ACCURATE SEQUENTIAL WEIGHTED LEAST SQUARES ALGORITHM FOR WIRELESS SENSOR NETWORK LOCALIZATION

Frankie K. W. Chan and H. C. So

Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong
email: k.w.chan@student.cityu.edu.hk, hcso@ee.cityu.edu.hk

ABSTRACT

Estimating the positions of sensor nodes is a fundamental and crucial problem in *ad hoc* wireless sensor networks (WSNs). In this paper, an accurate node localization method for WSNs is devised based on the weighted least squares technique with the use of time-of-arrival measurements. Computer simulations are included to evaluate the performance of the proposed approach by comparing with the classical multidimensional scaling method and Cramér-Rao lower bound.

1. INTRODUCTION

Recent technological advances in wireless communications and microsystem integration have enabled the development of small, inexpensive, low-power sensor nodes which are able to collect surrounding data, perform small-scale computations and communicate among their neighbors. These wirelessly connected nodes, when working in a collaborative manner, have great potential in numerous remote monitoring and control applications [1] such as asset management, habitat monitoring, health caring, building automation, battlefield surveillance as well as environment observation and forecasting. Since sensor nodes are often arbitrarily placed with their positions being unknown, sensor positioning is a fundamental and crucial issue for the wireless sensor network (WSN) operation and management.

Node localization methods can be generally classified as the deterministic [2]-[9] and probabilistic approaches [10]-[11]. The simplest deterministic technique is to exploit the connectivity information — who is within the communication range of whom — to derive the node positions with the use of the anchor nodes subject to the proximity constraints imposed by the known connections, but it only provides coarse-grain location estimates. Mathematically, this can be formulated as a linear programming or semi-definite programming problem [3]. Apart from connectivity, range-based schemes utilize node-to-node or hop distances and/or angles, which are obtained from the pair-wise time-of-arrival (TOA), time-difference-of-arrival, received signal strength and/or angle-of-arrival measurements, for sensor positioning with higher location accuracy, although it is possible to use the average hop length and hop counts between indirectly connected nodes to deduce distance information [4] as well. Assuming that the range measurements errors are Gaussian distributed, the maximum likelihood (ML) methods for node localization correspond to the nonlinear least squares problem [5]-[7] which must be solved iteratively. In spite of attaining optimum estimation performance, the ML approach requires centralized data processing with intensive computations and sufficiently precise initial estimates for global convergence. Alternatively, the range-based measurements can also be converted into linear equations where the node positions are easily solved even in a distributed manner but at the expense of error accumulation [8]-[9]. Another computationally attractive range-based positioning technique is to employ multidimensional scaling (MDS) [2] which transforms the pair-wise distance information into the relative coordinates of nodes. On the other hand, particle filtering [10]-[11] is a representative example of the probabilistic approach, where each sensor stores a conditional density on its own coordinates based on its measurements and the conditional density of its neighbors for node localization, has a high potential of tracking purposes at the cost of excessive computational requirements. In this paper, a sequential algorithm for WSN positioning, which belongs to the deterministic category, is developed based on the weighted least squares (WLS) localization approach suggested in [12] and [13].

The rest of the paper is organized as follows. In Section 2, the simple and efficient localization approach of [12] and [13] will be firstly reviewed and then the development of the sequential weighted least squares (SWLS) algorithm is presented. Simulation results are included in Section 3 to evaluate the estimator performance by comparing with the classical MDS [2] method and Cramér-Rao lower bound (CRLB). Conclusions and future works are provided in Section 4.

2. ALGORITHM DEVELOPMENT

In this section, we are going to derive a computationally attractive node localization method. Before proceeding, we review the two-step weighted least squares (TSWLS) algorithm for position estimation of a single source [12]-[13], which acts as a key component of our estimation approach.

2.1 Review of Two-step Weighted Least Squares Localization Algorithm for Single Mobile Terminal

The TSWLS algorithm in [12] is shown to have close-to-CRLB performance when the noise is sufficiently small. In this subsection, its basic operation will be reviewed. Suppose there are $M \geq 3$ base stations (BSs) whose locations are known *a priori* and a single mobile terminal (MT) with unknown location. Let (x,y) be the MT position which is to be determined and the known coordinates of the ith BS be (x_i, y_i), $i = 1,2,\ldots,M$. The Euclidean distances between the MT and BSs can be easily determined from the corresponding TOA measurements, which are modelled as

$$r_i \quad = \quad d_i + n_i + q_i \mathbb{U}(\alpha - p) \quad i = 1,2,\ldots,M \qquad (1)$$

where $d_i = \sqrt{(x-x_i)^2 + (y-y_i)^2} = ct_i$ is the ideal distance with c and t_i are the speed of light and corresponding noise-free TOA, respectively. The second and third components represent the line-of-sight (LOS) error and possible non-line-of-sight (NLOS) error, respectively. Let $\mathbf{n} = [\ n_1 \ \cdots \ n_M \ ]^T$ be the LOS noise vector which is distributed as $\mathbf{n} \sim \mathcal{N}(\mathbf{0}, \mathbf{C})$ where $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Gamma})$ means the Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Gamma}$ and $\mathbf{C}$ is assumed to be known up to a proportionality constant. For the NLOS counterpart, $q_i \sim \mathcal{U}(0,R)$, R is the maximum NLOS distance, $\mathcal{U}(a,b)$ stands for the uniform distribution with a and b respectively the starting and ending points, $p \sim \mathcal{U}(0,1)$, $\alpha \in [0,1]$ is the probability of obtaining NLOS distance measurement and $\mathbb{U}(p)$ denotes the unit step function. Without measurement errors, (1) becomes

$$r_i = d_i = \sqrt{(x-x_i)^2 + (y-y_i)^2}, \quad i = 1, \cdots, M \tag{2}$$

Squaring both sides of (2) yields

$$\begin{aligned} r_i^2 &= x_i^2 + y_i^2 - 2x_i x - 2y_i y + x^2 + y^2 \\ \Rightarrow -2x_i x - 2y_i y + R &= r_i^2 - K_i, \quad i = 1, \cdots, M \end{aligned} \tag{3}$$

where $K_i \triangleq x_i^2 + y_i^2$ and $R \triangleq x^2 + y^2$. In doing so, (3) is reorganized into a set of linear equations in x, y and R. In the presence of noise, we define the first step estimation error $\mathbf{e}_1$ of the form:

$$\mathbf{e}_1 = \mathbf{h}_1 - \mathbf{G}_1 \mathbf{z} \tag{4}$$

where

$$\begin{aligned} \mathbf{h}_1 &= \begin{bmatrix} r_1^2 - K_1 \\ \vdots \\ r_M^2 - K_M \end{bmatrix} \\ \mathbf{G}_1 &= \begin{bmatrix} -2x_1 & -2y_1 & 1 \\ \vdots & \vdots & \vdots \\ -2x_M & -2y_M & 1 \end{bmatrix} \\ \mathbf{z} &= [\ x \ \ y \ \ R \ ]^T \end{aligned}$$

with the superscript T denotes the matrix transposition. In the first step, x, y and R are assumed to be independent and hence their WLS estimates are computed as

$$\hat{\mathbf{z}} = \left(\mathbf{G}_1^T \mathbf{W}_1 \mathbf{G}_1\right)^{-1} \mathbf{G}_1^T \mathbf{W}_1 \mathbf{h}_1 \tag{5}$$

where $\mathbf{W}_1^{-1} = \mathbb{E}\left(\mathbf{e}_1 \mathbf{e}_1^T\right) \approx \mathbf{BCB}$, $\mathbb{E}$ is the expectation operator, $\mathbf{B} = \operatorname{diag}(r_1, \cdots, r_M)$ is the diagonal matrix of range measurements. In the second step, the relationship of x, y and R is utilized and we define the resultant estimation error $\mathbf{e}_2$:

$$\mathbf{e}_2 = \mathbf{h}_2 - \mathbf{G}_2 \mathbf{z}_p \tag{6}$$

where

$$\begin{aligned} \mathbf{h}_2 &= [\ \hat{z}_1^2 \ \ \hat{z}_2^2 \ \ \hat{z}_3 \ ]^T \\ \mathbf{G}_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \\ \mathbf{z}_p &= [\ x^2 \ \ y^2 \ ]^T \end{aligned}$$

and $\hat{z}_i$, $i = 1,2,3$, represent the ith element of $\hat{\mathbf{z}}$. By only considering the linear terms of $\mathbb{E}\left(\mathbf{e}_2 \mathbf{e}_2^T\right)$ which is valid for small noise conditions, the WLS estimate of $\mathbf{z}_p$ is given by

$$\mathbf{z}_p = \left(\mathbf{G}_2^T \mathbf{W}_2 \mathbf{G}_2\right)^{-1} \mathbf{G}_2^T \mathbf{W}_2 \mathbf{h}_2 \tag{7}$$

where $\mathbf{W}_2^{-1} = \mathbb{E}\left(\mathbf{e}_2 \mathbf{e}_2^T\right) \approx \mathbf{B}_1 \operatorname{cov}(\hat{\mathbf{z}}) \mathbf{B}_1$, $\operatorname{cov}(\hat{\mathbf{z}}) = \left(\mathbf{G}_1^T \mathbf{W}_1 \mathbf{G}_1\right)^{-1}$ and $\mathbf{B}_1 = \operatorname{diag}(\hat{z}_1, \hat{z}_2, 0.5)$. The final position estimate is

$$[\ x \ \ y \ ]^T = \operatorname{diag}(sgn([\hat{z}_1, \hat{z}_2])) \sqrt{\mathbf{z}_p} \tag{8}$$

where *sgn* stands for the signum function. For more details and variants of the algorithm, interested reader is referred to [12] and [13].

2.2 Development of Sequential Weighted Least Squares Algorithm for *Ad Hoc* Wireless Sensor Networks

In this subsection, the TSWLS algorithm is extended for WSN localization. Let M be the total number of sensors and denote the position of the ith sensor by (x_i, y_i). For simplicity, the WSN is assumed to be fully-connected and the distance measurement between the ith and jth sensors is

$$r_{i,j} = d_{i,j} + n_{i,j} + q_{i,j} \mathbb{U}(\alpha - p) \quad i, j = 1, \cdots, M, \quad i < j \tag{9}$$

where $d_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$, $n_{i,j} \sim \mathcal{N}\left(0, \sigma_{i,j}^2\right)$, and $q_{i,j} \sim \mathcal{U}(0,R)$. The quantities $d_{i,j}$, $n_{i,j}$ and $q_{i,j}\mathbb{U}(\alpha - p)$ are respectively the noise-free distance, LOS error and NLOS error between the ith and jth sensors.

If all positions of the sensors are unknown, their position estimates can only be expressed in terms of a few ones. In this paper, concrete result for direct comparison and evaluation is preferred and thus relative position estimation is avoided. Without loss of generality, the positions of the first $k \geq 3$ sensors are assumed to be known in order to uniquely determine the locations of the other $(M-k)$ ones. As a result, all distance measurements among these sensors are free of noise, that is, $n_{i,j} = q_{i,j} = 0$, $i, j = 1, \cdots, k$, $i < j$.

In the first step, the distance information between unknown sensors will not be used. The position of the ith sensor, $i = k+1, \cdots, M$, is estimated using the distance measurements between the ith sensor and the k known sensors, that is, $r_{i,j}, \quad j = 1, \cdots, k$, by utilizing the TSWLS algorithm described in the previous subsection. In other words, we treat the k known sensors to be the BSs and the $(M-k)$ unknown ones to be the MTs, then apply the TSWLS one-by-one to get the position estimates of the $(M-k)$ sensors.

After obtaining the $(M-k)$ position estimates, the location of the ith sensor is re-estimated by assuming the position estimates of the other $(M-k-1)$ sensors to be the true locations. The TSWLS is applied again to estimate the position of the ith sensor using the distance measurements $r_{i,j}, \quad j = 1, \cdots, i-1, i+1, \cdots, M$. In a nutshell, we treat the ith sensor to be the MT and the other $M-1$ sensors to be the BSs, then update the $(M-k)$ sensors sequentially by the TSWLS. The main difference between this step and the previous step is that the number of "BSs" increases from k to $M-1$.

Finally, the second step is repeated again by treating the updated position estimates of the $(M-k)$ unknown sensors to be the true locations. The recursion is terminated to produce the finalized position estimates when the difference of results between successive iterations is sufficiently small.

To summarize, the SWLS works as follows:

1. Apply the TSWLS localization algorithm described in Section 2.1 to obtain the position of the ith sensor, $i = k+1, \cdots, M$, using the distance measurements $r_{i,j}$, $j = 1, \cdots, k$.

2. After all the positions of the $(M-k)$ unknown sensors have been estimated, apply the TSWLS to estimate the positions of the ith sensor, $i = k+1, \cdots, M$, using the distance measurements $r_{i,j}$, $j = 1, \cdots, M$, $j \neq i$.
3. Repeat Step 2 until convergence of position estimates.

It is noteworthy that the SWLS algorithm can be applied even when the WSN is not fully-connected as long as each sensor has at least three distance measurements. In that case, step 1 should be modified as:

1. Apply the TSWLS to obtain the position of the ith sensor, $i \in \{k+1, \cdots, M\}$ which is connected to at least 3 of the k known sensors using the corresponding distance measurements. After that, treat the estimated sensors to be the known sensors and estimate the positions of the remaining sensors. This process is repeated until the positions of all the $(M-k)$ sensors have been obtained.

Compared to other methods which require the WSN to be fully-connected or divide the WSN into smaller fully-connected regions, namely cliques, our algorithm is certainly more flexible and adapted to practical environment. Furthermore, the SWLS algorithm is distributed in nature, that is, it allows each sensor to share the computational burden and is not necessary to have a centralized computer to process the computations.

3. NUMERICAL EXAMPLES

Computer simulation has been conducted to evaluate the performance of the proposed TOA-based WSN positioning approach. We compare the mean square position errors (MSPEs) of the SWLS algorithm with the classical MDS as well as CRLB in WSN localization. The noise power of $n_{i,j}$ is obtained by $\sigma_{i,j}^2 = d_{i,j}^2/\mathrm{SNR}$ where SNR is the signal-to-noise ratio. In the following simulations, the total number of sensors M and the number of known sensors k are set to 14 and 4, respectively. The 10 unknown sensors are located in a 100 m $\times$100 m area and the 4 known sensors are located at the four corners as shown in Figure 1. The SWLS method is iterated 5 times for each trial and all results are averages of 1000 independent runs. The WSN is fully-connected and the average of the 10 MSPEs corresponding to the 10 unknown sensors is plotted to show the overall performance.

In the first scenario, all the distance measurements are considered as LOS paths and thus $\alpha = 0$. It can be seen from Figure 2 that the MSPEs of the MDS method are higher than the CRLB by over 8 dBm^2 while the MSPEs of the SWLS method are about 1 dBm^2 more than the CRLB, which indicates its approximate optimality.

In the second scenario, the effects of NLOS propagation are investigated. The probability of obtaining NLOS distance measurements and the maximum NLOS distance are set to $\alpha = 0.1$ and $R = 5$, respectively while the other settings are the same as the first scenario. It can be seen from Figure 3 that the MSPEs of the SWLS method are at least 5 dBm^2 less than those of the MDS method. It demonstrates that the superiority of the SWLS over the MDS method in NLOS propagation environment.

In the third scenario, the positions of the 10 unknown sensors are located randomly but limited to the 10000 m^2 area in each independent run. The CRLBs computed in the 1000 runs are averaged and all the other simulation parameters are the same as the first scenario. From Figure 4, it can be observed that the MSPEs of the SWLS method are about 2 dBm^2 above the CRLB while the difference between the MDS and the CRLB is around 7 dBm^2. It also demonstrates that the SWLS is generally more superior than the MDS method.

Finally, the third test is repeated with NLOS propagation. By comparing the MSPEs of the SWLS and MDS methods, the robustness of the SWLS is again shown in Figure 10, which indicates that in general, the SWLS method outperforms the MDS method in NLOS propagation situation.

4. CONCLUSIONS AND FUTURE WORKS

A node localization algorithm has been developed for *ad hoc* wireless sensor networks based on weighted least squares technique. Simulation results show that the performance of the proposed method is better than the classical multidimensional scaling method in both line-of-sight and non-line-of-sight environments.

From the simulations, it is observed that the SWLS algorithm is suboptimal. Therefore, future work will be carried out to polish up the SWLS algorithm to produce optimal results. Moreover, methods such as M-estimate technique in the literature can be applied to further increase the robustness of the SWLS method in NLOS propagation situation. On the other hand, the computational burden of computing matrix inversion can be lessen by using fast algorithms in the scientific computing literature, which is expected to make the SWLS more computationally attractive.

REFERENCES

[1] N. Patwari, J.N. Ash, S. Kyperountas, A.O. hero, III, R.L. Moses and N.S. Correal, "Locating the nodes: cooperative localization in wireless sensor networks," *IEEE Signal Processing Magazine*, pp. 54-69, July 2005.

[2] X. Ji and H. Zha, "Robust sensor localization algorithm in wireless ad-hoc sensor networks," *Proc. Computer Communications and Networks*, pp. 527-532, Oct. 2003.

[3] L. Doherty, K S.J. Pister, L. El Ghaoui, "Convex position estimation in wireless sensor networks," *Proc. IEEE INFOCOM*, pp. 1655-1663, 2001.

[4] D. Niculescu and B. Nath, "Ad hoc positioning system (APS)," *Proc. IEEE GlobeCom*, vol. 5, pp. 2926-1931, San Antonio, AZ, Nov. 2001.

[5] R.L. Moses, D. Krishnamurthy and R.M. Patterson, "A self-localization method for wireless sensor networks," *EURASIP Journal on Applied Signal Processing*, vol. 4, pp. 348-358, 2003.

[6] N. Patwari, A.O. Hero, III, M. Perkins, N.S. Correal and R.J. O'Dea, "Relative location estimation in wireless sensor networks, " *IEEE Trans. Signal Processing*, vol. 51, pp. 3521-2525, Sept, 2004.

[7] M. Meurer, P.W. Baier, T.Weber, C.A. Jotten and S. Heilmann," SPIDER: enhanced distance based localization of mobile radio terminals," *Proc. VTC 2004-Fall*, vol. 5, pp. 3521-3525, Sept. 2004.

[8] A. Savvides, C.-C. Han and M.B. Srivastava, "Dynamic fine-grained localization in ad-hoc networks of sensors," *Proc. 7th Annual Int. Conf. Mobile Computing*

and Networking, pp. 166-179, ACM Press, Rome, Italy, July 2001.

[9] C. Savarese, J.M. Rabaey and J. Beutel, ”Location in distributed ad-hoc wireless sensor network,” *Proc. ICASSP 2001*, vol. 4,pp. 2037-2040, Salt Lake City, UT, may 2001.

[10] M. Coates, ”Distributed particle filters for sensor networks,” *Proc. IEEE Information Processing in Sensor networks*, pp. 225-233, Apr. 2004.

[11] T. Ihler, J.W. Fisher, III R.L. Moses and A.S. Willsky, ”Nonparametric belief propagation for self-localization of snesor networks,” *IEEE J. Selected Areas in Communications*, vol. 23, pp. 809-819, April 2005.

[12] Y.T. Chan and K.C. Ho, ”A simple and Efficient Estimator for Hyperbolic Location,” *IEEE Trans. Signal Processing*, vol. 42, pp. 1905-1915, 1994.

[13] X. Wang, Z. Wang and B. O’Dea, ”A TOA-based location algorithm reducing the errors due to non-line-of-sight (NLOS) propagation,” *IEEE Trans. Vehicular Technology*, vol. 52, pp. 112-116, 2003.

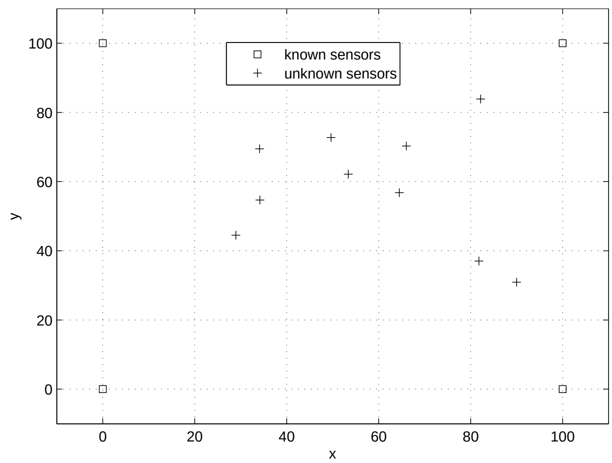

Figure 1: Positions of the sensors

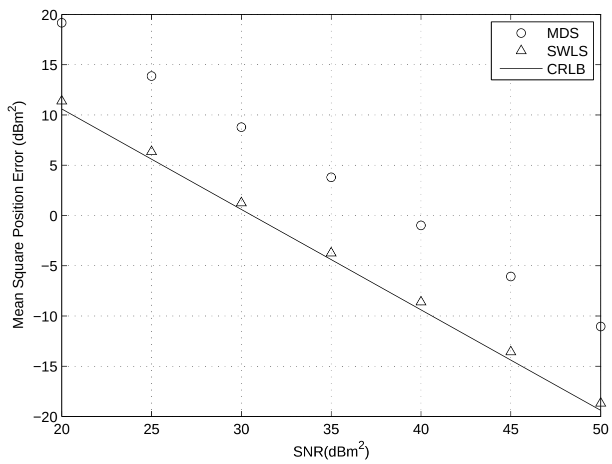

Figure 2: Mean square position error versus SNR at LOS environment with fixed-position sensors

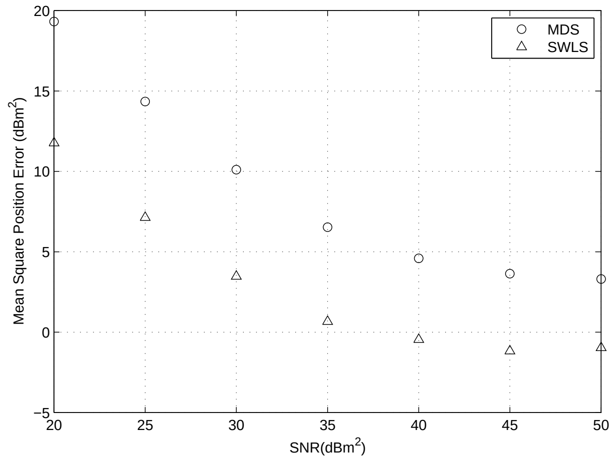

Figure 3: Mean square position error versus SNR at NLOS environment with fixed-position sensors

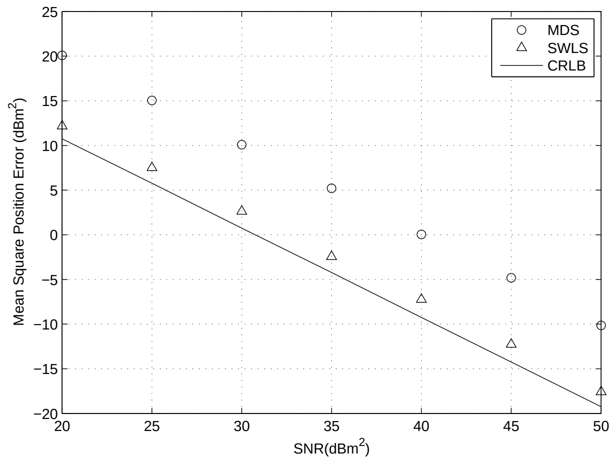

Figure 4: Mean square position error versus SNR at LOS environment with random-position sensors

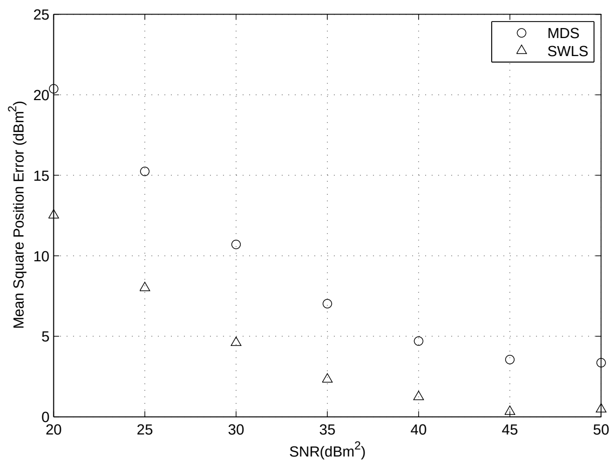

Figure 5: Mean square position error versus SNR at NLOS environment with random-position sensors