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NEW MARKOV RANDOM FIELD MODEL BASED ON NAKAGAMI DISTRIBUTION FOR MODELING ULTRASOUND RF ENVELOPE

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ABSTRACT

The aim of this paper is to propose a new Markov Random Field (MRF) model for the backscattered ultrasonic echo in order to retrieve information about backscatter characteristics, such as the density, the scatterer amplitude, the scatterer spacing and the direction of interaction. The model combines the Nakagami distribution that describes the envelope of backscattered echo with spatial interaction using MRF. We first construct the Nakagami-MRF model and illustrate the role of its parameters by some synthetic simulations. Then, to enhance the ability of this MRF model to retrieve information on the spatial backscatter distribution, we compare the parameter values estimated on simulated radio-frequency (RF) envelope image for different tissue scatterers characteristics (density, amplitude, spacing, spatial orientation). It follows that the first parameter is related to the density and the amplitude, and, the interaction parameters are related to the scatterer spacing and the orientation.

1. INTRODUCTION

Many researchers have used stochastic models to describe the envelope of the backscattered echo of tissues, called the radio-frequency (RF) envelope. The parameters of these distributions depend on some characteristics such as the density (number of scatterers[1] within the resolution cell of the transducer), and scatterer amplitude related to the size of the scatterers. We can mention : Rayleigh distributions (square root of an exponential distribution), K-distribution [1] (square root of the product of a Gamma distribution with an exponential distribution), and Nakagami (square root of a Gamma distribution). The Rayleigh model is commonly employed [2], but under some conditions, such as the presence of a large number of randomly located scatterers. Wagner [3] classifies the other models according to their Signal to Noise Ratio (SNR) compared with the SNR of Rayleigh distribution. The first class called pre-Rayleigh $(SNR < 1.91)$ describes heterogenous texture. The second, called Rayleigh (SNR=1.91), appears as homogenous texture class. The third corresponding to the periodic texture is the post-Rayleigh class $(SNR > 1.91)$.

The K-distribution was shown to model pre-Rayleigh and Rayleigh texture [4, 5]. The two parameters of K-distribution, provide information on the number of scatterers, the variation in the scattering amplitude and the average scattering amplitude. But it is not general enough to describe the statistics of the backscattered echo from range cell containing a periodic alignment of scatterers giving rise to post-Rayleigh. The much simpler model based on the Nakagami distribution was proposed in [6, 7] to characterize the ultrasonic tissue. In addition to scattering amplitude and density, This model can take into account the regularity of the scatterer spacing [8].

However, as the previous ones, this distribution can't describe any anisotropic property of the texture. The Markov Random Field is a powerful tool to model the probability of spatial interactions in an image and has been extensively applied to extract texture features for image characterization and classification. We found in literature, many attempts to model spatially these images using Markov Random Fields. The most common is the Gaussian MRF [9, 10], but does not fit to the envelope distribution of the backscattered echo. In [11, 12] it has been shown that the K-MRF model based on the K-distribution locally guarantees better fit to data but it can't respect post-Rayleigh statistics. So, we propose in this paper a new Markov Random Field for textured ultrasound envelope image based on Nakagami distribution which is a priori more relevant than the Gaussian MRF or K-MRF. The construction of the model is based on some properties of MRF and is introduced in the following section. To evaluate and understand the parameters role in the model, we use an ultrasound RF simulator that realistically models the physical process in RF signal generation, and uses the density, spacing and the amplitudes for the scattering process.

The paper is organized as follows. First, we introduce the Nakagami-MRF model and its parameters. Second, we simulate the model for different values of the interaction parameters and study the case of a very similar histogram. Next, some experiments with a realistic Ultrasound (US) RF simulator [6, 13] are processed with the estimation of the MRF parameters by Conditional Least Square (CLS) method. The link with the backscatter characteristics is established. The analysis, the discussion, and some concluding remarks close up the paper.

2. STATISTICAL MODEL: NAKAGAMI DISTRIBUTION

The density function of the envelope of the backscattered signal can be described in terms of the Nakagami distribution. With parameters (m,Ω), its density function of the amplitude

[1] scatterers are defined as small structures in tissue, reflecting and scattering the incoming wave

X of the RF envelope at x is given by:

$$f_{m,\Omega}(x) = \frac{2m^m}{\Gamma(m)\Omega^m} x^{2m-1} \exp(-\frac{m}{\Omega}x^2) \quad \forall x \in \mathbb{R}_+, \qquad (1)$$

here $\Gamma(.)$ is Gamma function. The parameter $\Omega > 0$ is scaling parameter. The shape parameter m is constrained to be greater to 0.5 ($m \geq 0.5$) [14]. Note that for m=0.5, the density function is half Gaussian, while for m=1, the density function is Rayleigh. For $m > 1$ the density appears to be similar to Rician [6]. For values of $0.5 < m < 1$, the density function can be described as pre-Rayleigh.

The Nakagami parameters (m,Ω) can be obtained from the moments of the envelope as follows:

$$\begin{cases} \Omega = E\{X^2\} \\ m = \dfrac{\Omega^2}{E\{(X^2-\Omega)^2\}} \end{cases} \qquad (2)$$

It is possible to see that the Nakagami distribution can be identified as belonging to the class of density functions such as Gamma distribution. If we define a new random variable A of Gamma distribution with parameter (α,β) and $X = \sqrt{A}$, the probability density function of X, $f_{\alpha,\beta}(x)$ can be shown to be:

$$f_{\alpha,\beta}(x) = \frac{2\beta^\alpha}{\Gamma(\alpha)} x^{2\alpha-1} \exp(-\beta x^2) \qquad (3)$$

which is Nakagami distribution with parameter $(m = \alpha, \Omega = \frac{\alpha}{\beta})$. For convenient, we use the density given by the equation 3 as the density function of the Nakagami distribution with paramters (α,β). The second moment of this distribution is then given by:

$$E\{X^2\} = \frac{\alpha}{\beta} \qquad (4)$$

3. NAKAGAMI-MRF MODEL

Our goal is to model the RF envelope image by a spatial model based on the Nakagami distribution. Therefore, we model the envelope amplitude image X by a Markov random field which allows us to take into account the spatial information between each pixel s of the pixel set S of the image.

3.1 Spatial Model: Nakagami-MRF Presentation and Features

Readers are referred to [9, 15], for details of MRF models, a recognized technic for modeling image textures. In the two dimensional image lattice S, the pixel values $x = \{x_s/s \in S\}$ are a realization of random variables $X = \{X_s/s \in S\}$. So, we suppose that at each pixel s of S, the envelope amplitude X_s given $X_{V_s} = (x_r)_{r\in V_s}$, the envelope amplitude of the pixels of the neighbourhood V_s of s, centered at s, follows Nakagami distribution with parameters depending on X_{V_s}

$$(X_s/X_r = x_r, r \in V_s) \propto Nakagami(\alpha_s, \beta) \qquad (5)$$

where α_s is defined as follows:

$$\alpha_s = \frac{1}{2}\left(a_s + 1 + \sum_{r\in V_s} b_{sr} \ln x_r\right) \qquad (6)$$

which (a_s, b_{sr}, β) are the parameters of the model with $\beta \in \mathbb{R}_+^*$ and $(a_s, b_{sr}) \in \mathbb{R}^2$.

As often, we consider here a stationary field of order 2. Therefore, first, $b_{sr} = 0$ for all r outside of the 8 nearest neighbor pixel of s, defining the pixel set of neighbourhood of s. Second, at each pixel $s \in S$, $a_s = a$ and the interaction parameters $b_{sr} = b_i$ of pixel pair $< s,r >$ when the pixel r (resp. s) is located at the relative position $i \in \{1,2,3,4\}$ from s (resp. r) as shown in table 1. In order to understand

3	2	4
1	s	1
4	2	3

Table 1: Relative position

the role of the parameters of this Markovian model, some simulations have been done. Simulations in figure (1.a,.b,.c) permit to understand the role of interaction parameters of the field. Indeed, we note that when an interaction parameter (b_i) is positive, then the neighbord pixels according to the direction have similar intensities (attraction). The images in figure (1.a',.b',.c') show the value of α_s calculated for each s according to equation (6). The table (2) shows the values of (α,β) obtained by fitting the histogram of the images of figure (1.a,.b,.c) by Nakagami distribution by using the moment method. We can conclude that for different configurations of interactions b_i we can preserve the parameters of the Nakagami (α,β).

	fig.1.a	fig.1.b	fig.1.c
α	1.42	1.43	1.40
α/β	150	152	149

Table 2: Nakagami Parameter estimation of the examples in (Figure 1.a,.b,.c)

3.2 Construction of the model

At any pixel $s \in S$, let denote f_s the density function of X_s given the amplitude observations of its neighbourhood (V_s), $(X_r)_{r\in V_s} = (x_r)_{r\in V_s}$. The construction of our Markovian model use the following property [15] :

Property *If the Markovian random field X belongs to exponential family:*

$$\ln f_s(x_s) = A_s((x_r)_{r\in V_s}) B_s(x_s) + C_s(x_s) + D_s((x_r)_{r\in V_s}), \quad (7)$$

with $B_s(1) = C_s(1) = 0$.

Then, it exists a_s *et* b_{sr} *defined:*

$$A_s((x_r)_{r\in V_s}) = a_s + \sum_{r\in V_s} b_{sr} B_r(x_r). \qquad (8)$$

The global energy function is defined by:

$$U(x) = \sum_{s\in S}(a_s B_s(x_s) + C_s(x_s)) + \sum_{r\neq s, <s,r>} b_{sr} B_s(x_s) B_r(x_r). \qquad (9)$$

As the Nakagami distribution belonging to exponential family, we use this property in order to construct our Nakagami-MRF. After identification of the Log of $f_{\alpha,\beta}$ distribution of

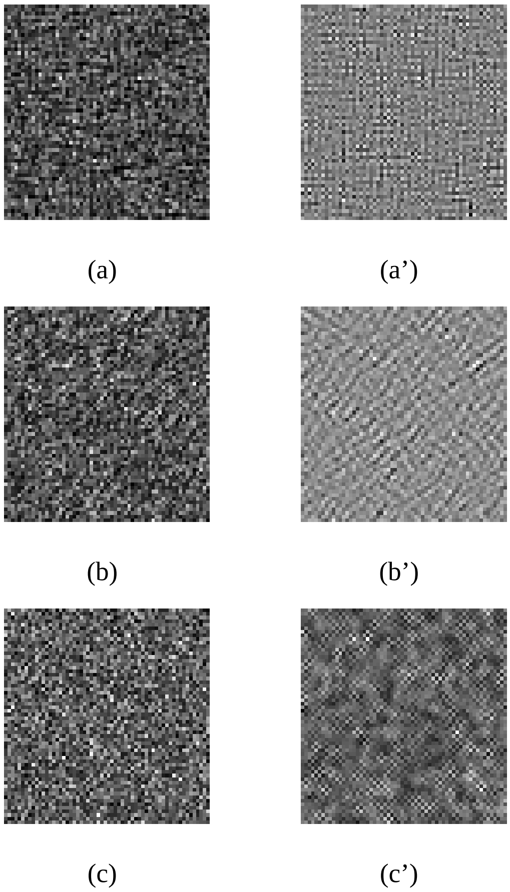


Figure 1: Three example realizations of Nakagami-MRF (a,b,c) using the energy function of the Nakagami model (eq.11) and their images α_s (a',b',c'). The visual perception is fundamentally different due to the different parameter vectors θ = (2, 0.5, 0.5,-0.5,-0.5,0.01)) (a), θ = (2,0,0,-0.5,0.5,0.01)(b) and θ = (6.5,-0.5,-0.5,0,0,0.01) (c).

parameters $(\alpha,\beta) \in \mathbb{R}_+^{*\,2}$ defined in equation (3) we have:

$$\begin{cases} A_s((x_r)_{r\in V_s}) = 2\alpha - 1 \\ B_s(x_s) = \ln x_s \\ C_s(x_s) = -\beta x_s^2 + \beta \\ D_s((x_r)_{r\in V_s}) = \ln(2) + \alpha \ln\beta - \Gamma(\alpha) - \beta \end{cases} \qquad (10)$$

while introducing the parameters a_s et b_{sr} in the expression of $A_s((x_r)_{r\in V_s})$, the value of α at each s,noted here α_s is then given by equation (6). while β remains constant and independent of the site s.

In this model, the energy function is defined as:

$$U(X=x) = \sum_{s\in S}(a_s \ln x_s - \beta x_s^2 + \beta) + \sum_{r\neq s, <s,r>} b_{sr} \ln x_s \ln x_r \qquad (11)$$

3.3 MRF-Parameter Estimation

Due to its computation efficiency, the Conditional least squares estimate (CLS) method has been commonly accepted to estimate the parameters of MRF models. The estimate of the parameter vector $\theta = (a_s, b_{sr}, \beta)$ is obtained by minimising the sum on all $s \in S$ of the quadratic difference between the squared amplitude x_s^2 and its conditional average :

$$\hat{\theta} = arg \min_{\theta} \sum_{s\in S} \left(x_s^2 - E\{X_s^2/V_s\}\right)^2 \qquad (12)$$

It follows from the equation (4) that the second moment of Nakagami-MRF is :

$$E\{X_s^2/X_r = x_r, r\in V_s\} = \frac{1}{2\beta}\left(a_s + 1 + \sum_{r\in V_s} b_{sr} \ln x_r\right) \qquad (13)$$

Then, we obtain the following equation:

$$2\beta x_s^2 = a_s + 1 + \sum_{r\in V_s} b_{sr} \ln x_r, \quad \forall s \in S \qquad (14)$$

The conditional least squares estimate of θ can be started forward.

To conclude, we set up the table (3) to compare the values of the parameters used for simulation (a, b_i, β) and the estimated parameters: $(\hat{a}, \hat{b_i}, \hat{\beta})$. According to the table, the estimation method show acceptable resemblance between used and estimated parameters.

		a	b_1	b_2	b_3	b_4	β
Fig.1.a	θ	2	0.5	0.5	-0.5	-0.5	0.01
	$\hat{\theta}$	1.96	0.48	0.48	-0.54	-0.46	0.009
Fig.1.b	θ	2	0	0	-0.5	0.5	0.01
	$\hat{\theta}$	1.97	-0.01	0.04	-0.53	0.46	0.009
Fig.1.c	θ	6.5	-0.5	-0.5	0	0	0.01
	$\hat{\theta}$	6.23	-0.48	-0.50	-0.09	0.10	0.009

Table 3: Parameters of the examples in (Figure 1) and their estimation using CLS.

4. EVALUATION ON SIMULATED RF ENVELOPE IMAGE

4.1 Backscatter characteristics

The three backscatter characteristics are density, spacing, and scatterer cross section (scatterer amplitude). Density is a measure of the average number of scatterers in the resolution cell of the US transducer. Spacing (or placement) refers to the randomness or regularity of the distances between scatterers. The scattering amplitudes or scattering cross section show the amplitude variations caused by many phenomenon like attenuation, absorption, diffraction.

Some RF simulators take into account these characteristics and consider them as being stochastic. Indeed, the simulator given in [6, 13] takes into account the randomness of the amplitudes by the signal to noise rate, SNR_a of the scattering cross-section. In this case, the amplitudes are gamma-distributed with shape parameter a_A^2 and with unit scale parameter. The scatterer spacing was also characterized. In [8], the gamma distribution has been shown to accurately describe scatterer spacing. By denoting the mean spacing between scatterers as $\underline{d}$, the spacing distribution is also described by the gamma distribution, $\gamma(u,v)$, with shape parameter u and scale parameter $v = \underline{d}/u$. For large u, the spacing is regular. For $u = 1$ the scatterers are randomly spaced (diffuse). When $u < 1$, the scatterers are clustered together [8]. Figure (2) demonstrates this behavior for various values of u when the mean inter-scatterer distance $\underline{d}$ is maintained at unity.

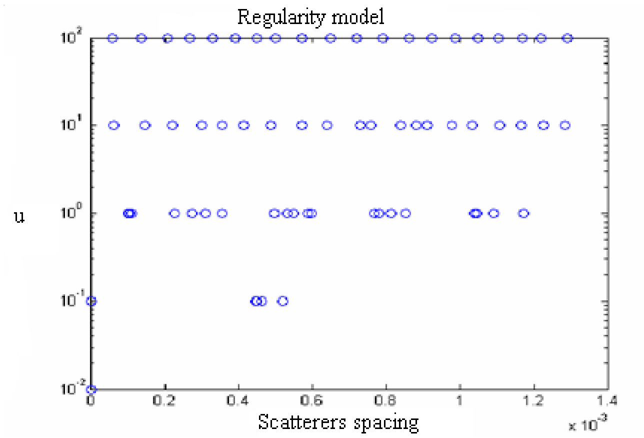


Figure 2: Spatial scatterer organization with variation of u

4.2 RF envelope image Simulator

The ultrasound RF simulator introduced in [6, 13] is used to generate echo envelopes. This simulator realistically models the actual physical process in RF signal generation, and uses the density (N), spacing (u), and amplitudes (SNR_a) to describe the scattering process. The parameters for the simulator are as follows: f_0 (center frequency)=3.5 MHz, B (bandwidth)= 0.8 MHz, v (velocity of sound) = 1446 m/s, sampling window size = 3,7 cm. In the simulation, the RF backscattered signal consisting of 100 A lines (1-D RF signals) was sampled at 40 MHz. Therefore, we obtained 2048 samples in each line. To ensure uncorrelated samples, every second sample in every A line is used. The received RF backscattered signal is demodulated at f_0 using the setup shown in figure (3), resulting in inphase and quadrature components X and Y, respectively. A 10th order 2 MHz Butterworth lowpass filter is applied to both components, and the echo envelope is computed as $\sqrt{X^2+Y^2}$ [13].

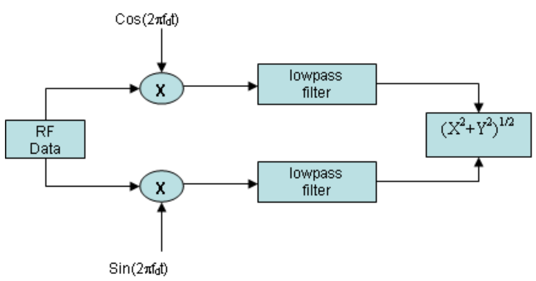
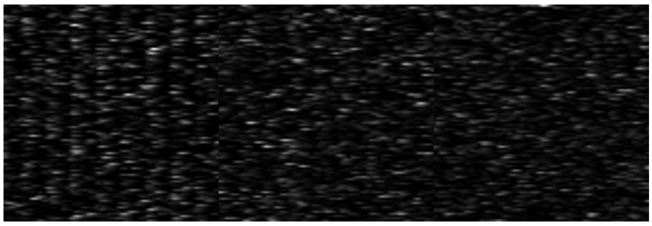


Figure 3: Block diagram of the processing path from RF signal to the envelope of the signal.

4.3 Evaluation Methods

We use the RF simulator described above to generate echo envelope with different configurations of the triplet (N, SNR_a, u). Our goal is to evaluate the ability of the Nakagami-MRF parameters to characterize backscatter characteristics, and then, to provide information on the number of the scatterers as well as the scattering amplitude, scatterer spacing and direction. We consider three examples. We suggest through these examples, first, to characterize the spacing of reflectors by the interaction parameter b_i, second, to represent the density and the scatterer amplitudes by the first parameter a_s, and finally to show the ability of the model to detect the direction of the regularity.

First example: we generate different scatterer spacing values u, but we maintain constant the density of reflectors $N = 15$ and the value $SNR_a = 0.4$ of the scattering cross-section. This particular case is done to show the influence of scatterers repartition and regularity on our spatial model. We generate three regions corresponding to three values of $u \in \{0.1, 1, 10\}$. Figure (4) shows these regions. To show



(a) (b) (c)

Figure 4: Three types of textures for three various organizations: region (a): less regular, $u = 0.1$, region (b): $u = 1$, region (c): more regular, $u = 10$.

the ability of our spatial model, we use the model parameters as textural features for evaluation of scatterers regularity. Briefly, we apply the model in each region of the simulated RF envelope image and we estimate the parameters by the CLS method mentioned above. The table (4) shows the parameters estimated for the three regions: (a),(b) and (c). For comparison, we notice that the values of $\frac{a+1}{2\beta}$ are constants, but the values of $\frac{b_1}{2\beta}$ increase from region (a) to region (c). We note that horizontal interaction is mentioned by b_1, because the chosen RF simulator excludes all interactions except the horizontal one. That's why, the others b_i, $i \in \{2,3,4\}$ are very weak. From region (a) to (c), the interaction $\frac{b_1}{2\beta}$ increases due to the regularity of the spacing between reflectors.

Regions	$\frac{a+1}{2\beta}$	$\frac{b_1}{2\beta}$	$\frac{b_2}{2\beta}$	$\frac{b_3}{2\beta}$	$\frac{b_4}{2\beta}$	β
u=0.1	2.01	0.3562	0.05	-0.00	-0.02	0.261
u=1	2.04	0.564	0.04	-0.01	-0.03	0.348
u=10	2.03	0.632	-0.02	0.01	0.03	0.376

Table 4: Parameters of the Nakagami-MRF estimated by CLS on the 3 regions displayed in figure 4.

second example: In this example $N = 20$ and $u = 1$, while using 4 different scattering amplitudes $SNR_a \in \{0.4, 0.8, 2, 5\}$. Table (5) shows the estimation values of $\frac{a+1}{2\beta}$

and $\frac{b_1}{2\beta}$ for these configurations. We observe that the values of $\frac{a+1}{2\beta}$ decrease well $\frac{b_1}{2\beta}$ rest unchanged. We conclude from this behavior, that the first parameter of the model a_s should be related to the scattering amplitudes.

SNR_a	$\frac{a+1}{2\beta}$	$\frac{b_1}{2\beta}$	β
0.4	1.96	0.610	0.413
0.8	1.59	0.652	0.667
2	1.42	0.636	0.882
5	1.41	0.633	0.910

Table 5: Parameters of the Nakagami-MRF estimated by CLS for configurations of $SNR_a = (0.4, 0.8, 2, 5)$. The others parameters of the RF simulator are constants $(N = 20, u = 1)$.

Third example: Now $SNR_a = 0.8$, $u = 1$, while using different density scatterers $N \in \{7, 15, 20, 50\}$. Table (6) shows the estimation values of $\frac{a+1}{2\beta}$ and $\frac{b_1}{2\beta}$ for these configurations. We observe that the estimated parameter $\frac{a+1}{2\beta}$ decrease and the estimated value of $\frac{b_1}{2\beta}$ are unchanged. We conclude from this behavior, that the first parameter of the model a should be related to the density of scatterers.

N	$\frac{a+1}{2\beta}$	$\frac{b_1}{2\beta}$	β
7	1.80	0.572	0.427
15	1.65	0.651	0.615
20	1.59	0.652	0.667
50	1.458	0.657	0.850

Table 6: Parameters of the Nakagami-MRF estimated by CLS for $N = (7, 15, 20, 50)$. The others parameters of the RF simulator are constants $(SNR_a = 0.8, u = 1)$.

4.4 Results

It is clear that the value of Nakagami distribution parameter α gives information about the homogeneity of region but not the direction of the interactions. The interaction parameters of our model is able to take into account this effect. The interaction parameter b_i of Nakagami-MRF model permits to distinguish between spacing regularity for scatterers. The information given by a concerns the density of the scatterers (N) and the scatterers amplitudes (SNR_a).

5. CONCLUSION AND DISCUSSION

A spatial Markov random field model is used to characterize ultrasound backscatter characteristics. Experimental results on simulated RF envelope image show the behavior of every parameter of the model. Then, the first parameter a_s of the Nakagami-MRF model is related to the effective number of the scatterers α and so, it characterize the density and the amplitude of the scatterers. Here, We preserve the properties of the statistical model: Nakagami-distribution. For the interaction parameters of the MRF model b_{sr}, it indicates the spacing scatterers. So, for a regular spacing, the interaction is strong and b_{sr} is important, and for irregular spacing, the parameter b_{sr} is weak.

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