ANALYZING EFFECT OF NOISE ON LMS-TYPE APPROACHES TO BLIND ESTIMATION OF SIMO CHANNELS: ROBUSTNESS ISSUE

Md. Kamrul Hasan[1] *and Patrick A. Naylor*[2]

[1]Department of Electrical and Electronic Engineering, Bangladesh University of Engineering and Technology, Dhaka-1000, Bangladesh
[2]Department of Electrical and Electronic Engineering, Imperial College London, UK
e-mail: khasan@eee.buet.ac.bd, p.naylor@imperial.ac.uk

ABSTRACT

An analysis of the noise effect on the convergence characteristic of the least-mean-squares (LMS) type adaptive algorithms for blind channel identification is presented. It is shown that the adaptive blind algorithms misconverge in the presence of noise. A novel technique for ameliorating such misconvergence characteristic, using a frequency domain energy constraint in the adaptation rule, is proposed. Experimental results demonstrate that the robustness of the blind adaptive algorithms can be significantly improved using such constraints.

1. INTRODUCTION

Blind channel identification (BCI) is a common issue in diverse fields of science and engineering. Signals transmitted from the source are adversely affected by the propagating medium/channel. The channel identification, therefore, is required to remove its detrimental effect from the received signal often by inversion. In communications, the problem is to equalize the channel effect on the received signal to obtain the transmitted signal. In geophysics, the reflectivity of the earth layers is explored by extracting seismic wavelets from the sensor signals. In speech processing, particularly in acoustic dereverberation, the problem is to separate the sound source from the received microphone signals.

Several single and multichannel BCI schemes are reported in the literature. Multichannel identification schemes, however, are recognized to be more effective in removing the unknown channel effects than their single channel counterparts. Among the various techniques reported so far, e.g. least-squares approach [1], subspace method [2], maximum-likelihood method [3], Newton algorithm [4], the LMS algorithm [4] is simple and efficient. Among all of its variants, it has been shown in [5] that the normalized multichannel frequency domain LMS (NMCFLMS) algorithm is more computationally efficient and effective for identifying long acoustic channels which are of particular interest for dereverberation. However, the LMS-type blind adaptive algorithms lack robustness to additive noise. It is shown using numerical examples in [6] that the NMCFLMS algorithm misconverges if the observations are corrupted by noise. A method for improving robustness has been also described in that paper. The implementation, however, requires knowledge of the positions and amplitudes of some dominant components of the impulse responses which make the algorithm nonblind and thus limits its use in practical situations.

In this paper, we give an analysis of the noise effect on the convergence characteristic of the LMS-type blind adaptive algorithms and show that they misconverges due to nonzero gradient of the noisy error performance surface. The nonzero gradient has the effect of attenuating the high frequency components of the channel estimate and thus leads to a misconverged solution in the end. To ameliorate misconvergence characteristics of such algorithms, we propose a novel technique that plays the role of enhancing the high frequency components and thus attempts to counter balance the detrimental effect of nonzero gradient of the cost function.

2. PROBLEM FORMULATION

Consider a speech signal recorded inside a non anechoic room using a linear array of microphones. The channel outputs and observed signals are then given by

$$y_i(n) = s(n) * h_i(n) = \sum_{k=0}^{L-1} h_{i,k}(n) s(n-k) \quad (1)$$

$$x_i(n) = y_i(n) + v_i(n), \quad i = 1, 2, \cdots, M \quad (2)$$

where M is the number of microphones, $s(n)$, $y_i(n)$, $x_i(n)$, $v_i(n)$ and $h_{i,k}(n)$ denote, respectively, the clean speech, reverberant speech, the reverberant speech corrupted by background noise, observation noise, and impulse response of the source to ith microphone. It is assumed that the additive noise on M channels is uncorrelated white random sequence, i.e., $E\{v_i(t)v_j(t)\} = 0$ for $i \neq j$ and $E\{v_i(t)v_i(t-t')\} = 0$ for $t' \neq 0$. It is also assumed that $v_i(n)$ are uncorrelated with $s(n)$.

A blind channel identification algorithm estimates $\mathbf{h}_i = [h_{i,0} \;\; h_{i,1} \cdots h_{i,L-1}]^T$, $i = 1, 2, \cdots, M$ solely from the observations $x_i(n)$, $n = 1, 2, \cdots, N$. The identifiability conditions commonly stated are: i) The channel transfer functions don't contain any common zeros, ii) The autocorrelation matrix of the source signal is of full rank. In this paper, we explore the reason behind misconvergence of the LMS-type algorithms reported in [4] and [5], and propose to attach a spectral constraint in the adaptation rule in order to improve their robustness to the blind identification of time-invariant $\mathbf{h}_i$ from the noise corrupted sequence $x_i(n)$.

3. THE NMCFLMS ALGORITHM

In this section, the NMCFLMS algorithm is briefly summarized as it is known to be more computationally efficient and effective among the LMS-type algorithms for identifying long acoustic channels. From (1), we deduce the following relationship:

$$y_i(n) * h_{j,k} - y_j(n) * h_{i,k} = 0 \quad (3)$$

However, in presence of noise an error function may be defined as

$$e_{ij}(n) = x_i(n) * \widehat{h}_{j,k} - x_j(n) * \widehat{h}_{i,k} \quad (4)$$

The frequency domain instantaneous squared error at the m-th block minimized by the NMCFLMS algorithm is given by

$$J_f(m) = \sum_{i=1}^{M-1} \sum_{j=1+1}^{M} \underline{e}_{ij}^H(m) \underline{e}_{ij}(m) \quad (5)$$

where $\underline{e}_{ij}(m)$ is the frequency-domain block error signal between the i-th and j-th channels. As reported in [5], the algorithm is summarized below:

$$\begin{aligned} \widehat{\underline{\mathbf{h}}}_k^{10}(m+1) &= \widehat{\underline{\mathbf{h}}}_k^{10}(m) - \mu[\mathbf{p}_k(m) + \delta \mathbf{I}_{2L\times 2L}]^{-1} \\ &\quad \times \sum_{i=1}^{M} D_{x_i}^*(m) \underline{\mathbf{e}}_{ik}^{01}(m), \quad k = 1, 2, \cdots, M \end{aligned} \quad (6)$$

where

$$\widehat{\underline{\mathbf{h}}}_k^{10}(m) = \mathbf{F}_{2L\times 2L} \begin{bmatrix} \widehat{\mathbf{h}}_k(m) \\ \mathbf{0} \end{bmatrix} \quad (7)$$

$$\underline{\mathbf{e}}_{ik}^{01}(m) = \mathbf{F}_{2L\times 2L} \begin{bmatrix} \mathbf{0} \\ \mathbf{F}_{L\times L}^{-1} \underline{\mathbf{e}}_{ik}(m) \end{bmatrix} \quad (8)$$

$$\begin{aligned} \mathbf{p}_k(m) &= \lambda \mathbf{p}_k(m-1) + (1-\lambda) \sum_{i=1, i\neq k}^{M} D_{x_i}^*(m) D_{x_i}(m), \\ &\quad k = 1, 2, \cdots, M. \end{aligned} \quad (9)$$

Here m is the frame index and $\mathbf{F}$ denotes the discrete Fourier transform (DFT) matrix. The frequency-domain error function $\underline{\mathbf{e}}_{ik}(m)$ is given by

$$\underline{\mathbf{e}}_{ik}(m) = D_{x_i}(m) \widehat{\underline{\mathbf{h}}}_k(m) - D_{x_k}(m) \widehat{\underline{\mathbf{h}}}_i(m) \quad (10)$$

The diagonal matrix $D_{x_i}(m)$ is the DFT of the mth frame data block for the ith channel, i.e.,

$$\begin{aligned} D_{x_i}(m) &= diag(\mathbf{F}\{\mathbf{x}_i(m)_{2L\times 1}\}) \\ \mathbf{x}_i(m)_{2L\times 1} &= [x_i(mL-L) \;\; x_i(mL-L+1) \\ &\quad \cdots x_i(mL+L-1)]^T \end{aligned} \quad (11)$$

and the estimate of the kth channel coefficient vector is defined as $\widehat{\mathbf{h}}_k(m) = [\widehat{h}_{k,0}(m) \;\; \widehat{h}_{k,1}(m) \cdots \widehat{h}_{k,L-1}(m)]^T$.

4. NOISE EFFECT ON THE ADAPTIVE BCI ALGORITHMS

In this section, we present a time-domain analysis of noise effect on the LMS-type adaptive BCI algorithms. To investigate the convergence characteristic of such algorithms in presence noise, we rewrite the error function in (4) as

$$\begin{aligned} e_{ij}(n) &= [y_i(n) * h_{j,k} - y_j(n) * h_{i,k}] \\ &\quad + [v_i(n) * h_{j,k} - v_j(n) * h_{i,k}] = e_{yij}(n) + e_{vij}(n) \end{aligned} \quad (12)$$

From (12), we see that under noisy condition the error function, $e_{ij}(n)$, consists of two parts, namely $e_{yij}(n)$ and $e_{vij}(n)$. What happens if the filter coefficients are derived by minimizing $e_{ij}(n)$ in the mean squared error (MSE) sense is a concern of this paper. The cost function is defined as

$$J_x = E\{J_x(n)\} = E\left\{\sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \varepsilon_{ij}^2(n)\right\} \quad (13)$$

where $E\{\cdot\}$ denotes the expectation operator and

$$\varepsilon_{ij}(n) = \begin{cases} \frac{e_{ij}(n)}{||\widehat{\mathbf{h}}_{ij}||^2}, & i \neq j, i, j = 1, 2, \cdots, m \\ 0, & i = j, i, j = 1, 2, \cdots, m \end{cases} \quad (14)$$

The error $e_{ij}(n)$ is divided by $||\widehat{\mathbf{h}}_{ij}||$ to avoid the trivial estimate. Using (12) and neglecting the crosscorrelation between noise and signal, we can write (13) as

$$\begin{aligned} J_x &= E\left\{\sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \frac{e_{yij}^2(n)}{||\widehat{\mathbf{h}}_{ij}||^2}\right\} + E\left\{\sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \frac{e_{vij}^2(n)}{||\widehat{\mathbf{h}}_{ij}||^2}\right\} \\ &= J_y + J_v \end{aligned} \quad (15)$$

where J_y and J_v denote the mean squared error due to signal and noise, respectively. The mean squared error estimate of the channel impulse response is given by

$$\widehat{\mathbf{h}} = \arg\min_{\widehat{\mathbf{h}}} (J_y + J_v), \text{ subject to } ||\widehat{\mathbf{h}}|| = 1 \quad (16)$$

The minimization of the first part, i.e. J_y, with respect to the adaptive filter coefficients is equivalent to that of the noise-free case. The second part may be viewed as a constraint attached to that minimization process with a built-in Lagrange multiplier.

The LMS algorithm finds the desired solution by moving along the opposite direction of the performance surface at each iteration:

$$\mathbf{h}(n) = \mathbf{h}(n-1) - \mu \frac{\partial J_x}{\partial \mathbf{h}(n-1)} \quad (17)$$

Using (15), the gradient of the cost function may be obtained as

$$\nabla J_x = \nabla J_y + \nabla J_v \quad (18)$$

where ∇J_y and ∇J_v denote the gradients of the noise-free and noise only cost function, respectively. Substituting (18) into (17), we obtain

$$\mathbf{h}(n) = \mathbf{h}(n-1) - \mu(\nabla J_y + \nabla J_v) \quad (19)$$

It can be shown that the MSE J_v due to the noise only term is

$$\begin{aligned} J_v = E\{J_v(n)\} &= \frac{1}{||\widehat{\mathbf{h}}||^2}\Big[\sum_{i\neq 1} \sigma_{v_i}^2 \widehat{\mathbf{h}}_1^T \widehat{\mathbf{h}}_1 + \sum_{i\neq 2} \sigma_{v_i}^2 \widehat{\mathbf{h}}_2^T \widehat{\mathbf{h}}_2 + \cdots \\ &\quad + \sum_{i\neq M} \sigma_{v_i}^2 \widehat{\mathbf{h}}_M^T \widehat{\mathbf{h}}_M\Big] \end{aligned} \quad (20)$$

Then the gradient of J_v is given by

$$\nabla J_v = E\{\nabla J_v(n)\} = \frac{2}{||\widehat{\mathbf{h}}||^2}[\mathbf{R}_v - J_v \mathbf{I}_{ML\times ML}]\widehat{\mathbf{h}} \quad (21)$$

and that of J_y is given by

$$\nabla J_y = E\{\nabla J_y(n)\} = \frac{2}{||\widehat{\mathbf{h}}||^2}[\mathbf{R}_y - J_y \mathbf{I}_{ML\times ML}]\widehat{\mathbf{h}} \quad (22)$$

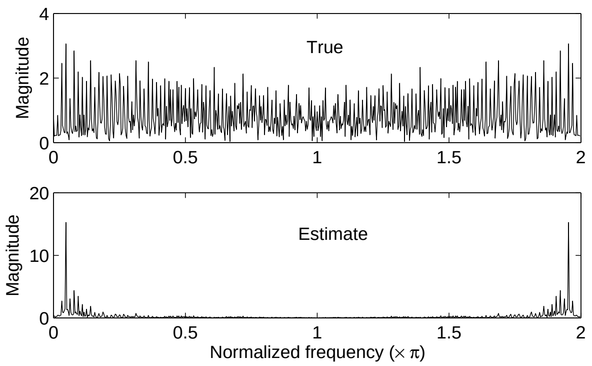

Figure 1: Magnitude spectrum of the misconverged NMCFLMS estimates of AIRs at SNR=15 dB. The concatenated true AIRs are shown in Fig. 2 (a).

For the convergence of the LMS type algorithms it is necessary that ∇J_x approaches zero with iterations, i.e.,

$$\nabla J_x = \nabla J_y + \nabla J_v = 0 \tag{23}$$

The total gradient can only be zero if and only if $\nabla J_y = -\nabla J_v$ because $\nabla J_v \neq 0$ as can be seen from (22) for unequal noise powers on the channels. It can also be shown that $\nabla J_y \neq -\nabla J_v$ in general. This findings leads to an important conclusion that the presence of noise in blind multichannel identification algorithms makes the signal plus noise MSE surface non-concave though the signal MSE surface has a global minimum. Thus the deleterious consequence of noise effect is the non-zero gradient of the cost function (J_x).

It has been observed that this effect is equivalent to spectral attenuation of the estimated impulse response after the decay of the initial transient. Analytical work to study this in more detail is underway. Thus the misconverged solution is nothing but the spectrally attenuated version of an intermediate estimate during the iteration. The spectral attenuation can be verified by observing the magnitude spectrum of the misconverged AIRs shown in Fig. 1.

5. THE ENERGY CONSTRAINED NMCFLMS ALGORITHM

We now consider modifying the adaptive algorithm such that the estimated impulse response Fourier domain energy is approximately uniformly distributed. We derive a robust adaptation rule by attaching a frequency domain constraint calculated solely from the estimated impulse response at each iteration to the original cost function. The constraint is of interest in practical applications since we can assume that the energy in the frequency domain is evenly distributed for acoustic/random channels. For unit norm constraint, i.e., $||\widehat{\mathbf{h}}|| = 1$, the frequency-domain energy of the concatenated impulse response is also unity following the Parseval's relation.

The convergence characteristic can be ameliorated by minimizing the constrained cost function defined as

$$\begin{aligned} &\text{minimize} \quad J = E[J_f(m)] \\ &\text{subject to} \quad \sum_{k=k_1}^{k_2} |\widehat{H}(k)|^2 = \gamma\mathscr{E} \end{aligned} \tag{24}$$

where $\widehat{H}(k)$ denotes the ML-th point DFT coefficient of $\widehat{\mathbf{h}}(m)$ and $\mathscr{E} = 1/(ML)\sum_{k=0}^{ML-1} |\widehat{H}(k)|^2$. Using the Lagrange multiplier β, the instantaneous cost function can be reformulated as

$$J(m) = J_f(m) + \beta J_P(m) \tag{25}$$

where the penalty term $J_p(m)$ is given by

$$J_p(m) = \left(\gamma\mathscr{E} - \sum_{k=k_1}^{k_2} |\widehat{H}(k)|^2\right)^2 \tag{26}$$

where k_1 and k_2 determines the region of $\widehat{H}(k)$ to be emphasized to prevent misconvergence. The typical values for k_1 and k_2 are $ML/4$ and $3ML/4-1$, respectively when ML is an integer multiple of 4. In this case, γ can be set to 0.5, i.e. the total energy is assumed to be equally divided in the low and high frequency subbands. A more general penalty function may be obtained using a symmetrical weighting function $0 \leq W(k) \leq 1$:

$$J_p(m) = \left(\gamma\mathscr{E} - \frac{1}{ML}\sum_{k=0}^{ML-1} W(k)|\widehat{H}(k)|^2\right)^2 \tag{27}$$

In this case, energy distribution factor γ can be obtained as $\gamma = \sum_{k=0}^{ML-1} W(k)/ML$. The summation term in (27) can be written in vector-matrix form as

$$\sum_{k=0}^{ML-1} W(k)|\widehat{H}(k)|^2 = \underline{\widehat{\mathbf{h}}}^H(m)\mathbf{W}\underline{\widehat{\mathbf{h}}}(m) \tag{28}$$

where $\mathbf{W}$ is the diagonal matrix with diagonal entries representing the weights, $W(k)$, of the DFT coefficients. Using (28), we can rewrite (27) as

$$J_p(m) = \left(\gamma\mathscr{E} - \frac{1}{ML}\underline{\widehat{\mathbf{h}}}^H(m)\mathbf{W}\underline{\widehat{\mathbf{h}}}(m)\right)^2 \tag{29}$$

The weighting matrix $\mathbf{W}$ balances energy distribution among the low and high frequency DFT coefficients. The lowpass effect of the gradient descent LMS type algorithms can be counter balanced by properly choosing $\mathbf{W}$.

The gradient of the penalty term $J_p(m)$ with respect to $\underline{\widehat{\mathbf{h}}}^*$ can be obtained as

$$\begin{aligned} \nabla J_p(m) &= \left[\left(\frac{\partial J_p}{\partial \underline{\widehat{\mathbf{h}}}_1^*}\right)^T \cdots \left(\frac{\partial J_p}{\partial \underline{\widehat{\mathbf{h}}}_k^*}\right)^T \cdots \left(\frac{\partial J_p}{\partial \underline{\widehat{\mathbf{h}}}_M^*}\right)^T \right]^T \\ &= \frac{-2(\gamma\mathscr{E} - \frac{1}{ML}\underline{\widehat{\mathbf{h}}}^H(m)\mathbf{W}\underline{\widehat{\mathbf{h}}}(m))}{ML}\mathbf{W}\underline{\widehat{\mathbf{h}}}(m) \end{aligned} \tag{30}$$

With the gradient vector computed, we can now define the parameter update equation for the constrained NMCFLMS algorithm. The update equation for the proposed algorithm will contain an additional term due to the penalty function as compared to the original one, and is given by

$$\begin{aligned} \underline{\widehat{\mathbf{h}}}_k^{10}(m+1) &= \underline{\widehat{\mathbf{h}}}_k^{10}(m) - \mu[\mathbf{p}_k(m) + \delta\mathbf{I}_{2L\times 2L}]^{-1} \\ &\times \sum_{i=1}^{M} D_{x_i}^*(m)\underline{\mathbf{e}}_{ik}^{01}(m) \\ &-\beta\mu \mathrm{F}_{2L\times 2L}^* \begin{bmatrix} \mathrm{F}_{L\times L}^{-1}\left(\frac{\partial J_p(m)}{\partial \underline{\widehat{\mathbf{h}}}_k^*}\right) \\ \mathbf{0}_{L\times 1} \end{bmatrix} \end{aligned} \tag{31}$$

6. SIMULATION RESULTS

In this section, we present computer simulation results to investigate the effectiveness of the proposed constrained algorithm to blind channel estimation problems. The dimension of the room was taken to be $(5\times4\times3)$ m. A linear array consisting of $M=5$ microphones with uniform separation of $\tau=0.2$ m was used in the experiment. The first microphone and source were positioned at $(1.0,1.5,1.6)$ m and $(2.0,1.2,1.6)$ m, respectively. The positions of the other microphones can be obtained by adding $\tau=0.2$ m successively with the y-coordinate of the first microphone. The impulse responses were generated using the image model reported in [7] for reverberation time $T_{60}=0.1$ s and then truncated so as to make the length $L=128$. In all cases, the source signal was Gaussian white noise, and λ was fixed to $[1-1/(3L)]^L$.

The performance index used for measurement of improvement is the normalized projection misalignment defined as

$$\mathrm{NPM}(m)=20\log_{10}\left(\frac{1}{||\mathbf{h}||}\left|\left|\left[\mathbf{h}-\frac{\mathbf{h}^T\widehat{\mathbf{h}}(m)}{\widehat{\mathbf{h}}^T(m)\widehat{\mathbf{h}}(m)}\widehat{\mathbf{h}}(m)\right]\right|\right|\right) \tag{32}$$

where $||\cdot||$ is the l_2 norm. Using this index, the performance of the proposed algorithm is compared with the conventional NMCFLMS. We feel not fair, even though the results are similar, to compare our proposed fully blind constrained algorithm with the one reported in [6] as it is not a blind algorithm in totality for the reason explained in Section 1.

The results of estimated AIRs using the conventional NMCFLMS algorithm for $\mu=0.5$ and SNR=15 dB are shown in Fig. 2. As can be seen, the sparse nature of the true impulse responses shown in Fig. 2 (a) are not visible in the estimates depicted in Fig. 2 (b). It is interesting to observe that the nature of the estimates of different channels at the misconvergence are also very similar. This result indicates that the lowpass filters acting on each channel have very similar bandwidth with narrowband characteristic. On the contrary, the proposed constrained algorithm accurately estimates the AIRs. Comparative results on the convergence rate of the NMCFLMS algorithm with and without constraints are shown in Fig. 3 for $\mu=0.5$ and SNR=15 dB. As expected, the convergence of the conventional NMCFLMS algorithm under noisy condition is followed by misconvergence. In contrast, the constrained NMCFLMS algorithm proposed in this paper is asymptotically stable at the preliminary convergence which is though somewhat biased due to noise. The bias gets lower as the noise level decreases.

7. CONCLUSIONS

In this paper, we have investigated the performance of the NMCFLMS algorithm in the identification of AIRs when observations are corrupted by noise. We have demonstrated that the presence of additive noise leads to the misconvergence of the conventional NMCFLMS algorithm. The reason behind misconvergence has been shown to be nonzero gradient of the noise error surface. It is argued that the effect of nonzero gradient is the lowpass filtering on the AIRs. A novel method has been proposed to stop misconvergence and thus to ameliorate convergence characteristic of the blind NMCFLMS algorithm by attaching a constraint on the energy of the high frequency DFT coefficients of the estimated impulse response. The constraint plays the role of a highpass filter to counter balance the cause of misconvergence. In support of the theory, the numerical tests have also demonstrated noise robustness of the proposed algorithm.

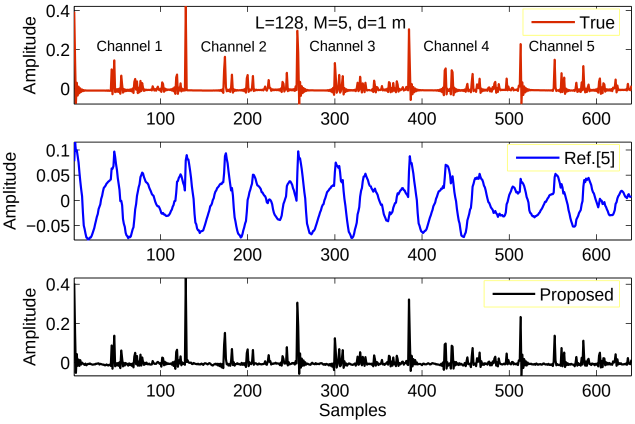

Figure 2: Results on channel estimation at SNR=15 dB using the NMCFLMS; (a) True AIRs, (b) Estimated AIRs using [5], and (c) Estimated AIRs using the proposed method.

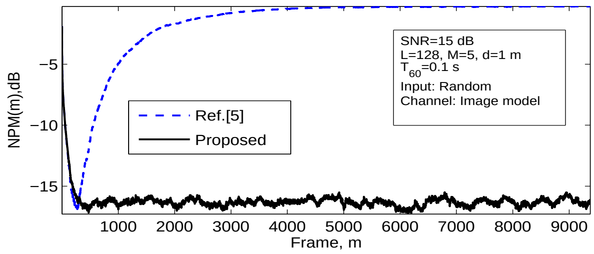

Figure 3: Comparative results using NPM at SNR=15 dB.

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