the simulation results in Fig. 9.

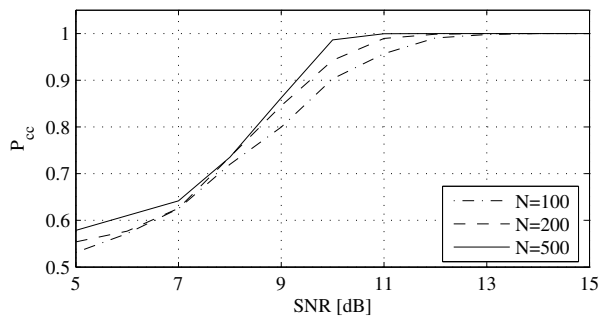


Figure 9: Averaged performance of COUNT classifier for various sample sizes.

To store the signal plane partitionings, which change with SNR, lookup tables can be used. A two-bit value can be used for each entry in a lookup table to identify the three possible constellations. The memory requirements then depend on the granularity or resolution of the partitioning. For example, we used a resolution of 0.01 for partitioning and considered values in the range  $[-2 \dots 2]$ . Making use of inherent symmetry, only 15 kB of memory are then required to store a complete lookup table for each particular SNR value.

## 7. TWO-STAGE METHODS

In the previous section, we have used the COUNT method to classify all three constellation candidates. While the performance was good for high SNR, we would like to have better performance for low SNR as well. To this end, we combine the HOS classification method from section 4 with the COUNT method in a two-stage process. We first try to estimate whether the sample vector belongs to the 16-QAM via the HOS method. If not, we apply COUNT to differentiate between 4-QAM and 64-QAM.

The result is shown in Fig. 10. The performance is considerably improved. This two-stage classification method, denoted HOS-COUNT in the following, achieves results impressively close to those achieved by the optimum ML method (see Fig. 5) at much lower computational complexity.

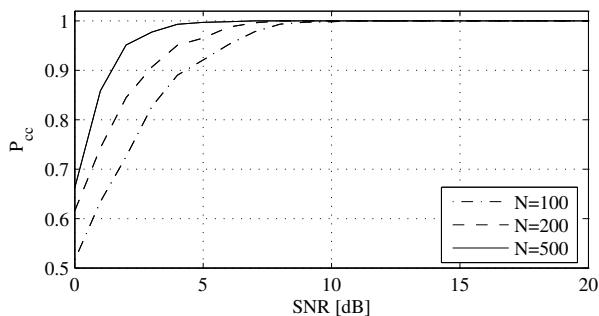


Figure 10: Averaged performance of two-stage classifier HOS-COUNT.

Finally, one can also combine the SQUARE method with HOS classification. In this case, 16-QAM is classified by the HOS classifier and the SQUARE classifier differentiates between 4-QAM and 64-QAM. Results are shown in Fig. 11.

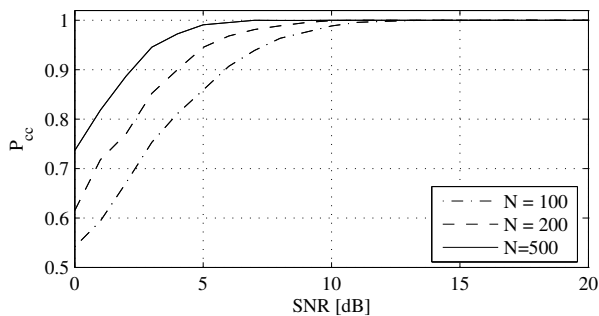


Figure 11: Averaged performance of two-stage classifier HOS-SQUARE.

While HOS-SQUARE has slightly poorer performance than HOS-COUNT, it has the additional advantage that no lookup tables are required.

## 8. CONCLUSION

We investigated the problem of AMC for three QAM constellation candidates. In the context of multicarrier systems, it is mandatory to use low-complexity methods for AMC. We have proposed and evaluated several low-complexity methods to solve the AMC problem. It could be shown that while each method has some shortcomings, combining them in a two-stage classification process achieves very good performance for the SNR ranges typically encountered in the considered transmission environments.

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