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GLOBAL CALIBRATION OF CDMA-BASED ARRAYS

A. M. Mengot and A. Manikas

Communications and Signal Processing Research Group
Department of Electrical and Electronic Engineering
Imperial College London
SW7 2BT, United Kingdom
a.mengot@imperial.ac.uk and a.manikas@imperial.ac.uk

ABSTRACT

In this paper, a calibration approach capable of handling simultaneously location, gain and phase uncertainties, as well as mutual coupling (global calibration), is proposed for asynchronous CDMA-based antenna arrays in the presence of multipath. The manifold vector is modelled based on a first order Taylor series expansion, to encompass the errors. The calibration technique involves a hybrid combination of pilot calibration and self calibration techniques, and requires the code sequence of a reference user. This method employs the concept of the STAR (Spatio-Temporal ARray) manifold vector and a subspace type preprocessor to provide estimates of the path delays and directions, as well as estimating the array manifold, location, gain and phase errors taking mutual coupling effects into consideration.

NOTATION

a, A	Scalar
$\underline{a}$, $\underline{A}$	Column Vector
$\mathbb{A}$, $\mathbf{A}$	Matrix
$\exp(\underline{A})$	Element by element exponential of $\underline{A}$
$\angle(\cdot)$	Angle of the argument
$\text{abs}(\cdot)$	Absolute value of the argument
$(\cdot)^T$	Transpose
$(\cdot)^H$	Conjugate transpose
$\otimes$	Kronecker product
$\odot$	Hadamard product
$\oslash$	Hadamard division
$(\cdot)^\dagger$	Pseudo-inverse function
$\mathbb{I}_N$	Identity matrix of $N \times N$ dimension
$\mathbb{J}_N$	$N \times N$ downshifting matrix
$\mathcal{L}[\mathbb{A}]$	Subspace spanned by columns of $\mathbb{A}$
$\mathcal{L}^\perp[\mathbb{A}]$	Complement subspace of $\mathbb{A}$
$P[\mathbb{A}]$	Projection operator onto $\mathcal{L}[\mathbb{A}]$, i.e. $P[\mathbb{A}] = \mathbb{A}\left(\mathbb{A}^H\mathbb{A}\right)^{-1}\mathbb{A}$
$P^\perp[\mathbb{A}]$	Projection operator onto $\mathcal{L}^\perp[\mathbb{A}]$, i.e. $P^\perp[\mathbb{A}] = \mathbb{I} - P[\mathbb{A}]$
$\text{eig}_{\text{min}}(\mathbb{A})$	Minimum eigenvalue of $\mathbb{A}$
$\det(\mathbb{A})$	Determinant of $\mathbb{A}$

1. INTRODUCTION

The demand for wireless communication services has increased considerably over the years, and with this increase comes the need for techniques capable of providing the high data rate services. Multiple access techniques, such as CDMA (Code Division Multiple Access), are some of those employed to achieve these high data rate service requirements.

Antenna arrays have the inherent ability to process signals in both space and time, spatially separating and isolating signals from other co-channel signals and interferences present. This leads to improved performance in signal detection and source separation, due to their detection and resolution capabilities. Hence, arrays are increasingly being used in both civilian and military applications such as mobile communication systems, sensor networks, geological surveys, radar, sonar and navigation systems.

However, array uncertainties due to deviation of the true array characteristics or parameters from their nominal values introduce errors into the system which, if ignored, would degrade the performance of this system either slowly or abruptly. The uncertainties/errors result from a number of factors such as ageing of sensor components, drift, mutual coupling, thermal effects, changes in the environmental conditions, imposed movements, etc. Note that mutual coupling occurs when the antennas of the array are not completely isolated from each other and re-radiate part of their received signal to neighboring antennas, and this effect can be taken into account by modelling a Mutual Coupling Matrix (MCM).

Calibration techniques devised to estimate the array parameters can be described as pilot and self calibration techniques. Pilot calibration methods use the known parameters of the received signals in estimating the array parameters. In [1] an algorithm was proposed for jointly estimating geometrical and electrical uncertainties, as well as mutual coupling. Self calibration methods employ iterative minimization algorithms in estimating both the array and source parameters. The technique proposed in [2] calibrates the array in the presence of errors due to gain, phase and mutual coupling.

In CDMA systems, the user's transmitted signal is spread by a unique spreading sequence. These systems often encounter multipath, and the multipath signals can be used in the calibration process. There has been some research into the calibration of CDMA systems [3, 4, 5, 6]. However, most have only considered gain and phase errors.

The proposed technique in this paper employs a preprocessor [7], and involves a combination of pilot calibration and self calibration methods. This new approach is applied for the calibration of CDMA-based arrays in the presence of sensor location, gain and phase errors, as well as mutual coupling and multipath. The method requires the code sequence of one user and assumes knowledge of the parameters of one sensor.

2. ARRAY SIGNAL AND MANIFOLD MODEL

Consider an M-user asynchronous CDMA system. Assume that the i^{th} user's signal arrives at an N element array via K_i distinct paths, and that the j^{th} path of the i^{th} user arrives at the array reference point from direction $\{\theta_{ij},\phi_{ij}\}$, where θ_{ij} and ϕ_{ij} represent the azimuth and elevation, respectively.

The baseband received signal vector due to the M-users at the array can be modelled as

$$\underline{x}(t)=\sum_{i=1}^{M}\sum_{j=1}^{K_i}\mathbb{C}\underline{S}_{ij}m_i(t-\tau_{ij})\beta_{ij}+\underline{n}(t) \tag{1}$$

where $\underline{S}_{ij}$ denotes the manifold vector of the j^{th} path of the i^{th} user, $\mathbb{C}$ represents the Mutual Coupling Matrix (MCM), β_{ij} denotes the path coefficient, τ_{ij} denotes the path delay, $\underline{n}(t)$ represents the noise vector (assumed to be white Gaussian noise), and $m_i(t)$ is the baseband transmitted signal encompassing the binary data $\mathrm{a}_i[n]$ of the i^{th} user as

$$m_i(t)=\sum_{n=-\infty}^{\infty}\mathrm{a}_i[n]\,c_{i,PN}(t-nT_{cs}) \tag{2}$$

where $c_{i,PN}(t)$ is one period of the i^{th} user's pseudo-noise (PN) spreading sequence waveform of length $\mathcal{N}_c=\frac{T_{cs}}{T_c}$, with T_{cs} being the symbol duration and T_c the chip interval.

The array manifold vector, defined as $\underline{S}_{ij}\triangleq\underline{S}(\theta_{ij},\phi_{ij})$, for the j^{th} path of the i^{th} user is modelled as

$$\underline{S}_{ij}=\underline{\gamma}\odot\exp\left(j\underline{\psi}\right)\odot\exp\left(-j\mathbf{r}^T\underline{k}_{ij}\right) \tag{3}$$

where $\mathbf{r}=[\underline{r}_1,\underline{r}_2,\cdots,\underline{r}_N]$ represents the Cartesian coordinates of the array sensor locations in units of half wavelengths, $\underline{\gamma}=[\gamma_1,\gamma_2,\ldots,\gamma_N]^T$ and $\underline{\psi}=[\psi_1,\psi_2,\ldots,\psi_N]^T$ denote the array gain and phase vectors, respectively. The parameter $\underline{k}_{ij}$ is the wavenumber vector in the direction $\{\theta_{ij},\phi_{ij}\}$ defined as $\underline{k}_{ij}\triangleq\pi[\cos\theta_{ij}\cos\phi_{ij},\sin\theta_{ij}\cos\phi_{ij},\sin\phi_{ij}]^T$. Without any loss of generality, the array is assumed coplanar with the sources and ϕ_{ij} is assumed equal to zero for every (i,j), i.e. all users are located on the (x,y) plane. This implies that $\underline{k}_{ij}=\pi[\cos\theta_{ij},\sin\theta_{ij},0]^T$ and $\underline{S}_{ij}=\underline{S}(\theta_{ij})$. The array characteristics are rarely perfectly known in realistic situations. The true parameters are the sum of the nominal values (denoted by the symbol $\widehat{\ }$) and the uncertainty or error (denoted as $\widetilde{\ }$). For example $\mathbf{r}=\widehat{\mathbf{r}}+\widetilde{\mathbf{r}}$.

The effects of mutual coupling between the sensor elements are taken into account by the matrix $\mathbb{C}$, and the manifold vector is multiplied by this matrix, i.e. $\mathbb{C}\underline{S}_{ij}$. This matrix can be modelled as [1]

$$\mathbb{C}=\mathbb{A}\odot\mathbb{L}\odot\mathbf{\Gamma}\odot\exp(j\mathbf{\Phi})\odot\exp(j\pi\mathbb{D}) \tag{4}$$

where

$$\mathbb{A}_{qk}=\begin{cases}\sqrt{1-\alpha_k'} & \text{for } q=k\\ \sqrt{\alpha_k} & \text{otherwise}\end{cases}$$

$$\mathbb{D}_{qk}=\begin{cases}0 & \text{for } q=k\\ d_{qk}=\|\underline{r}_q-\underline{r}_k\| & \text{otherwise}\end{cases}$$

$$\mathbb{L}_{qk}=\begin{cases}1 & \text{for } q=k\\ \frac{1}{2\pi d_{qk}} & \text{otherwise}\end{cases}$$

$$\mathbf{\Gamma}_{qk}=\begin{cases}1 & \text{for } q=k\\ \gamma_q\gamma_k\exp\left(j\left(\psi_q+\psi_k\right)\right) & \text{otherwise}\end{cases}$$

$$\mathbf{\Phi}_{qk}=\begin{cases}0 & \text{for } q=k\\ \varphi_k & \text{otherwise}\end{cases}$$

with α_k' being the power dissipated by the k^{th} array element and α_k being the power of its signal re-radiated to the other elements, d_{qk} denotes the relative location between the k^{th} and q^{th} sensors. The matrix $\mathbb{L}$ models the free space propagation losses, matrix $\mathbf{\Gamma}$ is dependent on the array electrical (gain and phase) characteristics, and the matrix $\mathbf{\Phi}$ models the random phase φ_k introduced by the k^{th} sensor.

The matrix $\mathbb{C}$ is dependent on the array gain, phase and sensor positions which in turn are subject to uncertainties; hence, in the presence of geometrical and electrical uncertainties, the true MCM becomes $\mathbb{C}=\widehat{\mathbb{C}}+\widetilde{\mathbb{C}}$.

The output from each antenna of the array is sampled at chip rate and passed through a bank of N tapped delay line (TDL) of length $2\mathcal{N}_c$. The outputs of the TDLs are concatenated to form the $2\mathcal{N}_cN$ dimensional discretized signal vector given as

$$\underline{x}[n]=\left[\underline{x}_1[n]^T,\underline{x}_2[n]^T,\cdots,\underline{x}_N[n]^T\right]^T \tag{5}$$

where $\underline{x}_k[n]$ denotes the output from the k^{th} antenna.

However, due to the multipath delay and lack of synchronization, each TDL will contain contributions from not only the current but the previous and next symbols. In order to include this, the array manifold vector is extended to the spatio-temporal array (STAR) manifold vector. Taking into account the effects of mutual coupling, and with $\underline{S}_{ij}$ approximated by a first order Taylor series to encompass the array errors, the STAR manifold vector is modelled for the j^{th} path of the i^{th} user as

$$\begin{aligned}\underline{\mathfrak{h}}_{ij}&=\mathbb{C}\underline{S}_{ij}\otimes\left(\mathbb{J}^{l_{ij}}\underline{\mathfrak{c}}_i\right)\\&=\mathbb{C}\left(\underline{\widehat{S}}_{ij}+\underline{D}_{\theta_{ij}}\widetilde{\theta}_{ij}\right)\otimes\left(\mathbb{J}^{l_{ij}}\underline{\mathfrak{c}}_i\right)\end{aligned} \tag{6}$$

where $\underline{D}_{\theta_{ij}}$ is the first derivative of the manifold vector with respect to θ_{ij}, evaluated at the known parameters, i.e. the first derivative of $\underline{\widehat{S}}_{ij}$, $\widetilde{\theta}_{ij}$ denotes the perturbation in the DOA, $l_{ij}=\left\lceil\frac{\tau_{ij}}{T_c}\right\rceil$ is the discretized equivalent of the path delay, and $\underline{\mathfrak{c}}_i$ represents one period of the i^{th} user's PN sequence padded with $\mathcal{N}_c$ zeros at the end

$$\underline{\mathfrak{c}}_i=\left[[\alpha_i[0],\alpha_i[1],\ldots,\alpha_i[\mathcal{N}_c-1]],\underline{0}_{\mathcal{N}_c}^T\right]^T. \tag{7}$$

$\mathbb{J}$ is a $2\mathcal{N}_c\times2\mathcal{N}_c$ matrix, modelled as

$$\mathbb{J}=\begin{bmatrix}\underline{0}_{2\mathcal{N}_c-1}^T & 0\\ \mathbb{I}_{2\mathcal{N}_c-1} & \underline{0}_{2\mathcal{N}_c-1}\end{bmatrix} \tag{8}$$

such that every time $\mathbb{J}^l$ (or $\left(\mathbb{J}^T\right)^l$) operates on a column vector, the contents of the vector are down-shifted (or upshifted) by l elements, with zeros being added to the top (or bottom) of the vector.

Without loss of generality, we will assume that the first user is the desired user and the received data vector can be written in terms of the desired signal, Inter-Symbol Interference (ISI), Multiple Access Interference (MAI) and noise components as

$$\underline{x}[n] = \mathrm{a}_1[n]\,\mathbb{H}_1\underline{\beta}_1 + \mathrm{ISI}[n] + \mathrm{MAI}[n] + \underline{n}[n] \qquad (9)$$

where the first term represents contributions from the desired user, $\underline{n}[n]$ is the sampled noise vector,

$$\begin{aligned}\mathrm{ISI}[n] &= \left[\mathbb{H}_{1,\text{prev}}\underline{\beta}_1, \mathbb{H}_{1,\text{next}}\underline{\beta}_1\right]\begin{bmatrix}\mathrm{a}_1[n-1]\\ \mathrm{a}_1[n+1]\end{bmatrix}\\ \mathrm{MAI}[n] &= \sum_{i=2}^{M}\left[\mathbb{H}_{i,\text{prev}}\underline{\beta}_i, \mathbb{H}_i\underline{\beta}_i, \mathbb{H}_{i,\text{next}}\underline{\beta}_i\right]\begin{bmatrix}\mathrm{a}_i[n-1]\\ \mathrm{a}_i[n]\\ \mathrm{a}_i[n+1]\end{bmatrix}\end{aligned}$$

with $\mathbb{H}_i = \left[\underline{\mathfrak{h}}_{i1}, \underline{\mathfrak{h}}_{i2}, \cdots, \underline{\mathfrak{h}}_{iK_i}\right]$ being a matrix with columns the K_i STAR manifold vectors of the i^{th} user, $\mathbb{H}_{i,\text{prev}} = \left(\mathbb{I}_N \otimes \left(\mathbb{J}^T\right)^{\mathcal{N}_c}\right)\mathbb{H}_i$ and $\mathbb{H}_{i,\text{next}} = \left(\mathbb{I}_N \otimes \mathbb{J}^{\mathcal{N}_c}\right)\mathbb{H}_i$ represent contributions from the previous and next data symbol, respectively, and $\underline{\beta}_i = \left[\beta_{i1}, \beta_{i2}, \cdots, \beta_{iK_i}\right]^T$.

It can be observed that the STAR manifold vectors of the K_i paths of the desired user are linearly combined by the fading coefficient vector $\underline{\beta}_i$, making these paths indistinguishable in their contribution to the signal subspace associated with the covariance matrix $\mathbb{R}_{xx}$ of $\underline{x}[n]$. This looks like the well known coherence problem of subspace estimation techniques, and as such, signal subspace techniques (such as MUSIC) cannot be used to estimate the desired user's spatio-temporal channel parameters (path delays and directions). This problem needs to be considered when calibrating CDMA arrays.

3. CHANNEL AND ARRAY PARAMETER ESTIMATION

For the calibration process, it is common to regard one sensor as 'reference' and assume that its characteristics are known. In this paper, the first sensor is regarded as the 'reference' sensor, the first user is the desired user while its multipaths are used as the calibration signals. Hence for the rest of the paper, the symbol ℓ_j will be used to represent the j^{th} path of the desired user, i.e. $\ell_j \triangleq l_{1j}$ (first subscript 1 is dropped).

3.1 Channel Estimation

The signal $\underline{x}[n]$ is applied to a bank of preprocessors. The objective of the ℓ^{th} preprocessor is to remove the contributions of all multipath components, except the one with delay ℓ, by projecting them on to their compliment subspace. The ℓ^{th} preprocessor is modelled as a matrix [7] defined as

$$\mathbb{P}_\ell \triangleq \mathbb{I}_N \otimes P^{\perp}[\mathbf{C}_{1\ell}] \in \mathcal{R}^{2\mathcal{N}_cN\times 2\mathcal{N}_cN} \qquad (10)$$

where the matrix $\mathbf{C}_{1\ell}$ is formed by removing the ℓ^{th} column from the matrix $\mathbf{C}_1 = \left[\mathbb{J}^1\underline{\mathfrak{c}}_1, \ldots, \mathbb{J}^\ell\underline{\mathfrak{c}}_1, \ldots, \mathbb{J}^{\mathcal{N}_c}\underline{\mathfrak{c}}_1\right]$.

The preprocessed spatio-temporal received signal is

$$\underline{y}_\ell[n] = \mathbb{P}_\ell\underline{x}[n]. \qquad (11)$$

By forming the covariance matrix $\mathbb{R}_{yy}^\ell$ of the preprocessed signal $\underline{y}_\ell[n]$ and partitioning the observation space into the signal and noise subspaces, the intersection of the array manifold with the overall signal-subspace of $\mathbb{R}_{yy}^\ell$ will provide the desired user's parameters. This is equivalent to solving a multidimensional search of a cost function involving the array uncertainties and expressed as follows

$$\xi_1\left(\ell,\theta,\beta,\widetilde{\theta}\right) = \frac{\underline{p}^H\widehat{\mathbb{M}}^H\widehat{\mathbb{C}}^H\mathbb{B}_\ell^H\widehat{\mathbb{E}}_{n,\ell}\widehat{\mathbb{E}}_{n,\ell}^H\mathbb{B}_\ell\widehat{\mathbb{C}}\widehat{\mathbb{M}}\underline{p}}{\underline{p}^H\widehat{\mathbb{M}}^H\widehat{\mathbb{C}}^H\mathbb{B}_\ell^H\mathbb{B}_\ell\widehat{\mathbb{C}}\widehat{\mathbb{M}}\underline{p}} \qquad (12)$$

where

$$\begin{cases}\widehat{\mathbb{M}} \triangleq \widehat{\mathbb{M}}(\theta) = \left[\underline{\widehat{S}}(\theta), \underline{D}_\theta\right]\\ \mathbb{B}_\ell \triangleq \mathbb{I}_N \otimes \left(P^{\perp}[\mathbf{C}_{1\ell}]\right)\mathbb{J}^\ell\underline{\mathfrak{c}}_1\\ \underline{p} \triangleq \underline{p}\left(\widetilde{\theta},\beta\right) = \left[\beta,\ \widetilde{\theta}\beta\right]^T\end{cases} \qquad (13)$$

Furthermore, in Eqn. (12), $\widehat{\mathbb{E}}_{n,\ell}$ is the estimated noise subspace obtained from the eigenvalue decomposition of the covariance matrix $\widehat{\mathbb{R}}_{yy}^\ell$ over a given observation, and the matrix $\widehat{\mathbb{C}}$ is obtained by some prior knowledge of the MCM or an estimate obtained with the nominal array parameters.

However, an alternative and more efficient approach to this complex minimisation process is what is proposed in this paper and described as follows.

Since $\widehat{\mathbb{M}}\underline{p} \neq \underline{0}$, the path delays $\widehat{\ell}_1, \widehat{\ell}_2, \cdots, \widehat{\ell}_K$ can be estimated based on the following optimisation

$$\begin{aligned}&\left(\widehat{\ell}_1, \widehat{\ell}_2, \cdots, \widehat{\ell}_K\right) =\\ &\arg\min_{\forall\ell}\left\{\mathrm{eig}_{\min}\left(\mathbb{B}_\ell^H\widehat{\mathbb{E}}_{n,\ell}\widehat{\mathbb{E}}_{n,\ell}^H\mathbb{B}_\ell\left(\mathbb{B}_\ell^H\mathbb{B}_\ell\right)^{-1}\right)\right\} \end{aligned}\qquad (14)$$

The associated eigenvectors $\underline{\widehat{E}}_{\min,1}, \underline{\widehat{E}}_{\min,2}, \cdots, \underline{\widehat{E}}_{\min,K}$ corresponding to the minimum eigenvalues, are also obtained.

In addition, since the vector $\underline{p} \neq \underline{0}$, it can be proven that the directions $\widehat{\theta}_1, \widehat{\theta}_2, \cdots, \widehat{\theta}_K$ can be estimated by a 1 dimensional search of the following cost function

$$\xi_2(\theta) = \frac{\chi - \sqrt{\chi^2 - 4\det(\widehat{\mathbb{F}}(\theta))\det(\widehat{\mathbb{G}}(\theta))}}{2\det(\widehat{\mathbb{G}}(\theta))} \qquad (15)$$

$$\text{i.e.}\quad \left(\widehat{\theta}_1, \widehat{\theta}_2, \cdots, \widehat{\theta}_K\right) = \arg\min_{\forall\theta}\{\xi_2(\theta)\} \qquad (16)$$

where the matrices $\widehat{\mathbb{F}} \triangleq \widehat{\mathbb{F}}(\theta)$ and $\widehat{\mathbb{G}} \triangleq \widehat{\mathbb{G}}(\theta)$ are expressed as follows

$$\begin{cases}\widehat{\mathbb{F}} = \begin{bmatrix}f_{11} & f_{12}\\ f_{21} & f_{22}\end{bmatrix} = \widehat{\mathbb{M}}^H\widehat{\mathbb{C}}^H\mathbb{B}_\ell^H\widehat{\mathbb{E}}_{n,\ell}\widehat{\mathbb{E}}_{n,\ell}^H\mathbb{B}_\ell\widehat{\mathbb{C}}\widehat{\mathbb{M}}\\ \widehat{\mathbb{G}} = \begin{bmatrix}g_{11} & g_{12}\\ g_{21} & g_{22}\end{bmatrix} = \widehat{\mathbb{M}}^H\widehat{\mathbb{C}}^H\mathbb{B}_\ell^H\mathbb{B}_\ell\widehat{\mathbb{C}}\widehat{\mathbb{M}}\end{cases} \qquad (17)$$

and $\chi \triangleq (f_{11}g_{22} - f_{12}g_{21}) + (f_{22}g_{11} - f_{21}g_{12})$.

However, there is no need to estimate the vector $\underline{p}$ explicitly, but $\widetilde{\theta}_1, \widetilde{\theta}_2, \cdots, \widetilde{\theta}_K$ can be obtained based on the following expression

$$\widetilde{\theta} = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \tag{18}$$

evaluated for all paths of the desired user, where

$$\begin{cases} A = f_{22}(g_{12}+g_{21}) - g_{22}(f_{12}+f_{21}) \\ B = 2g_{11}f_{22} - 2f_{11}g_{22} \\ C = g_{11}(f_{12}+f_{21}) - f_{11}(g_{12}+g_{21}) \end{cases} \tag{19}$$

with the parameters as obtained in Eqn. (17).

Hence, for the j^{th} path, the direction $\widehat{\theta}_j$ is updated as

$$\widehat{\theta}_j = \widehat{\theta}_j + \widetilde{\theta}_j. \tag{20}$$

3.2 Estimation of Array Parameters and MCM

In addition to the estimated DOAs of the multipaths of the desired signal, the manifold vector $\widehat{\underline{S}}_j$ of these paths should be estimated, based on the eigenvectors $\underline{E}_{\min,\ell}$ obtained in the previous section. Indeed,

$$\widehat{\underline{S}}_j = \frac{\gamma_1 \exp(j\psi_1)\,\widehat{\mathbb{C}}^{-1}\widehat{\underline{E}}_{\min,\ell_j}}{\underline{w}^T\widehat{\mathbb{C}}^{-1}\widehat{\underline{E}}_{\min,\ell_j}} \tag{21}$$

where $\underline{w} = [1, 0, \cdots, 0]^T$ is an N-dimensional column vector, γ_1 and ψ_1 represent the gain and phase parameters, respectively, of the 1^{st} (reference) sensor, and $\gamma_1 = 1$ and $\psi_1 = 0$. Any deviation of this estimate from the true manifold vector would have been introduced due to $\widehat{\mathbb{E}}_{n,l}$ (as mentioned in the previous section), and the errors in $\widehat{\mathbb{C}}$.

The gain parameters can be estimated as

$$\widehat{\underline{\gamma}} = \frac{1}{K}\sum_{j=1}^{K} \text{abs}\left(\widehat{\underline{S}}_j\right). \tag{22}$$

The sensor locations and phase parameters can be estimated by

$$\left[-\widehat{\mathbf{r}}, \widehat{\underline{\psi}}\right] = \left(\left[\widehat{\mathbb{K}}, \underline{1}_N\right]^{\dagger}\mathbb{V}\right)^T \tag{23}$$

where $\widehat{\mathbb{K}} = \left[\widehat{\underline{k}}_1, \ldots, \widehat{\underline{k}}_K\right]^T$, with $\widehat{\underline{k}}_j$ formed from $\widehat{\theta}_j$, and $\mathbb{V} = [\underline{v}_1, \ldots, \underline{v}_K]^T$ with

$$\underline{v}_j = -\widehat{\mathbf{r}}^T\widehat{\underline{k}}_j + \widehat{\underline{\psi}}. \tag{24}$$

Note that $\underline{v}_j$ is the exponential part of $\widehat{\underline{S}}_j$, i.e $\underline{v}_j = \angle\left(\widehat{\underline{S}}_j\right)$.

The MCM $\widehat{\mathbb{C}}$ is updated with the estimated array location, gain and phase parameters based on Eqn. (4). Only the location, gain and phase dependent components of $\widehat{\mathbb{C}}$ are updated. However, the effects of the errors (if there be any) in the remaining components are small and can be ignored.

3.3 Estimation Procedure

The calibration process iterates between direction estimation and parameter estimation, until acceptable results are obtained. In order to obtain a better initial estimate of the directions, the gain parameter can be estimated before estimating the directions.

The steps in the calibration procedure are outlined as follows:

1. Formulate $\underline{y}_\ell$, using Eqn. (11), and form $\widehat{\mathbb{R}}^{\ell}_{yy}$.
2. Estimate the path delays $\widehat{\ell}_j$ for $j = 1, \cdots, K$ using Eqn. (14).
3. Using Eqn. (21), estimate $\widehat{\underline{S}}_j$.
4. Estimate the gain parameters using Eqn. (22).
5. Formulate matrices $\mathbb{F}$ and $\mathbb{G}$ using Eqn. (17).
6. Estimate the directions $\widehat{\theta}_1, \widehat{\theta}_2, \cdots, \widehat{\theta}_K$ using Eqn. (15).
7. Using Eqns. (18) and (20), estimate $\widetilde{\theta}_j$ and update $\widehat{\theta}_j$.
8. Estimate the sensor location and phase parameters using Eqn.(23).
9. Update the MCM $\widehat{\mathbb{C}}$ using Eqn. (4).
10. Evaluate ξ_3, where

$$\xi_3 = \sum_{j=1}^{K} \frac{\widehat{\underline{S}}(\theta_j)^H\,\widehat{\mathbb{C}}^H\mathbb{B}^H_{\ell_j}\widehat{\mathbb{E}}_{n,\ell_j}\widehat{\mathbb{E}}^H_{n,\ell_j}\mathbb{B}_{\ell_j}\widehat{\mathbb{C}}\widehat{\underline{S}}(\theta_j)}{\widehat{\underline{S}}(\theta_j)^H\,\widehat{\mathbb{C}}^H\mathbb{B}^H_{\ell_j}\mathbb{B}_{\ell_j}\widehat{\mathbb{C}}\widehat{\underline{S}}(\theta_j)} \tag{25}$$

with $\widehat{\underline{S}}(\theta_j)$ formed from the estimated direction and array parameters. If $\left\|\xi_{3,\text{previous}} - \xi_{3,\text{current}}\right\|^2$ >threshold, then repeat the Steps 3-5 and 7-10, else stop (convergence achieved).

4. SIMULATION RESULTS

An 8-user environment is simulated, and a six sensor circular array of half-wavelength spacing employed to collect data over an observation interval equivalent to 200 data symbols. Each user is assigned a unique 31 length spreading code. The first (desired) user has 5 multipaths of delays $2T_c$, $3T_c$, $5T_c$, $7T_c$, $9T_c$, arriving from directions $(20°, 0°)$, $(50°, 0°)$, $(80°, 0°)$, $(110°, 0°)$, $(130°, 0°)$, respectively. The other users have 2 multipaths each. The errors are assumed within 15% of the nominal values and the input SNR is 30dB.

The method is evaluated for two cases. In the first case, all path directions, $\theta_1, \theta_2, \cdots, \theta_K$, of the desired user are unknown. In the second case, the first path (e.g. direct path) of the desired user is known. Note that the path delays as well as the directions of the other multipaths of the desired user are unknown and will still need to be estimated.

The plots in Figs. 1, 2 and 3 illustrate the errors of the direction, gain and phase estimates, respectively, versus the number of iterations. From these figures, it can be observed that for the first case where the direction of the desired user is unknown, although the gain and phase estimates come close to the true values, the direction estimates do not. However, in the case where the direction of the first path is known, both the direction and array parameter estimates obtained are close to the true values.

Fig. 4 depicts the estimates of the sensor positions before and after calibration, while the plots in Figs. 5 and 6 illustrate the path delay and direction estimation before and after calibration. The results show the improved performance of the method.

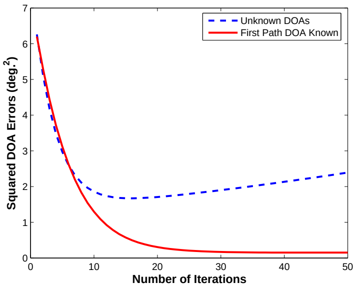

Figure 1: DOA error estimation vs number of iterations

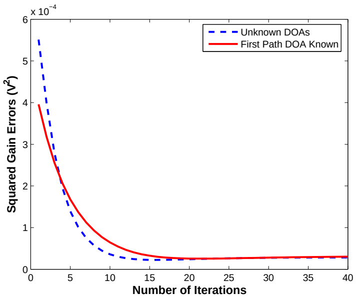

Figure 2: Gain error estimation vs number of iterations

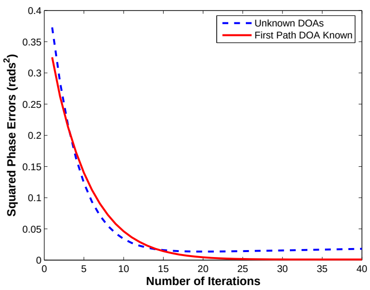

Figure 3: Phase error estimation vs number of iterations

5. CONCLUSION

In this paper, an algorithm is proposed for the global calibration of CDMA-based arrays operating in the presence of of location, gain and phase uncertainties, as well as mutual coupling. The simulation results illustrate the performance of the proposed technique, and also depict a better performance when a pilot signal of known direction is employed.

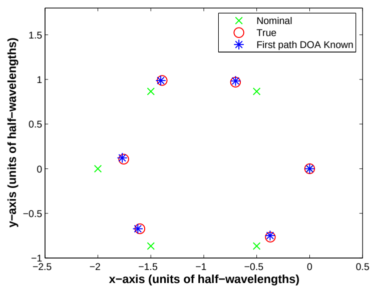

Figure 4: Sensor Position Estimates

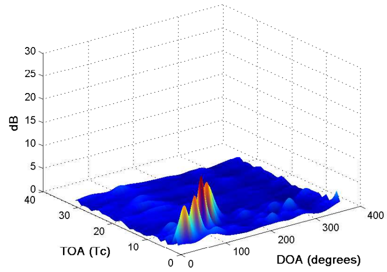

Figure 5: Path delay and direction estimation with uncalibrated array

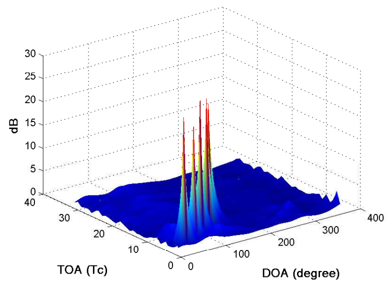

Figure 6: Path delay and direction estimation with calibrated array

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