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MINIMIZING THE DELAY ESTIMATION ERROR OF A SPREAD SPECTRUM SIGNAL FOR SATELLITE POSITIONING

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ABSTRACT

As is known, satellite positioning is based on measuring the delay experienced by a Spread Spectrum (SS) signal that propagates from the satellite to the receiver. In such a scenario, the more accurate the delay estimation is, the more precise user position computation will be. This paper derives a criterion to improve position accuracy, based on minimizing the variance of time-delay estimation at the receiver. In particular, it focuses on designing sequences with specified constraints on the aperiodic auto-correlation sequence. The techniques used to meet such constraints are based on difference sets obtained from power residue classification.

1. INTRODUCTION

Satellite positioning is based on estimating in the receiver the propagation times of a set of Spread Spectrum (SS) signals broadcast by multiple satellites at known locations. By performing at least four such measurements, the receiver can uniquely obtain its own spatial coordinates and time reference [1].

In such a scenario, position accuracy thus depends on delay estimation accuracy. In other words, the variance of the delay error must be kept as low as possible, so that receiver operation can be cast into a conventional parameter estimation problem, to be tackled with the tools of estimation theory [2], and, particularly, signal synchronization [3]. Acquisition and tracking issues for ranging codes are well documented in the literature [4], but the fundamental limits of these functions and the techniques to get close to such limits are relatively less investigated.

This paper deals with pseudorandom sequences which modulate the navigation data. In particular, this contribution is concerned with the design of new sets of spreading codes, which minimize the Cramér-Rao Bound (CRB) on the variance of delay estimation. Our approach follows cyclotomic theory and power residue classification.

After this introduction, Section 2 investigates for the CRB, identifying the goals to be attained in improving time-delay estimation, whereas Section 3 describes the design of optimum spreading sequences. Results are shown in 3.3, while some conclusions are eventually drawn in Section 4.

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2. STATEMENT OF THE PROBLEM

The basic format of a bandpass SS signal for positioning is

$$x_{BP}(t) = \sqrt{2C}\Re\left\{\sum_{k=-\infty}^{+\infty} c_k g\left(t-kT_c\right)e^{j(2\pi f_0 t+\theta)}\right\}, \quad (1)$$

where C is the average power of the signal, $\Re\{\cdot\}$ denotes the real part of a complex-valued argument, f_0 is the carrier frequency, θ is the carrier phase, T_c is the chip time, and $g(t)$ is a real-valued shaping pulse with energy T_c. The sequence $\mathbf{c} = \{c_k = \pm 1\}_{k=0}^{N-1}$, also referred to as *ranging code*, is a pseudorandom binary sequence. For the sake of simplicity, data modulation in (1) has been neglected.

Assuming ideal coherent demodulation, baseband-equivalent of the received signal[1] can be modeled as

$$z(t) = x(t-\tau)+n(t) = \sqrt{2C}\sum_{k=-\infty}^{+\infty} c_k g\left(t-\tau-kT_c\right)+n(t), \quad (2)$$

where τ is the time delay experienced by the SS signal when propagating from the satellite to the receiver (as seen in the time reference frame of the receiver), and $n(t)$ represents complex-valued additive white Gaussian noise with two-sided power spectral density $2N_0$.

As is known, the ranging code $\mathbf{c}$ has spectral properties similar to a random binary sequence, but is actually deterministic [5]. As stated in the introduction, the problem of accurate positioning can be cast into a *parameter estimation* problem. Therefore, optimization deals with the fundamental accuracy bounds on parameter estimation. The Cramér-Rao Bound (CRB) [2], which is a lower bound on the error variance of any unbiased estimate, can provide a useful benchmark for positioning accuracy.

The CRB is formulated in terms of the likelihood function of the scalar parameter to be estimated. Let $r(t)$ be a segment on N consecutive chip intervals T_c of the noisy received signal (2), i.e.

$$r(t) = \sqrt{2C}\sum_{k=0}^{N-1} c_k g\left(t-kT_c-\tau\right)+n(t). \quad (3)$$

Since $\mathbf{c}$ is known to the receiver, the only unknown parameter is the delay τ. We can process the observed signal $r(t)$ with some *unbiased estimator*, to derive an estimate $\hat{\tau}$ of the signal delay τ. Let

$$\varepsilon \triangleq \frac{\hat{\tau}-\tau}{T_c}$$

[1] Unless otherwise stated, only baseband equivalents of bandpass signals will be considered.

be the normalized timing error. The variance σ_ε^2 of *any* unbiased estimator of ε (the so-called *jitter variance*) is lower bounded by [2], [3]

$$\sigma_\varepsilon^2 = E_{\mathbf{r}}\{\varepsilon^2\} \geq CRB(\varepsilon) \triangleq \left[E_{\mathbf{r}} \left\{ \left[T_c \frac{\partial}{\partial \tau} \ln p(\mathbf{r}|\tau) \right]^2 \right\} \right]^{-1}, \tag{4}$$

where $\mathbf{r}$ is a vector representation of $r(t)$ on a complete orthonormal basis [2], $p(\mathbf{r}|\tau)$ is the conditional probability density function (pdf) of $\mathbf{r}$ for a given τ (the *likelihood function* of τ), and $E_{\mathbf{r}}\{\cdot\}$ denotes statistical expectation with respect to $p(\mathbf{r})$.

Equation (4) gives a criterion to optimize the performance in terms of position accuracy by optimizing delay estimation accuracy. It can be shown [3] that the estimation variance of the simple Delay Locked Loop (DLL) attains the CRB. So it makes sense trying to find the particular ranging codes $\mathbf{c}$'s that minimize the CRB (4) for delay estimation.

3. MINIMIZATION OF $CRB(\varepsilon)$ UPON THE CODE

3.1 Optimization criterion

In this section, the relation between ranging codes and $CRB(\varepsilon)$ is investigated. The dependence of $CRB(\varepsilon)$ on the ranging code $\mathbf{c}$ is not apparent in (4). Nonetheless, $r(t)$ and thus $\mathbf{r}$ depend on $\{c_k\}$ as in (3), and so $CRB(\varepsilon)$ is a function of the particular values of $\{c_k\}$.

After some manipulations, $CRB(\varepsilon)$ can be rewritten as

$$CRB(\varepsilon) = \frac{N_0}{T_c \int_{T_{obs}} \left| \frac{\partial x(t-\tau)}{\partial \tau} \right|^2 dt}, \tag{5}$$

where T_{obs} is the observation interval. Therefore, to increase accuracy, the optimum ranging code $\mathbf{c}_{opt}$ should be such that

$$\mathbf{c}_{opt} = \underset{\mathbf{c}}{\operatorname{argmax}} \left\{ \int_{T_{obs}} \left| \frac{\partial x(t-\tau)}{\partial \tau} \right|^2 dt \right\}. \tag{6}$$

Let $T_{obs} = NT_c$, with $N \gg 1$. Neglecting the constant term $\sqrt{2C}$ in (3) and taking into account the boundary effect in (6),

$$\int_{T_{obs}} \left| \frac{\partial x(t-\tau)}{\partial \tau} \right|^2 dt \approxeq \sum_{\ell=0}^{N-1} \sum_{k=0}^{N-1} c_\ell c_k \int_{-\infty}^{+\infty} \dot{g}(t-\ell T_c-\tau)\dot{g}(t-kT_c-\tau)dt, \tag{7}$$

with $\dot{g}(t) \triangleq \partial g(t)/\partial t$. Let $m[\ell] \triangleq \frac{\partial^2}{\partial t^2}[g(t) \otimes g(-t)]|_{t=\ell T_c}$. In view of Parseval's theorem, (7) can be rewritten as

$$\int_{T_{obs}} \left| \frac{\partial x(t-\tau)}{\partial \tau} \right|^2 dt = -T_c \sum_{\ell=0}^{N-1} \sum_{k=0}^{N-1} c_\ell c_k m[k-\ell].$$

As can be verified, for each finite-energy pulse $g(t)$ there exists ℓ' such that $|m[\ell]/m[0]| \ll 1$ for $|\ell| > \ell'$, with $m[0] < 0$ and $m[-\ell] = m[\ell]$. Thus, $\mathbf{c}_{opt}$ can be well approximated by

$$\mathbf{c}_{opt} \approxeq \underset{\mathbf{c}}{\operatorname{argmin}} \left\{ \sum_{\ell=1}^{\ell'} \operatorname{sgn}\{m[\ell]\} R_c^{(a)}(\ell) \right\}, \tag{8}$$

where $R_c^{(a)}(\ell)$ is the *aperiodic* auto-correlation of $\mathbf{c}$

$$R_c^{(a)}(\ell) \triangleq \frac{1}{N} \sum_{n=0}^{N-1-\ell} c_n c_{n+\ell},$$

and

$$\operatorname{sgn}\{x\} \triangleq \begin{cases} +1, & x \geq 0, \\ -1, & x < 0. \end{cases}$$

Our aim is to find a set $\mathscr{C}$ of optimum codes to be possibly assigned to different satellites. So far, no considerations were made about off-peak auto-correlation and cross-correlation properties, which play a central role in systems engineering [6]. Let

$$R_{c^{(s)}}^{(p)}(\ell) \triangleq \frac{1}{N} \sum_{n=0}^{N-1} c_n^{(s)} c_{n+\ell \,(\mathrm{mod}\, N)}^{(s)} \tag{9}$$

and

$$R_{c^{(s)}c^{(t)}}^{(p)}(\ell) \triangleq \frac{1}{N} \sum_{n=0}^{N-1} c_n^{(s)} c_{n+\ell \,(\mathrm{mod}\, N)}^{(t)}$$

be the *periodic* auto-correlation sequence of $\mathbf{c}^{(s)}$ and *periodic* cross-correlation sequence between $\mathbf{c}^{(s)}$ and $\mathbf{c}^{(t)}$, respectively. Ideally, our sequences should belong to a set $\mathscr{C}$, where the following two properties hold:

- for each sequence $\mathbf{c}^{(s)} \in \mathscr{C}$, $\left| R_{c^{(s)}}^{(p)}(\ell) \right| \ll 1$ for $\ell \neq 0$;
- for each pair $\mathbf{c}^{(s)}, \mathbf{c}^{(t)} \in \mathscr{C}$, $\left| R_{c^{(s)}c^{(t)}}^{(p)}(\ell) \right| \ll 1$ $\forall \ell$.

In particular, these two conditions can be expressed as

$$\begin{cases} \left| R_{c^{(s)}}^{(p)}(\ell) \right| \leq \lambda_a, & \forall \ell \neq 0, \forall \mathbf{c}^{(s)} \in \mathscr{C}, \quad \text{(10a)} \\ \left| R_{c^{(s)}c^{(t)}}^{(p)}(\ell) \right| \leq \lambda_c, & \forall \ell, \forall \mathbf{c}^{(s)}, \mathbf{c}^{(t)} \in \mathscr{C}. \quad \text{(10b)} \end{cases}$$

Let us now go back to the main issue. It can be shown [8] that

$$R_c^{(p)}(\ell) = R_c^{(a)}(\ell) + R_c^{(a)}(\ell - N).$$

For small values of ℓ, as in (8), $R_c^{(a)}(\ell - N)$ cannot be large, as the number of correlated symbols is small. In particular, $|R_c^{(a)}(\ell - N)| \leq \ell/N$. Thus,

$$R_c^{(p)}(\ell) - \frac{\ell}{N} \leq R_c^{(a)}(\ell) \leq R_c^{(p)}(\ell) + \frac{\ell}{N},$$

and we can use such (lower or upper) bounds for $R_c^{(a)}(\ell)$ to minimize (8) according to $\operatorname{sgn}\{m[\ell]\}$.

Under the constraint given by (10a), $\mathbf{c}_{opt}$ may be chosen such that

$$R_{c_{opt}}^{(p)}(\ell) = -\operatorname{sgn}\{m[\ell]\} \cdot \lambda_a, \qquad 1 \leq \ell \leq \ell'.$$

Unfortunately, this method generates a very low number of sequences. To increase the cardinality of $\mathscr{C}$, suboptimal sequences can be designed, where an allowed interval is chosen for each $R_c^{(p)}(\ell)$, $1 \le \ell \le \ell'$. If $\mathrm{sgn}\{m[\ell]\} = -1$, $R_c^{(p)}(\ell)$ is allowed to stay in the interval $[\theta(\ell), \lambda_a]$, where $0 \le \theta(\ell) \le \lambda_a$. At the same time, if $\mathrm{sgn}\{m[\ell]\} = +1$, $R_c^{(p)}(\ell)$ is allowed to stay in the interval $[-\lambda_a, -\theta(\ell)]$.

The approach discussed in the following first considers constraints on auto-correlation and then tests the designed sequences for acceptable cross-correlation properties.

3.2 Design Algorithm Description

Once the optimization criterion is settled, we have to find the solution to the minimization problem (8). The method proposed for designing sequences with auto-correlation properties as in (8) requires a little number theory background, which is the theoretical study of residue classes [7]. In particular, this algorithm applies only to codes whose lengths N are prime numbers.

The periodic auto-correlation sequence (9) can be rearranged as [8]

$$R_c^{(p)}(\ell) = \frac{1}{N}\Big[p_{++}(\ell) + p_{--}(\ell) - p_{+-}(\ell) - p_{-+}(\ell)\Big],$$

where $p_{\pm\pm}(\ell)$ is the number of agreements between symbols ± 1 of sequence $\mathbf{c}$ and symbols ± 1 of the replica code *cyclically shifted* by ℓ steps.

Let any integer e, such that $N \equiv 1 \pmod{e}$, be defined as the *number of residue classes* and let $f \triangleq (N-1)/e$. By viewing $p_{\pm\pm}(\ell)$ in terms of residue classes, any arbitrary periodic auto-correlation sequence can be written as

$$R_c^{(p)}(\ell) = \frac{1}{N}\Big[N - 4fq + 4\sum_{i=0}^{q-1}\sum_{j=0}^{q-1}(d_i - x_\ell, d_j - x_\ell)\Big], \quad (11)$$

where q is the number of the residue classes that are actually selected in the design of the code[2], x_ℓ is the identifier of the residue class of ℓ, d_i is the identifier of the $i-$th chosen class, belonging to a set

$$\mathscr{D} \triangleq \Big\{ \big(d_0, \cdots, d_{q-1}\big) : 0 \le d_0 < \cdots < d_{q-1} \le e-1 \Big\},$$

and $(d_i - x_\ell, d_j - x_\ell)$ are the *cyclotomic numbers* [9].

Calculating the cyclotomic numbers can be performed analytically only for specific N [9], [10], because the complexity of the system grows linearly with e. Thus, in this paper such numbers are obtained via computer search.

In view of (11), (10a) can be expressed as

$$\left|R_c^{(p)}(\ell)\right| = \left|\frac{1}{N}\Big[1 - ef + 4\sum_{i=0}^{q-1}\sum_{j=0}^{q-1}(d_i - x_\ell, d_j - x_\ell)\Big]\right| \le \lambda_a, \qquad 0 \le x_\ell \le e-1, \quad (12)$$

where typical values for λ_a are derived in Section 3.3.

From (12) we get immediately

$$\begin{cases} \displaystyle\sum_{i=0}^{q-1}\sum_{j=0}^{q-1}(d_i - x_\ell, d_j - x_\ell) \ge \eta_L(x_\ell), \\ \displaystyle\sum_{i=0}^{q-1}\sum_{j=0}^{q-1}(d_i - x_\ell, d_j - x_\ell) \le \eta_U(x_\ell), \end{cases}$$

with $0 \le x_\ell \le e-1$, where

$$\begin{aligned} \eta_L(x_\ell) &\triangleq \left\lceil \frac{ef - 1 - 2\psi f - N\lambda_a}{4} \right\rceil \\ \eta_U(x_\ell) &\triangleq \left\lfloor \frac{ef - 1 - 2\psi f + N\lambda_a}{4} \right\rfloor \end{aligned} \quad (13)$$

denote the lower and upper bounds for each x_ℓ, respectively, $\lceil\cdot\rceil$ and $\lfloor\cdot\rfloor$ denote ceiling and floor functions, respectively, and $\psi \equiv e \pmod{2}$.

For each ℓ, $1 \le \ell \le \ell'$, a corresponding $\theta(\ell)$ has to be established. To make sure that the constraint set by $\theta(\ell)$ is actually met, we can apply the following algorithm:

- $\forall\, x_\ell$, $0 \le x_\ell \le e-1$, initialize $\eta_L(x_\ell)$ and $\eta_U(x_\ell)$ as in (13).
- for each ℓ, $1 \le \ell \le \ell'$:
  if $\mathrm{sgn}\{m[\ell]\} = -1$ and $\left\lceil \frac{ef-1-2\psi f+N\theta(\ell)}{4} \right\rceil > \eta_L(x_\ell)$, then
  $$\eta_L(x_\ell) = \left\lceil \frac{ef - 1 - 2\psi f + N\theta(\ell)}{4} \right\rceil;$$
  if $\mathrm{sgn}\{m[\ell]\} = +1$ and $\left\lfloor \frac{ef-1-2\psi f-N\theta(\ell)}{4} \right\rfloor < \eta_U(x_\ell)$, then
  $$\eta_U(x_\ell) = \left\lfloor \frac{ef - 1 - 2\psi f - N\theta(\ell)}{4} \right\rfloor.$$
- amidst the $e!/[q!\cdot(e-q)!]$ possible sets $\mathscr{D}$'s of class identifiers, choose all the ones such that
  $$\eta_L(x_\ell) \le \sum_{i=0}^{q-1}\sum_{j=0}^{q-1}(d_i - x_\ell, d_j - x_\ell) \le \eta_U(x_\ell), \qquad 0 \le x_\ell \le e-1.$$
- find the spreading code associated to each $\mathscr{D}$, following
  $$c_k = \begin{cases} +1, & \text{if } x_k \in \mathscr{D}, \\ -1, & \text{if } x_k \notin \mathscr{D} \text{ or } k = 0. \end{cases}$$

The algorithm can be applied a number of times up to $\frac{N-1}{2}$ for each possible different value of e, in order to enlarge the set of sequences $\mathscr{C}_a$.

The last requirement to be verified is the constraint on the cross-correlation (10b) to come to a final set of *Minimum Jitter Sequences* (MJSs) $\mathscr{C} \subseteq \mathscr{C}_a$.

The periodic cross-correlation $R^{(p)}_{c^{(s)}c^{(t)}}(\ell)$ can be seen in terms of number of agreements between ± 1 of $\mathbf{c}^{(s)}$ and ± 1 of cyclically shifted $\mathbf{c}^{(t)}$. For sequences with different e's, it is easy to verify that $\left|R^{(p)}_{c^{(s)}c^{(t)}}(\ell)\right| \le \lambda_a \le \lambda_c$ for every ℓ. The same result can be observed for sequences with the same e, but only with $\ell \ne 0$. The cross-correlation $R^{(p)}_{c^{(s)}c^{(t)}}(0)$ for sequences with the same e may be on the contrary quite larger. This is because two sequences $\mathbf{c}^{(s)}$ and $\mathbf{c}^{(t)}$ with the same e bear a number of agreements proportional to the number of class identifiers that belong to both $\mathscr{D}^{(s)}$ and $\mathscr{D}^{(t)}$ plus those that (jointly) do not. By means of the relations between N, f, e and q, a sequence $\mathbf{c}^{(s)}$ finally belongs to $\mathscr{C}$ iff the relation

$$\left\lceil \frac{1}{4}\left(e - 2\psi - \frac{N\lambda_c}{f}\right) \right\rceil \le \chi \le \left\lfloor \frac{1}{4}\left(e - 2\psi + \frac{N\lambda_c}{f}\right) \right\rfloor$$

[2] To balance the number of symbols $+1$ and -1, q is set to $e/2$ if e is even, and to $(e-1)/2$ if e is odd [7].

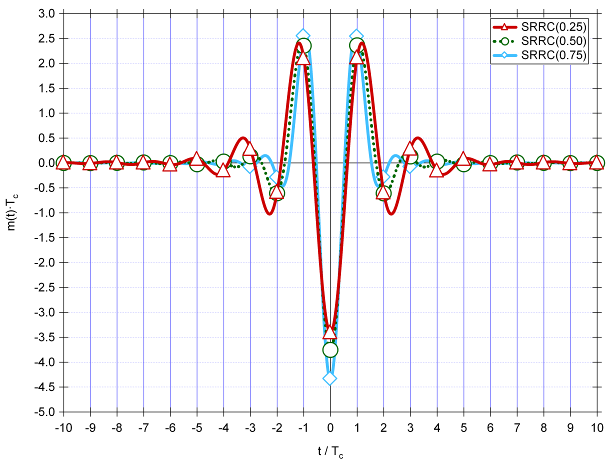

Figure 1: Shape of $m(t)$ with SRRC pulses.

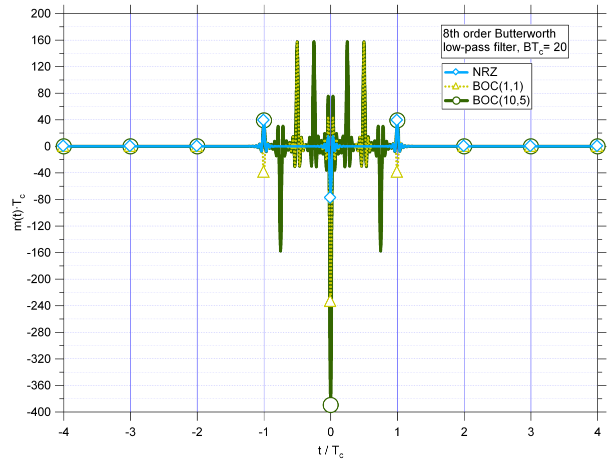

Figure 2: Shape of $m(t)$ with filtered NRZ and BOC pulses.

is accomplished $\forall \mathbf{c}^{(t)} \in \mathscr{C}$, $\mathbf{c}^{(t)} \neq \mathbf{c}^{(s)}$, where χ is the number of class identifiers belonging to both $\mathscr{D}^{(s)}$ and $\mathscr{D}^{(t)}$.

3.3 Numerical results

This section contains some numerical results, obtained following the method derived in Sections 3.1 and 3.2.

Figures 1 and 2 show the shape of $m(t)$ when using SRRC pulses with different roll-off factors α, and filtered NRZ or BOC pulses[3] [11] – [12], respectively. As can be seen, typical values are $\ell' = 3$ and $\ell' = 1$, respectively. It can easily be understood that the improvement that can be attained with MJSs is larger for larger values of ℓ', i.e. for strongly band-limited pulses. In the following, we will use a SRRC pulse with $\alpha = 0.25$ and $\ell' = 3$.

To provide a fair comparison with Gold codes for GPS (C/A), the chosen length is $N = 1021$, representing the closest prime number to 1023.

Values of λ_a and λ_c are chosen by means of probabilities of missed detection P_{MD} and false alarm P_{FA} during the acquisition stage in a typical scenario for GPS receivers. The observation length is supposed to be $L \cdot N$ chip times. In a first approximation, we model the interfering codes as Gaussian interference[4] independent of thermal noise, so that the variance of the total noise term W after correlation is

$$\sigma_W^2 = \frac{\sigma^2}{L \cdot N} + \frac{I}{N},$$

where I is the number of interferers and $\sigma^2 = N_0/(2E_c)$, with $E_c = C \cdot T_c$. Hence, P_{MD} can be computed as

$$P_{MD} = \Pr\{1 + W \leq \lambda\} \approxeq Q\left(\frac{1-\lambda}{\sigma_W}\right),$$

[3]Even though "theoretical" GPS, SBAS and Galileo signals are specified with rectangular (hence, infinite-bandwidth) pulses, some form of band-limitation is introduced by the satellite transponder. In Fig. 2, ideal pulses are thus filtered with Butterworth low-pass filters.

[4]This approximation is used only for choosing design parameters. Performance with real interference will be derived by simulation later on.

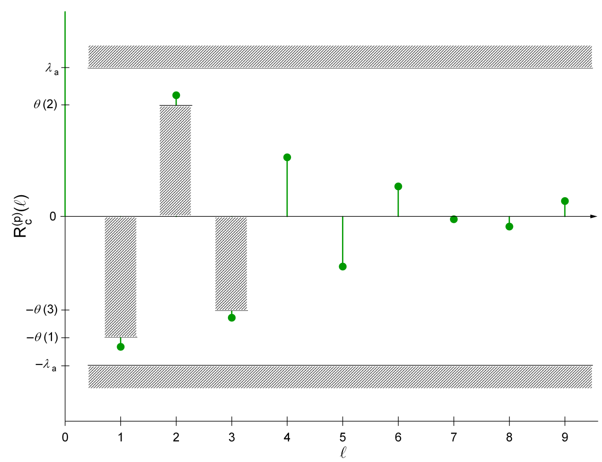

Figure 3: Auto-correlation mask.

where $Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-t^2/2} dt$, and P_{FA} is given by

$$P_{FA} = \Pr\{R_c^{(p)}(\ell) + W > \lambda | \ell \neq 0\} \approxeq Q\left(\frac{\lambda - R_c^{(p)}(\ell)}{\sigma_W}\right) \leq Q\left(\frac{\lambda - \lambda_a}{\sigma_W}\right).$$

By choosing $P_{MD} = 10^{-3}$ and $P_{FA} = 10^{-8}$, representing state-of-the-art values at such signal-to-noise ratio for commercial GPS receivers, and assuming $L = 20$, $C/N_0 = 40\,\text{dBHz}$, $T_c = 1\,\mu\text{s}$, $I = 7$, we get

$$\lambda_a \leq 1 - \sigma_W \cdot \left[Q^{-1}(P_{MD}) + Q^{-1}(P_{FA})\right] \approxeq 0.16.$$

We also set $\lambda_a = \lambda_c = 0.16$, $\theta(1) = 0.13$, $\theta(2) = 0.12$, and $\theta(3) = 0.1$, and we represent our constraints on the auto-correlation sequence as a "mask" depicted in Fig. 3 (which considers the particular values of $\text{sgn}\{m[l]\}$ for $1 \leq \ell \leq 3$, as in Fig. 1). Fig. 3 also reports the computed values $R_c^{(p)}(\ell)$ for a sequence resulting from our algorithm.

We now compute the performance of MJSs with respect to Gold codes as far as acquisition is concerned. Comparison is performed using a set whose cardinality ($|\mathscr{C}| = 42$)

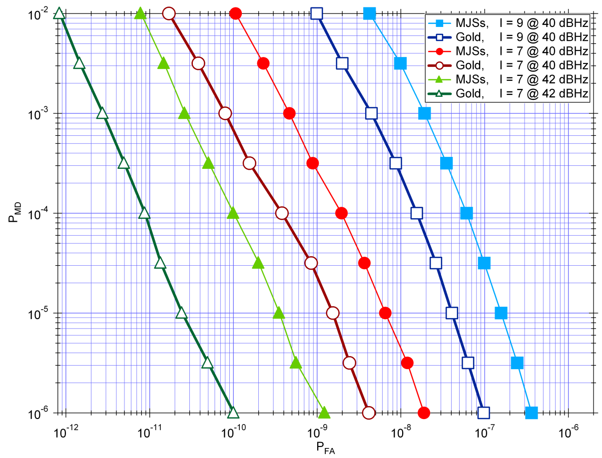

Figure 4: Performance in the acquisition stage.

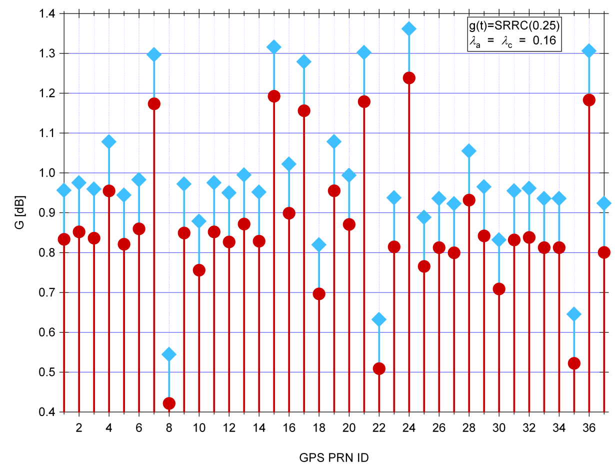

Figure 5: Gain of MJSs in terms of jitter variance.

is comparable with the number of Gold codes for GPS. Fig. 4 shows the pair (P_{MD}, P_{FA}) for a few values of the number of interfering codes I and for C/N_0 ratios. As can be seen, performance of proposed sequences is worse than the one achieved with Gold codes. This result could be foreseen owing to the properties of Gold codes, which show the minimum cross-correlation. However, as the number of visible satellites is typically $8 \div 10$, P_{FA}'s are still good in typical operating conditions for GPS receivers.

On the other hand, improvement in delay estimation variance during the tracking stage is directly connected to position accuracy, as stated in Section 2, and is our fundamental goal. Fig. 5 reports the gain

$$G \triangleq 10\log_{10}\left(\frac{CRB_{\mathbf{c}}(\varepsilon)}{CRB_{\mathbf{c}_i^g}(\varepsilon)}\right),$$

where $\mathbf{c} \in \mathscr{C}$, $\mathbf{c}_i^g$ is the Gold code of the GPS satellite with the $i-$th PRN ID, and CRB is computed as in (5). For each Gold code (from 1 to 37) we show the gain G with respect to the best-performing (diamond) and the worst-performing (circle) MJS. As can be seen, in many cases each MJS shows gains over 0.8 dB with respect to the Gold codes in terms of jitter variance. To evalute improvements in position accuracy using such shaping pulse, the standard deviation σ_ρ of each pseudorange can be reduced from 36.4 m to 32.7 m.

4. CONCLUSIONS

An algorithm to design Minimum Jitter Sequences (MJSs) to be used as binary ranging codes for satellite positioning systems has been presented. In particular, after discussing relationships between time-delay estimation variance and chip pattern of ranging codes, numerical techniques have been developed for minimizing such variance.

Results achieved show a trade-off between performance in the acquisition phase and during tracking. In practical situations, the MJSs provide good albeit not optimum performance in terms of probability of false alarm and missed detection. At the same time, they allow the variance of each pseudorange measurement to be reduced, thus improving position accuracy.

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