HIGHER ORDER DIRECTION FINDING WITH POLARIZATION DIVERSITY: THE PD-$2q$-MUSIC ALGORITHMS

Pascal Chevalier[(1)], *Anne Ferréol*[(1)], *Laurent Albera*[(2,3)] *and Gwenaël Birot*[(2,3)]

(1) Thales-Communications, EDS/SPM, 160 Bd Valmy – BP 82, Colombes, F-92704, France ;
(2) INSERM, U642, Rennes, F-35000, France ;
(3) Université de Rennes 1, LTSI, Rennes, F-35000, France ;
pascal.chevalier@fr.thalesgroup.com, anne.ferreol@fr.thalesgrooup.com
laurent.albera@univ-rennes1.fr, gwenael.birot@univ-rennes1.fr

ABSTRACT

Some $2q$-th ($q \geq 2$) order extensions of the MUSIC method, exploiting the information contained in the $2q$-th ($q \geq 2$) order statistics of the data and called $2q$-MUSIC methods, have been proposed recently for direction finding of non Gaussian signals. These methods are asymptotically robust to a Gaussian background noise whose spatial coherence is unknown and offer increasing resolution and robustness to modeling errors jointly with an increasing processing capacity as q increases. However, $2q$-MUSIC methods have been mainly developed for arrays with space diversity only and cannot put up with arrays of sensors diversely polarized. The purpose of this paper is to introduce, for arbitrary values of q ($q \geq 1$), three extensions of the $2q$-MUSIC methods able to put up with arrays having polarization diversity, which gives rise to the so-called PD-$2q$-MUSIC (Polarization Diversity $2q$-MUSIC) algorithms. These algorithms are shown to increase resolution, robustness to modeling errors and processing capacity of $2q$-MUSIC methods in the presence of diversely polarized sources from arrays with polarization diversity.

1. INTRODUCTION

Some $2q$-th ($q \geq 2$) order extensions of the MUSIC [13] method, exploiting the information contained in the $2q$-th ($q \geq 2$) order statistics of the data and called $2q$-MUSIC methods, have been proposed recently for direction finding (DF) of non Gaussian signals [4-5]. Note that the 4-MUSIC method proposed in [12] is a particular case of $2q$-MUSIC methods for $q = 2$. The $2q$-MUSIC methods with $q > 1$ are asymptotically robust to a Gaussian background noise whose spatial coherence is unknown and generate a virtual increase of both the number of sensors and the effective aperture of the considered array, introducing the higher order (HO) virtual array (VA) concept [2], which generalizes the fourth order (FO) one's introduced previously in [7] and [3]. A consequence of this property is that, despite of their higher variance [1], $2q$-MUSIC methods are shown in [5] to have resolution, robustness to modelling errors and processing capacity increasing with q. However, $2q$-MUSIC ($q \geq 2$) algorithms have been mainly developed for arrays of sensors with space diversity only (i.e. arrays with identical sensors at different locations), or for sources with the same polarization, and cannot put up with arrays of sensors having diverse polarizations. The exploitation of arrays with polarization diversity, possibly in addition to the space diversity, is very advantageous since for such arrays, multiple signals may be resolved on the basis of polarization as well as direction of arrival (DOA). However, most of methods which are currently available for DF from arrays with polarization diversity exploit only the information contained in the second order (SO) statistics of the observations, among which we find [8] [11]. HO methods of this kind are very scarce [10]. In order to increase the performance of the $2q$-MUSIC algorithms in the presence of sources having different polarizations, the purpose of this paper is to introduce, for arbitrary values of q ($q \geq 1$), three extensions of the $2q$-MUSIC methods able to put up with arrays having polarization diversity, which gives rise to the so-called PD-$2q$-MUSIC algorithms. For a given value of q, these algorithms are shown in this paper to increase the resolution, the robustness to modeling errors and the processing capacity of the $2q$-MUSIC methods in the presence of diversely polarized sources and from an array with polarization diversity.

2. HYPOTHESES AND DATA STATISTICS

2.1. Hypotheses

We consider an array of N narrow-band (NB) potentially different sensors and we call $\mathbf{x}(t)$ the vector of complex envelopes of the signals at the output of these sensors. Each sensor is assumed to receive the contribution of P zero-mean stationary NB sources, which may be statistically independent or not, corrupted by a noise. We assume that the P sources can be divided into G groups, with P_g sources in the group g, such that the sources in each group are assumed to be statistically dependent, but not perfectly coherent, while sources belonging to different groups are assumed to be statistically independent. Of course, P is the sum of all the P_g over all the groups. Under these assumptions, the observation vector can approximately be written as follows

$$x(t) \approx \sum_{i=1}^{P} m_i(t)\, a(\theta_i, \beta_i) + \eta(t) = \sum_{g=1}^{G} A_g\, m_g(t) + \eta(t) \quad (1)$$

where $\eta(t)$ is the noise vector, assumed zero-mean, stationary and Gaussian, $m_i(t)$ is the complex envelope of the source i, $\theta_i \triangleq (\theta_i, \varphi_i)$ where θ_i and φ_i are the azimuth and the elevation angles of source i, β_i is a (2 x 1) vector characterizing the state of polarization of source i and whose components will be defined hereafter, $a(\theta_i, \beta_i)$ is the steering vector of source i, A_g is the $(N \times P_g)$ matrix of the steering vectors of the sources belonging to group g, $m_g(t)$ is the associated (P_g x 1) vector with corresponding $m_i(t)$. In particular, in the absence of coupling between sensors, assuming a plane wave propagation, component n of vector $a(\theta_i, \beta_i)$, denoted $a_n(\theta_i, \beta_i)$, can be written, in the general case of an array with space and polarization diversity, as [6]

$$a_n(\theta_i, \beta_i) = f_n(\theta_i, \beta_i)$$
$$\exp\{j2\pi[x_n\cos(\theta_i)\cos(\varphi_i)+y_n\sin(\theta_i)\cos(\varphi_i)+z_n\sin(\varphi_i)]/\lambda\} \quad (2)$$

In (2), λ is the wavelength, (x_n, y_n, z_n) are the coordinates of sensor n of the array, $f_n(\theta_i, \beta_i)$ is a complex number corresponding to the response of sensor n to a unit electric field coming from the direction θ_i and having the state of polarization β_i [6]. Let β_{i1} and β_{i2} be two distinct polarizations for the source i (for example vertical and horizontal) and $a_1(\theta_i) \triangleq a(\theta_i, \beta_{i1})$ and $a_2(\theta_i) \triangleq a(\theta_i, \beta_{i2})$ be the corresponding steering vectors for DOA θ_i. We assume that the vectors $a_1(\theta)$ and $a_2(\theta)$ can be calculated analytically or measured by calibration whatever the value of θ. Considering an arbitrary polarization β_i for the source i, the complex electric field of the latter can be broken down into the sum of two complex fields, each arriving from the same direction, and having the polarizations β_{i1} and β_{i2} [6]. The steering vector $a(\theta_i, \beta_i)$ is then the weighted sum of the steering vectors $a_1(\theta_i)$ and $a_2(\theta_i)$ given by

$$a(\theta_i, \beta_i) = \beta_{i1} a_1(\theta_i) + \beta_{i2} a_2(\theta_i) \triangleq A_{12}(\theta_i)\, \beta_i \quad (3)$$

In (3), $A_{12}(\theta_i)$ is the $(N \times 2)$ matrix of the steering vectors $a_1(\theta_i)$ and $a_2(\theta_i)$, β_{i1} and β_{i2} are complex numbers such that $|\beta_{i1}|^2 + |\beta_{i2}|^2 = 1$ and β_i is the (2 x 1) vector with components β_{i1} and β_{i2}, which can be written, to within a phase term, as $\beta_i = [\cos\gamma_i,\ e^{j\phi_i}\sin\gamma_i]^T$, where γ_i and ϕ_i are two angles characterizing the polarization of source i and such that $(0 \leq \gamma_i \leq \pi/2, -\pi \leq \phi_i < \pi)$.

2.2. Data statistics

The $2q$-th ($q \geq 1$) order DF methods considered in this paper exploit the information contained in the $(N^q \times N^q)$ $2q$-th order covariance matrix, $C_{2q,x}$, whose entries are the $2q$-th order cumulants of the data, $\mathrm{Cum}[x_{i_1}(t),\dots, x_{i_q}(t), x_{i_{q+1}}(t)^*, \dots, x_{i_{2q}}(t)^*]$ ($1 \leq i_j \leq N$) ($1 \leq j \leq 2q$), where * corresponds to the complex conjugation. However, the previous entries can be arranged in $C_{2q,x}$ in different ways, indexed by an integer l such that $(0 \leq l \leq q)$, as it is explained in [2]

[5], and giving rise, under hypotheses of section 2.1, to the $C_{2q,x}(l)$ matrix given by [5]

$$C_{2q,x}(l) = \sum_{g=1}^{G} C_{2q,x_g}(l) + \eta_2\, V(l)\, \delta(q-1) \quad (4)$$

In (4), η_2 is the mean power of the noise per sensor, $V(l)$ is the $(N \times N)$ spatial coherence matrix of the noise for the arrangement indexed by l, such that $\mathrm{Tr}[V(l)] = N$, $\mathrm{Tr}[.]$ means Trace, $\delta(.)$ is the Kronecker symbol. The $(N^q \times N^q)$ matrix $C_{2q,x_g}(l)$ contains the $2q$-th order cumulants of $x_g(t)$ for the arrangement indexed by l, which can be written as

$$C_{2q,x_g}(l) \approx [A_g^{\otimes l} \otimes A_g^{*\otimes(q-l)}] C_{2q,m_g}(l)[A_g^{\otimes l} \otimes A_g^{*\otimes(q-l)}]^{\mathsf{H}} \quad (5)$$

In (5), $C_{2q,m_g}(l)$ is the $(P_g^q \times P_g^q)$ matrix of the $2q$-th order cumulants of $m_g(t)$ for the arrangement indexed by l, $^{\mathsf{H}}$ corresponds to the conjugate transposition, $\otimes$ is the Kronecker product and $A_g^{\otimes l}$ is the $(N^l \times P_g^l)$ matrix defined by $A_g^{\otimes l} \triangleq A_g \otimes A_g \otimes \dots \otimes A_g$ with a number of Kronecker product equal to $l - 1$. Note that it is shown in [2] and verified in this paper that the parameter l determines in particular the maximal processing power of PD-$2q$-MUSIC algorithms. In situations of practical interests, the $2q$-th order statistics of the data, $\mathrm{Cum}[x_{i_1}(t),\dots, x_{i_q}(t), x_{i_{q+1}}(t)^*, \dots, x_{i_{2q}}(t)^*]$, are not known a priori and have to be estimated from L samples of data, $x(k) \triangleq x(kT_e)$, $1 \leq k \leq L$, where T_e is the sample period, in a way that is completely described in [5] and which is not recalled here.

3. PD-2Q-MUSIC ALGORITHMS

3.1. Hypotheses

To develop PD-$2q$-MUSIC algorithms for the arrangement indexed by l, we need extra assumptions:

H1) $\forall\ 1 \leq g \leq G,\ P_g < N$

H2) $\forall\ 1 \leq g \leq G,\ A_g^{\otimes l} \otimes A_g^{*\otimes(q-l)}$ has full rank P_g^q

H3) $P(G, q) \triangleq \sum_{g=1}^{G} P_g^q < N^q$

H4) $\tilde{A}_{q,l} \triangleq [A_1^{\otimes l} \otimes A_1^{*\otimes(q-l)},\dots, A_G^{\otimes l} \otimes A_G^{*\otimes(q-l)}]$ has full rank $P(G, q)$

3.2. KP-PD-2q-MUSIC algorithm

Noting $r_{2q,m_g}(l)$ the rank of $C_{2q,m_g}(l)$ ($r_{2q,m_g}(l) \leq P_g^q$), we deduce from H1 and H2 that $C_{2q,x_g}(l)$ for $q > 1$ has also rank $r_{2q,m_g}(l)$. Hence, using H4 and for $q > 1$, matrix $C_{2q,x}(l)$ has a rank $r_{2q,x}(l)$ equal to the sum, over all the groups g, of $r_{2q,m_g}(l)$ and such that $r_{2q,x}(l) < N^q$ from H3. As matrix $C_{2q,x}(l)$ is Hermitian, but not positive definite, we deduce from the previous results that $C_{2q,x}(l)$ has $r_{2q,x}(l)$ non zero eigenvalues and $N^q - r_{2q,x}(l)$ zero eigenvalues for $q > 1$. The eigendecomposition of $C_{2q,x}(l)$, for $q > 1$, gives

$$C_{2q,x}(l)=U_{2q,s}(l)\Lambda_{2q,s}(l)U_{2q,s}(l)^{\mathsf{H}}+U_{2q,n}(l)\Lambda_{2q,n}(l)U_{2q,n}(l)^{\mathsf{H}}$$
$$(8)$$

where $\Lambda_{2q,s}(l)$ is the diagonal matrix of the non zero eigenvalues of $C_{2q,x}(l)$, $U_{2q,s}(l)$ is the unitary matrix of the associ-

ated eigenvectors, $\Lambda_{2q,n}(l)$ is the diagonal matrix of the zero eigenvalues of $C_{2q,x}(l)$ and $U_{2q,n}(l)$ is the unitary matrix of the associated eigenvectors. As $C_{2q,x}(l)$ is Hermitian, all the columns of $U_{2q,s}(l)$ are orthogonal to all the columns of $U_{2q,n}(l)$. Moreover, $\text{Span}\{U_{2q,s}(l)\} = \text{Span}\{\tilde{A}_{q,l}\}$ when the matrices $C_{2q,m_g}(l)$, $1 \leq g \leq G$, are full rank whereas $\text{Span}\{U_{2q,s}(l)\} \subset \text{Span}\{\tilde{A}_{q,l}\}$ otherwise. Defining $a_{q,l}(\theta, \beta) \triangleq a(\theta, \beta)^{\otimes l} \otimes a(\theta, \beta)^{*\otimes(q-l)}$ and noting $(\theta_{ig}, \beta_{ig})$ the DOA and polarization parameters of the ith source in the gth group, it can be easily verified that, in all cases, the vector $a_{q,l}(\theta_{ig}, \beta_{ig})$, always belongs to $\text{Span}\{U_{2q,s}(l)\}$. Consequently, all vectors $\{a_{q,l}(\theta_{ig}, \beta_{ig}), 1 \leq i \leq P_g , 1 \leq g \leq G\}$ are orthogonal to the columns of $U_{2q,n}(l)$ and are solutions of the following equation

$$a_{q,l}(\theta, \beta)^{\mathsf{H}} U_{2q,n}(l) U_{2q,n}(l)^{\mathsf{H}} a_{q,l}(\theta, \beta) = 0 \qquad (9)$$

Using (3) into (9), removing the redundancy of $\beta_{q,l} \triangleq [\beta^{\otimes l} \otimes \beta^{*\otimes(q-l)}]$ and normalizing the left hand side of (9) to obtain no minima in the absence of sources, the problem of sources DOA estimation by the PD-2q-MUSIC algorithm for the arrangement l then consists to find the P sets of parameters $(\hat{\theta}_i, \hat{\beta}_i) = (\hat{\theta}_i, \hat{\phi}_i, \hat{\gamma}_i, \hat{\phi}_i)$, $(1 \leq i \leq P)$, which are solution of the following equation

$$\frac{[\tilde{\beta}_l \otimes \tilde{\beta}_{q-l}^*]^{\mathsf{H}} Q_{q,l,1}(\theta)[\tilde{\beta}_l \otimes \tilde{\beta}_{q-l}^*]}{[\tilde{\beta}_l \otimes \tilde{\beta}_{q-l}^*]^{\mathsf{H}} Q_{q,l,2}(\theta)[\tilde{\beta}_l \otimes \tilde{\beta}_{q-l}^*]} = 0 \qquad (10)$$

In (10), the $((l+1)(q-l+1) \times (l+1)(q-l+1))$ $Q_{q,l,1}(\theta)$ and $Q_{q,l,2}(\theta)$ matrices are defined by

$$Q_{q,l,1}(\theta) \triangleq \qquad (11)$$
$$[B_l \otimes B_{q-l}]^{\mathsf{H}} A_{12,q,l}(\theta)^{\mathsf{H}} U_{2q,n}(l) U_{2q,n}(l)^{\mathsf{H}} A_{12,q,l}(\theta) [B_l \otimes B_{q-l}]$$

$$Q_{q,l,2}(\theta) \triangleq [B_l \otimes B_{q-l}]^{\mathsf{H}} A_{12,q,l}(\theta)^{\mathsf{H}} A_{12,q,l}(\theta)[B_l \otimes B_{q-l}] \quad (12)$$

where $A_{12,q,l}(\theta)$ is equal to $A_{12}(\theta)^{\otimes l} \otimes A_{12}(\theta)^{*\otimes(q-l)}$, $\tilde{\beta}_l$ is the $((l+1)\times 1)$ vector with components $\tilde{\beta}_l[j] = \beta_1^{l-j+1}\beta_2^{j-1}$, β_1 and β_2 are components of β and B_l is the $(2^l \times (l+1))$ matrix such that $\beta^{\otimes l} = B_l \tilde{\beta}_l$. For sources with unknown polarizations, the previous algorithm has to implement a searching procedure in both DOA and polarization, which is very complex. We thus limit the use of this algorithm to situations where the polarization of sources is known and we call this algorithm KP-PD-2q-MUSIC algorithm (Known Polarization PD-2q-MUSIC). In practical situations, $Q_{q,l,1}(\theta)$ is not known and has to be estimated from the eigenvalue decomposition of an estimate of $C_{2q,x}(l)$, by replacing $U_{2q,n}(l)$ by its estimate. In this case, the estimated left-hand side of (10) has to be minimized over θ.

3.3. UP-PD-2q-MUSIC algorithms

For sources with unknown polarizations, a simple way to remove the searching procedure with respect to the polarization parameter consists, for any fixed DOA, to minimize the left-hand side of equation (10) with respect to vector $\tilde{\beta}_{q,l} = [\tilde{\beta}_l \otimes \tilde{\beta}_{q-l}^*]$, as it is proposed in [8] for $q = 1$. This gives rise to the UP-PD-2q-MUSIC (Unknown Polarization PD-2q-MUSIC) algorithms whose first version for the arrangement

indexed by l, called UP-PD-2q-MUSIC(l)-1, consists to find DOA which cancel the pseudo-spectrum given by

$$P_{UP\text{-}PD\text{-}2q\text{-}Music(l)\text{-}1}(\theta) \triangleq \lambda_{q,l,\min}(\theta) \qquad (13)$$

In (13), $\lambda_{q,l,\min}(\theta)$ is the minimum eigenvalue of matrix $Q_{q,l,1}(\theta)$ in the metrics $Q_{q,l,2}(\theta)$. Note that the associated optimal vector $\tilde{\beta}_{q,l}$, noted $\tilde{\beta}_{q,l,\min}(\theta)$, corresponds to the associated eigenvector. Note that one way in which the eigenvalue $\lambda_{q,l,\min}(\theta)$ can be computed is by determining the minimum root of the following equation

$$\det[Q_{q,l,1}(\theta) - \lambda \, Q_{q,l,2}(\theta)] = 0 \qquad (14)$$

where $\det[X]$ means determinant of X. Thus, for each value of θ, searching in polarization space has been avoided by finding the roots of an equation of order $(l+1)(q-l+1)$, which corresponds to a substantial reduction in computation, at least for small values of q. We deduce from (14) and [9], that finding θ such that $\lambda = \lambda_{q,l,\min}(\theta)$ is zero is equivalent to find θ such that $\det[Q_{q,l,2}(\theta)^{-1}Q_{q,l,1}(\theta)] = \det[Q_{q,l,1}(\theta)] / \det[Q_{q,l,2}(\theta)] = 0$. A second version of the UP-PD-2q-MUSIC algorithm for the arrangement indexed by l, called UP-PD-2q-MUSIC(l)-2, consists to find DOA which cancel the pseudo-spectrum given by

$$P_{UP\text{-}PD\text{-}2q\text{-}Music(l)\text{-}2}(\theta) \triangleq \frac{\det[Q_{q,l,1}(\theta)]}{\det[Q_{q,l,2}(\theta)]} \qquad (15)$$

which allows a complexity reduction with respect to (13). Again, in practical situations, $Q_{q,l,1}(\theta)$ is not known and has to be estimated from the eigenvalue decomposition of an estimate of $C_{2q,x}(l)$. The problem then consists to find the P sets of parameters $\hat{\theta}_i = (\hat{\theta}_i, \hat{\phi}_i)$, $(1 \leq i \leq P)$, for which estimates of (13) or (15) are minimized.

4. IDENTIFIABILITY

The maximum number of sources to be processed by a given DP-2q-MUSIC algorithm is obtained for statistically independent sources. Under this assumption, assuming no coupling between the sensors, we deduce from the HO VA theory [2] that the KP-PD-2q-MUSIC algorithm for the arrangement indexed by l is able to process up to $P_{max} = N_{2q}^l - 1$ sources, provided that the associated VA has no ambiguities up to order $N_{2q}^l - 1$, where N_{2q}^l is the number of different virtual sensors of the associated VA. It has been shown in [2] that N_{2q}^l is a function of q, l and the true array of N sensors. Besides, table IV of [2] shows, for a general array with space and polarization diversities having sensors arbitrary located with different responses, the expression of an upper-bound, $N_{max}(2q,l)$, of N_{2q}^l as a function of N for $2 \leq q \leq 4$ and several values of l. A very useful case for practical situations, which was not studied in [2], corresponds to an array of $N = 2M$ sensors composed of two subarrays of M sensors having orthogonal polarizations. Two kinds of such arrays are considered in this section and correspond to arrays for which the sensors of the two subarrays are either collocated or not. Table 1 shows, for non collocated and collocated

subarrays respectively, the expression of $N_{max}(2q,l)$ as a function of N for $q = 2$ and several values of l. Note that this upper-bound corresponds to N_{2q}^l in most cases of array geometry with no particular symmetry, which is in particular the case for uniform circular array of M vectorial sensors with two components, when M is a prime number. In addition, table 1 shows the expression of N_{2q}^l as a function of $N=2M$ for $q = 2$ and several values of l, for an array composed of two collocated and orthogonally polarized Uniformly spaced Linear Array (ULA) of M identical sensors.

$2q$	l	Non-collocated subarrays $N_{max}(2q,l)$	collocated subarrays $N_{max}(2q,l)$	Collocated ULA subarrays N_{2q}^l
4	2	$N(N+1)/2$	$3N(N+2)/8$	$3(N-1)$
4	1	N^2-N+2	N^2-2N+4	$4(N-1)$

Table 1 – $N_{max}(2q, l)$ as a function of $N = 2M$ for $q = 2$ and several values of l, and for arrays with two orthogonally polarized subarrays

Under the same assumptions, following the same reasoning as that presented in [8], we deduce that the UP-PD-$2q$-MUSIC algorithms for the arrangement indexed by l are able to process up to $P_{max} = N_{2q}^l - (l+1)(q-l+1)$ sources, which corresponds to $N-2$ for $q = 1$, result obtained in [8].

5. SIMULATIONS

The results of the previous sections are illustrated in this section through computer simulations. To do so, two criterions [5] are computed, from 300 realizations, in the following in order to quantify the quality of the associated DOA estimation: a Probability of Non-Aberrant Results (PNAR) generated by a given method for a given source and the corresponding Root Mean Square Error (RMSE). The sources are assumed to have a zero elevation angle and to be zero-mean stationary sources corresponding to QPSK sources sampled at the symbol rate and filtered by a raise cosine pulse shaped filter with a roll-off = 0.3. Moreover, the statistical arrangement index used hereafter is l=1 and no modeling error is considered.
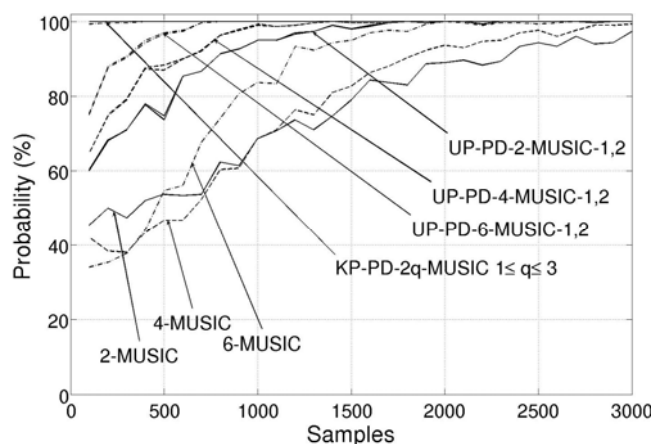

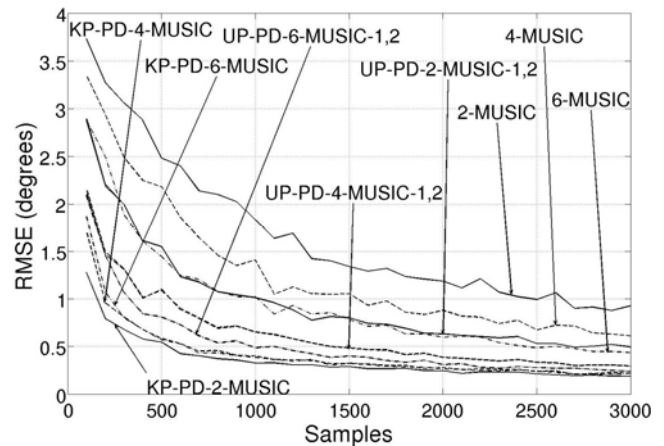Figure 1 – PNAR results of source 2 as a function of samples

Figure 2 – RMSE results of source 2 as a function of samples

5.1. Overdetermined mixtures of sources

We consider a Uniform Circular Array (UCA) of $N = 6$ crossed-dipoles with a radius r such that $r = 0.3 \lambda$. One dipole is parallel to the x-axis whereas the other is parallel to the z-axis. Three of these crossed-dipoles are combined to generate a right sense circular polarization in the y-axis while the three other dipoles are combined to generate a left sense circular polarization in the y-axis. The array is then composed of two orthogonally polarized overlapped (non-collocated) circular subarrays of $M = 3$ sensors so that adjacent sensors always have different polarizations. In this context, two QPSK sources with the same input SNR equal to 5 dB are received by the array. They are assumed to be weakly separated in both DOA and polarization, such that $(\theta_1, \gamma_1, \phi_1) = (50°, 45°, 0°)$ and $(\theta_2, \gamma_2, \phi_2) = (60°, 45°, 10°)$ respectively. Under these assumptions, figures 1 and 2 show the variations, as a function of the number of samples, of the PNAR criterion for the source 2, PNAR2, and the associated RMSE2 criterion (we obtain similar results for the source 1), at the output of the 12 methods mentioned on figure 1. For 2-MUSIC, 4-MUSIC and 6-MUSIC algorithms, the 6 sensors of the UCA are assumed to be identical with responses of sensor 1. Figures 1 and 2 show, at least for UP algorithms, better performance of methods exploiting both HO statistics and polarization discrimination and increasing performance with q for UP-PD-$2q$-MUSIC methods.

5.2. Underdetermined mixtures of sources

To illustrate the capability of PD-$2q$-MUSIC (q>1) algorithms to process underdetermined mixtures of sources, we limit the number of sensors of the previous circular array to $N = 3$ sensors. Under these assumptions, we deduce from tables II and IV of [2], and section 4 that KP-PD-4-MUSIC, UP-PD-4-MUSIC, KP-PD-6-MUSIC and UP-PD-6-MUSIC can process up to P_{max}=7, P_{max}=4, P_{max}=14 and P_{max}=9 sources, respectively. We deduce from tables VI and VII of [2] that 4-MUSIC and 6-MUSIC can process up to P_{max}=6 and P_{max}=11 sources, respectively. We assume now that the array receives 4 statistically independent QPSK sources with

the same input SNR equal to 15 dB and DOA and polarization parameters equal to $(\theta_1, \gamma_1, \phi_1) = (15°, 45°, -75°)$, $(\theta_2, \gamma_2, \phi_2) = (45°, 45°, 0°)$, $(\theta_3, \gamma_3, \phi_3) = (95°, 22.5°, 75°)$, $(\theta_4, \gamma_4, \phi_4) = (122.5°, 45°, 150°)$ respectively. Under these assumptions, figures 3 and 4 show the variations, as a function of the number of samples, of the lower PNAR and the highest RMSE results, among all the sources, at the output of the eight methods, namely the $2q$-MUSIC, the KP-PD-$2q$-MUSIC and the UP-PD-$2q$-MUSIC-m algorithms for $q \in \{2,3\}$ and for $m \in \{1,2\}$. Note the capability of PD-$2q$-MUSIC methods to process underdetermined mixtures of sources provided $P \leq P_{max}$. Note the poor performance of $2q$-MUSIC methods for the considered scenario due to the low input power of the weakest source at the output of the sensors. Better performance would be obtained for higher values of samples.
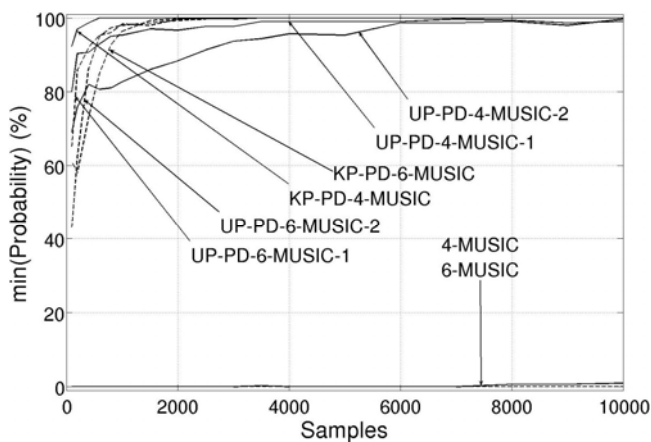

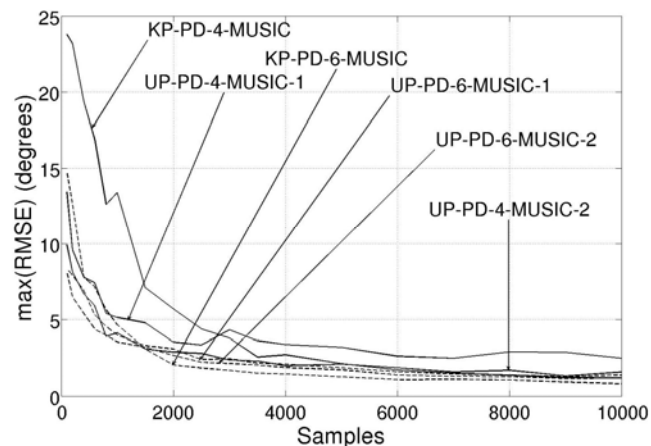Figure 3 – Minimal PNAR results of sources

Figure 4 – Maximal RMSE results of sources

6. CONCLUSION

Three versions of the $2q$-MUSIC ($q \geq 1$) algorithm able to put up with arrays having polarization diversity and giving rise to PD-$2q$-MUSIC algorithms have been presented in this paper for several arrangements of the $2q$-th order data statistics. The first version is well-suited for sources with known polarization whereas the two others do not assume any knowledge about the sources' polarization. For a given value of q, these algorithms have been shown to increase the resolution, the robustness to modeling errors (at least for several poorly angularly separated sources) and the processing capacity (at least for VA without any HO ambiguities) of the $2q$-MUSIC method in the presence of diversely polarized sources and from an array with polarization diversity. Moreover, despite a higher variance in the statistics estimation, performance of UP-PD-$2q$-MUSIC algorithms have been shown to generally increase with q when some resolution is required. This occurs in particular for sources which are poorly separated in both DOA and polarization. This result shows off for these scenarios the interest to jointly exploit polarization diversity and HO statistics for DF. Finally, identifiability issue of all of these methods has been addressed.

7. REFERENCES

[1] J.F. Cardoso, E. Moulines, "Asymptotic performance analysis of DF algorithms based on fourth-order cumulants", *IEEE Trans. Sig. Proc.*, vol. 43, pp. 214-224, January 1995.

[2] P. Chevalier, L. Albera, A. Ferréol, P. Comon, "On the Virtual Array concept for higher order array processing", *IEEE Trans. Sig. Proc.*, vol. 53, pp. 1254-1271, April 2005

[3] P. Chevalier, A. Ferréol, "On the virtual array concept for the fourth-order direction finding problem", *IEEE Trans. Sig. Proc.*, vol. 47, pp. 2592-2595, September 1999

[4] P. Chevalier, A. Ferréol, "Procédé et dispositif de goniométrie à haute resolution à un ordre pair arbitraire", *Patent* N°05.03180, April 2005.

[5] P. Chevalier, A. Ferréol, L. Albera, "High resolution direction finding from higher order statistics: the $2q$-MUSIC algorithm", *IEEE Trans. Sig. Proc.*, vol. 54, pp. 2986-2997, Aug. 2006.

[6] R.T. Compton, JR., "Adaptive Antennas - Concepts and Performance", Prentice Hall, Englewood Cliffs, New Jersey, 1988.

[7] M.C. Dogan, J.M. Mendel, "Applications of cumulants to array processing - Part I : Aperture extension and array calibration", *IEEE Trans. Sig. Proc.*, vol. 43, N°5, pp. 1200-1216, May 1995.

[8] E.R. Ferrara, Jr, T.M. Parks, "Direction finding with an array of antennas having diverse Polarizations", *IEEE Trans. Ant. Prop.*, vol. 31, pp. 231-236, March 1983

[9] A. Ferreol, E. Boyer, P. Larzabal, "Low-Cost algorithm for some bearing estimation methods in presence of separable nuisance parameters", *Electronics Letters*, vol. 40, pp. 966-967, July 2004.

[10] E. Gönen, J.M. Mendel, "Applications of cumulants to array processing - Part VI : Polarization and Direction of Arrival estimation with Minimally Constrained Arrays", *IEEE Trans. Sig. Proc.*, vol. 47, pp. 2589-2592, September 1999.

[11] S. Miron, N. Le Bihan, J.I. Mars, "Quaternion-MUSIC for vector-sensor array processing", *IEEE Trans. Sig. Proc.*, vol. 54, N°4, pp. 1218-1229, April 2006.

[12] B. Porat, B. Friedlander, "Direction finding algorithms based on higher order statistics", *IEEE Trans. Sig. Proc.*, vol. 39, pp. 2016-2024, September 1991.

[13] R.O. Schmidt, "Multiple emitter location and signal parameter estimation", *IEEE Trans. Ant. Prop.*, vol. 34, pp. 276-280, March 1986.