GLRT-BASED ARRAY RECEIVERS TO DETECT A KNOWN SIGNAL CORRUPTED BY NONCIRCULAR INTERFERENCES

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ABSTRACT

The problem of detecting a known signal with unknown parameters in the presence of interferences, whose second order statistics (SOS) are unknown, has received considerable attention these last decades. However, most of the available receivers assume second order (SO) circular interferences and become suboptimal in the presence of SO noncircular interferences, omnipresent in applications such as radar, satellite localization or radio communications. The scarce optimal receivers taking into account the potential SO noncircularity of the interferences have been developed under the limiting assumption of a known signal with known parameters or of a random signal. For this reason, following a generalized likelihood ratio test (GLRT) approach, we introduce different new array receivers for the detection of a known signal, with different sets of unknown parameters, corrupted by unknown noncircular interferences and we analyze their performances. Specifically, we show that these new non conventional (NC) detectors entail large gains in performance with respect to conventional (C) ones, depending on the a priori information available.

1. INTRODUCTION

Detecting a known signal with unknown parameters in the presence of noise plus interferences (called total noise in the following), whose covariance matrix is unknown, is an important problem, which concerns many applications such as radio communication networks, satellite location systems, Identification Friends and Foes (IFF) systems, radar and sonar. For this reason, this problem has received much attention these last decades, assuming the reception from an array of sensors and the knowledge of more or less a priori information about the propagation channel of the signal to be detected. More precisely, a spatio-temporal (ST) adaptive detector using the sampled covariance matrix estimate from secondary (signal free) data vectors is proposed in [2] [13] to detect a rank-one signal in the presence of an unknown and Gaussian total noise. This detector is modified in [12] to derive a constant false-alarm rate (CFAR) test called the adaptive matched filter (AMF) detector. This problem is reformulated in [8] by Kelly who proposes a GLRT-based detector to estimate the unknown parameters. An extension of Kelly's detector assuming that no signal free data vectors are available is presented in [15] for radar applications. Alternative detectors using no information about the useful propagation channel are presented in [3] [7] for time acquisition purposes. All the previous detectors assume implicitly or explicitly a SO circular [9] (or proper) total noise and become suboptimal in the presence of SO noncircular (or improper) interferences, such as AM, ASK, BPSK MSK, GMSK, OQPSK interferences, omnipresent in many applications. For this reason, some optimal detectors in SO noncircular contexts have been introduced, more or less recently, but under the restrictive conditions of either a known signal with known parameters [1] [17] or a random signal [14]. Despite these works, the major issue of practical use consisting to detect a known signal with unknown parameters in the presence of an arbitrary unknown SO noncircular total noise has been scarcely investigated up to now. In fact, references [4] [6] seem to be the only ones addressing this problem, for synchronization purposes and for a completely unknown useful propagation channel. However, this assumption is not always valid in practice, especially in radar applications. For this reason, following a GLRT approach, the purpose of this paper is to introduce some new array receivers, associated with various choices of the unknown signal parameters, and to analyze their performance, for the detection of a known signal corrupted by an unknown SO noncircular total noise. To simplify the analysis, only BPSK useful signals are considered in this paper. This assumption is not so restrictive since BPSK signals are used in numbers of practical systems like DS-CDMA radio communications networks, GPS, IFF or some radar systems. For such known waveforms, the new NC detectors implement optimal widely linear (WL) [10] filters instead of linear ones which are part of conventional detectors.

2. HYPOTHESES AND PROBLEM FORMULATION

2.1 Hypotheses

We consider an array of N Narrow-Band (NB) sensors receiving the contribution of a known BPSK signal and a total noise composed of some potentially SO noncircular interferences and a background noise. The complex envelope of the useful signal is given by

$$s(t) \;=\; \sum_{n=0}^{K-1} a_n\, v(t - nT) \tag{1}$$

where $a_n = \pm 1$ $(0 \leq n \leq K-1)$ are known transmitted symbols, T is the symbol duration and $v(t)$ is a raised cosine pulse shaped filter such that $r(nT) \triangleq v(t) \otimes v(-t)^* /_{t=nT} = 0$ for $n \neq 0$, where $\otimes$ and $*$ are the convolution and the complex conjugation operations respectively. Noting $\boldsymbol{x}(t)$ the vector of the complex envelopes of the signals at the output of these sensors, T_e the sampling period such that T/T_e is an integer q, $s_v(kT_e) \triangleq s(t) \otimes v(-t)^* /_{t=kT_e}$ and $\boldsymbol{x}_v(kT_e) \triangleq \boldsymbol{x}(t) \otimes v(-t)^* /_{t=kT_e}$ the sampled useful signal and observation vector at the output of a matched filtering operation to the pulse shaped filter $v(t)$, we obtain

$$\boldsymbol{x}_v(kT_e) \;\approx\; \mu_s\, e^{j\phi_s}\, s_v(kT_e)\, \boldsymbol{s} \;+\; \boldsymbol{b}_{Tv}(kT_e) \tag{2}$$

In (2), $\boldsymbol{b}_{Tv}(kT_e)$ is the sampled total noise vector at the output of $v(-t)^*$, assumed to be uncorrelated with $s_v(kT_e)$, μ_s and ϕ_s are real parameters controlling the amplitude and phase of the known signal on the first sensor respectively and $\boldsymbol{s}$ is the steering vector of the known signal. It is interesting to point out that model (2) assumes a free space propagation for the known signal but that it may still be used for useful propagation channels with delay spread by considering uncorrelated multipaths as interferences.

2.2 Second order statistics of the data

The SO statistics of the data considered in the following correspond to the first and second correlation matrices of $\boldsymbol{x}_v(kT_e)$, defined by $R_x(kT_e) \triangleq \mathrm{E}[\boldsymbol{x}_v(kT_e)\, \boldsymbol{x}_v(kT_e)^\dagger]$ and $C_x(kT_e) \triangleq \mathrm{E}[\boldsymbol{x}_v(kT_e)\, \boldsymbol{x}_v(kT_e)^\mathrm{T}]$ respectively, where $^\mathrm{T}$ and † correspond to the transposition and transposition and conjugation operations respectively. Under the assumptions of section 2.1, $R_x(kT_e)$ and $C_x(kT_e)$ can be written as

$$R_x(kT_e) \;\approx\; \pi_s(kT_e)\, \boldsymbol{s}\, \boldsymbol{s}^\dagger \;+\; R(kT_e) \tag{3}$$

$$C_x(kT_e) \;\approx\; e^{j2\phi_s}\, \pi_s(kT_e)\, \boldsymbol{s}\, \boldsymbol{s}^\mathrm{T} \;+\; C(kT_e) \tag{4}$$

where $\pi_s(kT_e) \triangleq \mu_s^2\, \mathrm{E}[|s_v(kT_e)|^2] = \mu_s^2\, |s_v(kT_e)|^2$ is the instantaneous power of the useful signal received by an omnidirectional sensor, $R(kT_e) \triangleq \mathrm{E}[\boldsymbol{b}_{Tv}(kT_e)\, \boldsymbol{b}_{Tv}(kT_e)^\dagger]$ and $C(kT_e) \triangleq \mathrm{E}[\boldsymbol{b}_{Tv}(kT_e)\, \boldsymbol{b}_{Tv}(kT_e)^\mathrm{T}]$ are the first and second correlation matrices of $\boldsymbol{b}_{Tv}(kT_e)$ respectively. Note that $C(kT_e) = 0\ \forall k$ for a SO circular total noise vector and that the previous statistics depend on the time parameter since both the known signal and the interferences are not stationary.

2.3 Problem Formulation

We consider the detection problem with two hypotheses H_0 and H_1, where H_0 and H_1 correspond to the presence of total noise only and signal plus total noise in the observation vector $\boldsymbol{x}_v(kT_e)$ respectively. Due to the matched prefiltering of the data by $v(-t)^*$, it is sufficient to work at the symbol rate. Then, under the two previous hypotheses, using (2), the observation vector $\boldsymbol{x}_v(nT)$ can be written as :

$$\mathrm{H}_1 : \;\; \boldsymbol{x}_v(nT) \;\approx\; \mu_s\, e^{j\phi_s}\, s_v(nT)\, \boldsymbol{s} \;+\; \boldsymbol{b}_{Tv}(nT) \tag{5}$$

$$\mathrm{H}_0 : \;\; \boldsymbol{x}_v(nT) \;\approx\; \boldsymbol{b}_{Tv}(nT) \tag{6}$$

The problem addressed in this paper then consists to optimally detect, from the GLRT point of view, the sampled known signal, $s_v(nT) = r(0)\, a_n$ $(0 \leq n \leq K-1)$, from the observation vectors $\boldsymbol{x}_v(nT)$ $(0 \leq n \leq K-1)$. To this aim, we assume that the total noise is potentially SO noncircular and that each of the parameters μ_s, ϕ_s, $\boldsymbol{s}$, $R(nT)$ and $C(nT)$ may be either known or unknown, depending on the application. We first address the unrealistic case of completely known parameters in section 3 while the cases of practical interest corresponding to some unknown parameters are addressed in sections 4 and 5. To compute these new receivers, some theoretical assumptions, not necessary verified neither required in practical situations, are made. These assumptions are not so critical in the sense that optimal receivers derived under these assumptions still provide good detection performance even if most of the latter are not verified in practice. Defining $\tilde{\boldsymbol{b}}_{Tv}(nT) \triangleq [\boldsymbol{b}_{Tv}(nT)^\mathrm{T}, \boldsymbol{b}_{Tv}(nT)^{\dagger}]^\mathrm{T}$, these assumptions correspond, for $0 \leq n,m \leq K-1$, to :

- A_1: the samples $\tilde{\boldsymbol{b}}_{Tv}(nT)$ are uncorrelated to each other
- A_2: the matrices $R(nT)$ and $C(nT)$ do not depend on the symbol indice n and are noted R and C respectively.
- A_3: the samples $\boldsymbol{b}_{Tv}(nT)$ are Gaussian
- A_4: the samples $\boldsymbol{b}_{Tv}(nT)$ are noncircular
- A_5: the samples $\boldsymbol{b}_{Tv}(nT)$ and $s_v(mT)$ are statistically independent

Assumption A_1 requires in particular propagation channels with no delay spread and may be verified for temporally white interferences while A_2 is true for cyclostationary interferences with symbol period T. A_3 is a theoretical assumption allowing to only exploit the SO statistics of the observations from a LRT or a GLRT approach while A_4 is true in the presence of SO noncircular interferences but is generally not exploited in detection problems. Finally A_5 is verified in particular for a useful propagation channel with no delay spread.

3. OPTIMAL RECEIVER FOR KNOWN PARAMETERS

In order to compute the best possible detector of a known signal in a SO noncircular total noise and to obtain a reference receiver for the following sections, we consider in this section that the parameters μ_s, ϕ_s, $\boldsymbol{s}$, R and C are known. According to the Neyman-Pearson theory of detection, the optimal receiver for the detection of the known samples $s_v(nT)$ from $\boldsymbol{x}_v(nT)$ over the known signal duration is the LRT receiver. It consists to compare to a threshold the function $\mathrm{LR}(\boldsymbol{x}_v, K)$ defined by

$$\mathrm{LR}(\boldsymbol{x}_v, K) \triangleq \frac{\mathrm{p}[\boldsymbol{x}_v(nT),\, 0 \leq n \leq K-1,\, /\, \mathrm{H}_1]}{\mathrm{p}[\boldsymbol{x}_v(nT),\, 0 \leq n \leq K-1,\, /\, \mathrm{H}_0]} \tag{7}$$

where $p[\boldsymbol{x}_v(nT), 0 \le n \le K-1, / H_i]$ ($i = 0, 1$) is the conditional probability density of $[\boldsymbol{x}_v(0), \boldsymbol{x}_v(T), .., \boldsymbol{x}_v((K-1)T)]^T$ under H_i. Under both the previous known parameters assumption and A_1 to A_5, using the probability density of the noncircular Gaussian total noise presented in [16], we deduce, after some algebraic manipulations, that a sufficient statistic for the previous detection problem consists to compare to a threshold the function $NC1(\boldsymbol{x}_v, K)$ defined by

$$\boxed{NC1(\boldsymbol{x}_v,K) \triangleq \mathrm{Re}[\tilde{\boldsymbol{s}}(\phi_s)^\dagger R_{\tilde{b}}^{-1}\, \hat{\boldsymbol{r}}_{\tilde{x}a}]} \tag{8}$$

where $\tilde{\boldsymbol{s}}(\phi_s) \triangleq [e^{j\phi_s}\boldsymbol{s}^T, e^{-j\phi_s}\boldsymbol{s}^{\dagger}]^T$, $R_{\tilde{b}} \triangleq R_{\tilde{b}}(nT) = E[\tilde{\boldsymbol{b}}_{Tv}(nT)\, \tilde{\boldsymbol{b}}_{Tv}(nT)^\dagger]$ is given by

$$R_{\tilde{b}} = \begin{pmatrix} R & C \\ C^* & R^* \end{pmatrix} \tag{9}$$

and $\hat{\boldsymbol{r}}_{\tilde{x}a}$ is the ($2N$ x 1) vector defined by

$$\hat{\boldsymbol{r}}_{\tilde{x}a} \triangleq \frac{1}{K}\sum_{n=0}^{K-1} \tilde{\boldsymbol{x}}_v(nT)\, a_n \tag{10}$$

with $\tilde{\boldsymbol{x}}_v(nT) \triangleq [\boldsymbol{x}_v(nT)^T, \boldsymbol{x}_v(nT)^{\dagger}]^T$. Let us introduce the filter $\tilde{\boldsymbol{w}}_{1o} \triangleq R_{\tilde{b}}^{-1}\tilde{\boldsymbol{s}}(\phi_s)$, which corresponds to the so-called WL Spatial Matched Filter (SMF) [5], i.e. the WL filter which maximizes the output signal to interference plus noise ratio (SINR), and whose output is a real quantity. Then, (8) corresponds to the correlation of the WL SMF's output with the known useful symbols, a_n, over the known signal duration. In the particular case of a SO circular total noise ($C = 0$), (8) is reduced to the conventional detector [1] defined by:

$$C1(\boldsymbol{x}_v,K) \triangleq 2\mathrm{Re}[e^{-j\phi_s}\, \boldsymbol{s}^\dagger R^{-1}\, \hat{\boldsymbol{r}}_{xa}] \tag{11}$$

where $\hat{\boldsymbol{r}}_{xa}$ is defined by (10) with $\boldsymbol{x}_v(nT)$ instead of $\tilde{\boldsymbol{x}}_v(nT)$.

4. OPTIMAL RECEIVER FOR A KNOWN SIGNAL'S STEERING VECTOR

4.1 Applications

In most of the situations of practical interest, the parameters μ_s, ϕ_s, R and C are unknown while, for some applications, the steering vector $\boldsymbol{s}$ is known. This is in particular the case for radar applications for which a Doppler and a range processing currently take place at the output of a beam which is mechanically or electronically steered in a given direction and scanned to monitor all the directions of space. In this case, the steering vector $\boldsymbol{s}$ is associated with the current direction of the beam. Another example corresponds to satellite localization for which the satellites positions are known and the vector $\boldsymbol{s}$ may be associated, in this case, with the direction of one of the satellites. Moreover, in some cases, some signal free observation vectors (called secondary observation vectors) sharing the same total noise SOS are available in addition to the observation vectors containing the signal to be detected plus the total noise (called primary observation vectors). For example, the secondary observation vectors may correspond to samples of data associated with other range than the range of the detected target. In such situations, we will say that a total noise alone reference (TNAR) is available. In other applications, a TNAR is difficult to built, due for example to the total noise nonstationarity or to the presence of multipaths. In this context, following a GLRT approach, several optimal (from a GLRT point of view) receivers for the detection of a known signal, with different sets of unknown parameters, corrupted by a SO noncircular total noise are introduced. More precisely these new receivers assume that the parameters μ_s and ϕ_s are unknown, the vector $\boldsymbol{s}$ is known and the matrices R and C are either known (section 4.2) or unknown, with (section 4.3) or without (section 4.4) any available TNAR in this latter case. Note that the non conventional (NC) receivers introduced in this section are completely new.

4.2 Known total noise

Under the assumptions A_1 to A_4, assuming known parameters R, C and $\boldsymbol{s}$ and unknown parameters μ_s and ϕ_s, the optimal receiver, from a GLRT approach, for the detection of the known real signal $s_v(nT)$ ($0 \le n \le K-1$) in a SO noncircular total noise characterized by R and C, is given by (7) where the unknown parameters μ_s and $e^{j\phi_s}$ appearing in (7) have to be replaced by their maximum likelihood (ML) estimates. In this context, using the probability density of the noncircular Gaussian total noise presented in [16], we deduce, after some algebraic manipulations, that a sufficient statistic for the previous detection problem consists to compare to a threshold the function $NC2(\boldsymbol{x}_v, K)$ defined by

$$\boxed{NC2(\boldsymbol{x}_v,K) \triangleq \hat{\boldsymbol{r}}_{\tilde{x}a}^{\dagger} R_{\tilde{b}}^{-1} M_s [M_s^{\dagger} R_{\tilde{b}}^{-1} M_s]^{-1} M_s^{\dagger} R_{\tilde{b}}^{-1}\, \hat{\boldsymbol{r}}_{\tilde{x}a}} \tag{12}$$

where M_s is the ($2N$ x 2) matrix defined by:

$$M_s \triangleq \begin{pmatrix} \boldsymbol{s} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{s}^* \end{pmatrix} \tag{13}$$

For a SO circular total noise ($C = 0$), (12) is reduced to the conventional receiver [12] defined by

$$C2(\boldsymbol{x}_v, K) \triangleq \frac{|\boldsymbol{s}^\dagger R^{-1}\hat{\boldsymbol{r}}_{xa}|^2}{\boldsymbol{s}^\dagger R^{-1}\boldsymbol{s}} \tag{14}$$

which is proportional to the square modulus of the correlation between the output of the SMF, $\boldsymbol{w}_{1o} \triangleq R^{-1}\boldsymbol{s}$, and the known useful symbols, a_n, over the known signal duration.

4.3. Unknown total noise with a TNAR

When the total noise is unknown and when a TNAR is available, R and C may be estimated from the secondary data only through a ML approach. This gives rise to the detector $NC3(\boldsymbol{x}_v, K)$, defined by

$$\boxed{NC3(\boldsymbol{x}_v,K) \triangleq \hat{\boldsymbol{r}}_{\tilde{x}a}^{\dagger} \hat{R}_{\tilde{b}}^{-1} M_s [M_s^{\dagger} \hat{R}_{\tilde{b}}^{-1} M_s]^{-1} M_s^{\dagger} \hat{R}_{\tilde{b}}^{-1}\, \hat{\boldsymbol{r}}_{\tilde{x}a}} \tag{15}$$

where $\hat{R}_{\tilde{b}}$ is defined by

$$\hat{R}_{\tilde{b}} \triangleq \frac{1}{K'} \sum_{n=0}^{K'-1} \tilde{\boldsymbol{b}}_{Tv}(nT)' \, \tilde{\boldsymbol{b}}_{Tv}(nT)'^{\dagger} \tag{16}$$

where $2N \leq K'$ and the vectors $\tilde{\boldsymbol{b}}_{Tv}(nT)'$ $(0 \leq n \leq K' - 1)$ are the secondary signal free extended data vectors such that $E[\tilde{\boldsymbol{b}}_{Tv}(nT)' \tilde{\boldsymbol{b}}_{Tv}(nT)'^{\dagger}] = R_{\tilde{b}}$. For a SO circular total noise ($C = 0$), (15) is reduced to the conventional receiver presented in [12] and defined by

$$\boxed{\mathrm{C3}(\boldsymbol{x}_v, K) \triangleq \frac{|\boldsymbol{s}^{\dagger} \hat{R}^{-1} \hat{\boldsymbol{r}}_{xa}|^2}{\boldsymbol{s}^{\dagger} \hat{R}^{-1} \boldsymbol{s}}} \tag{17}$$

where $\hat{R}$ is defined by (16) with $\boldsymbol{b}_{Tv}(nT)'$ instead of $\tilde{\boldsymbol{b}}_{Tv}(nT)'$ and $\hat{\boldsymbol{r}}_{xa}$ is defined by (10) with $\boldsymbol{x}_v(nT)$ instead of $\tilde{\boldsymbol{x}}_v(nT)$.

4.4. Unknown total noise without any TNAR

When the total noise in unknown and when no TNAR is a priori available, R and C may be estimated from the primary data with respect to the ML criterion. After algebraic computations, this gives rise to the detector NC4($\boldsymbol{x}_v$, K), defined by

$$\boxed{\mathrm{NC4}(\boldsymbol{x}_v, K) \triangleq \frac{\hat{\boldsymbol{r}}_{\tilde{x}a}^{\dagger} \hat{R}_{\tilde{x}}^{-1} M_s [M_s^{\dagger} \hat{R}_{\tilde{x}}^{-1} M_s]^{-1} M_s^{\dagger} \hat{R}_{\tilde{x}}^{-1} \hat{\boldsymbol{r}}_{\tilde{x}a}}{1 - \hat{\boldsymbol{r}}_{\tilde{x}a}^{\dagger} \hat{R}_{\tilde{x}}^{-1} \hat{\boldsymbol{r}}_{\tilde{x}a}}} \tag{18}$$

where $\hat{R}_{\tilde{x}}$ is defined by

$$\hat{R}_{\tilde{x}} \triangleq \frac{1}{K} \sum_{n=0}^{K-1} \tilde{\boldsymbol{x}}_{Tv}(nT) \, \tilde{\boldsymbol{x}}_{Tv}(nT)^{\dagger} \tag{19}$$

with $2N \leq K$. In particular, for a circular total noise ($C = 0$), (18) reduces to the conventional receiver presented in [15] and defined by

$$\boxed{\mathrm{C4}(\boldsymbol{x}_v, K) \triangleq \frac{|\boldsymbol{s}^{\dagger} \hat{R}_x^{-1} \hat{\boldsymbol{r}}_{xa}|^2}{\boldsymbol{s}^{\dagger} \hat{R}_x^{-1} \boldsymbol{s} \, (1 - \hat{\boldsymbol{r}}_{xa}^{\dagger} \hat{R}_x^{-1} \hat{\boldsymbol{r}}_{xa})}} \tag{20}$$

5. OPTIMAL RECEIVER FOR AN UNKNOWN SIGNAL'S STEERING VECTOR

5.1 Applications

In some applications, the steering vector of the useful signal is not known a priori. This may be the case for radio communications applications where the emitter location may not be known or in the presence of flat fading. It may also concern radar processing without beam scanning. In this case, the range, Doppler shift and direction of arrival of the target may be jointly estimated from an omnidirectional reception [11]. In this context, assuming that μ_s, ϕ_s and $\boldsymbol{s}$ are unknown, we introduce some new GLRT-based receivers for the detection of a known signal corrupted by either a known or an unknown SO noncircular total noise.

5.2 Known total noise

Under the assumptions A_1 to A_4, assuming that parameters R, C are known and that μ_s, ϕ_s and $\boldsymbol{s}$ are unknown, the GLRT-based receiver for the detection of the known real signal $s_v(nT)$ $(0 \leq n \leq K - 1)$ in a SO noncircular total noise characterized by R and C, is given by (7) where the unknown parameters appearing in (7) have to be replaced by their ML estimates. We deduce, after some algebraic manipulations, that the associated detector consists to compare to a threshold the function NC5($\boldsymbol{x}_v$, K) defined by

$$\boxed{\mathrm{NC5}(\boldsymbol{x}_v, K) \triangleq \hat{\boldsymbol{r}}_{\tilde{x}a}^{\dagger} R_{\tilde{b}}^{-1} \hat{\boldsymbol{r}}_{\tilde{x}a}} \tag{21}$$

For a SO circular total noise ($C = 0$), (21) is reduced to the conventional receiver defined by

$$\boxed{\mathrm{C5}(\boldsymbol{x}_v, K) \triangleq \hat{\boldsymbol{r}}_{xa}^{\dagger} R^{-1} \hat{\boldsymbol{r}}_{xa}} \tag{22}$$

5.3 Unknown total noise with a TNAR

When the total noise in unknown and when a TNAR is available, R and C may be estimated from the secondary data only through a ML approach. This gives rise, under assumptions of section 5, to the detector NC6 ($\boldsymbol{x}_v$, K), defined by

$$\boxed{\mathrm{NC6}(\boldsymbol{x}_v, K) \triangleq \hat{\boldsymbol{r}}_{\tilde{x}a}^{\dagger} \hat{R}_{\tilde{b}}^{-1} \hat{\boldsymbol{r}}_{\tilde{x}a}} \tag{23}$$

where $\hat{R}_{\tilde{b}}$ is defined by (16). For a circular total noise ($C = 0$), this receiver is reduced to

$$\boxed{\mathrm{C6}(\boldsymbol{x}_v, K) \triangleq \hat{\boldsymbol{r}}_{xa}^{\dagger} \hat{R}^{-1} \hat{\boldsymbol{r}}_{xa}} \tag{24}$$

where $\hat{R}$ is defined by (16) with $\boldsymbol{b}$ instead of $\tilde{\boldsymbol{b}}$.

5.4. Unknown total noise without any TNAR

When the total noise in unknown and no TNAR is a priori available, R and C may be estimated from the primary data with respect to the ML criterion. After some computations, this gives rise to the detector NC7($\boldsymbol{x}_v$, K), defined by

$$\boxed{\mathrm{NC7}(\boldsymbol{x}_v, K) \triangleq \hat{\boldsymbol{r}}_{\tilde{x}a}^{\dagger} \hat{R}_{\tilde{x}}^{-1} \hat{\boldsymbol{r}}_{\tilde{x}a}} \tag{25}$$

which corresponds to the detector introduced in [4] for synchronization purposes in the presence of a noncircular total noise. For a circular total noise ($C = 0$), (25) reduces to the conventional receiver presented in [3] and [7] and defined by

$$\boxed{\mathrm{C7}(\boldsymbol{x}_v, K) \triangleq \hat{\boldsymbol{r}}_{xa}^{\dagger} \hat{R}_x^{-1} \hat{\boldsymbol{r}}_{xa}} \tag{26}$$

6. SIMULATIONS

To illustrate the performance of the previous NC and conventional detectors, we consider a burst radio communications link for which a training sequence of $K = 64$ symbols is transmitted at each burst. The BPSK useful signal is assumed to be corrupted by a BPSK interference whose INR is always 20 dB above the SNR. We consider a linear array of N omnidirectional sensors equispaced half a wavelength apart. The phase of both the useful signal and the interference are constant over all the bursts and equal to $\phi_s = 0°$ and $\phi_1 = 45°$ respectively. The DOA of the two sources are given by $\theta_s = 0°$ and $\theta_1 = 20°$ respectively. The performance are evaluated over 100.000 bursts. Under these assumptions, Figures 1 and 2 show, for the previous NC and conventional detectors, the non-detection probability of the useful signal as a function of the input SNR, for a false alarm rate equal to 0.001 and for $N = 1$ and $N = 2$ respectively. Note the increasing performance as the a priori information about the useful signal improves and the better performance of optimal detectors with respect to conventional ones thanks to the ability of the optimal detectors to reject the interference through phase discrimination (Figure 1) as well as through phase and DOA discrimination (Figure 2).

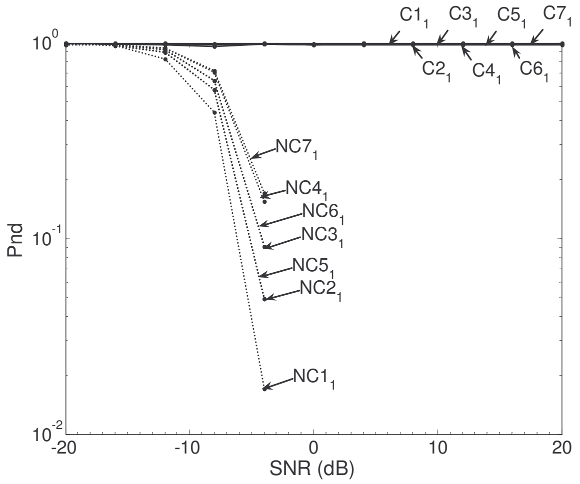

Figure 1 – *Non detection probability as a function of SNR K = 64, one BPSK interference, INR = SNR + 20 dB, $\theta_S = 0°$, $\theta_I = 20°$, $\phi_S = 0°$, $\phi_I = 45°$,. FAR = 0.001, 100000 runs, N = 1*

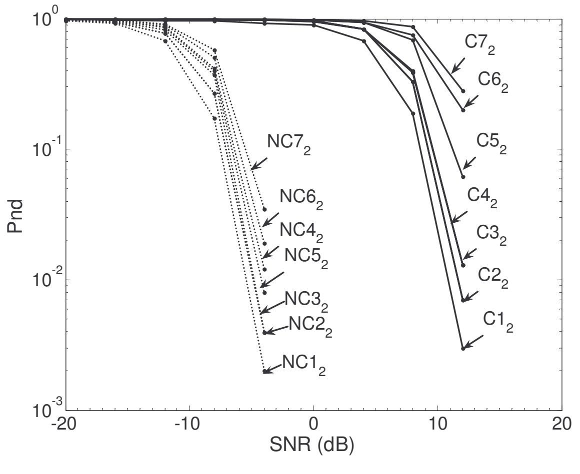

Figure 2 – *Non detection probability as a function of SNR K = 64, one BPSK interference, INR = SNR + 20 dB, $\theta_S = 0°$, $\theta_I = 20°$, $\phi_S = 0°$, $\phi_I = 45°$,. FAR = 0.001, 100000 runs, N = 2*

7. CONCLUSION

Several new receivers for the detection of a known BPSK signal, with different sets of unknown parameters, corrupted by noncircular interferences have been presented in this paper. It has been shown that taking into account the potential noncircularity property of the interferences may dramatically improve the performance of both mono and multi-channels receivers. In particular, the capability of the new detectors to do single antenna interference cancellation of rectilinear interferences, by exploiting the phase diversity between the sources in addition to the space diversity, has been verified for all the new detectors. It also puts forward that the more a priori information on the signal, the better the performance.

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