TIME-DELAY ESTIMATION OF SIGNALS IN NONSTATIONARY RANDOM NOISE VIA STATIONARIZATION AND WIGNER DISTRIBUTION-BASED APPROACH

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ABSTRACT

An effective method is proposed in this paper for the estimation problem of unknown time-delay of a signal which is received corrupted by the nonstationary random noise. The keys of the method are the stationarization of the nonstationary observation data and the introduction of Wigner distribution-based maximum likelihood function. The method is tested by simulations to show the efficacy.

1. INTRODUCTION

Recently, there has been an increasing interest in the estimation of time-delay of signals which are transmitted to a target and received with corrupting nonstationary random noise. It is needless to say that the need for determining the time-delay arises in sensor array systems. The problem of time-delay estimation for targets reduces to that of estimating the parameter associated with the received signal. The authors have been concentrating their attention to the problems of detecting signals and/or estimating time-delay as the parameter estimation, and they have developed an approach based on the maximum likelihood function which is constructed from the time-frequency realizations of Wigner distribution (WD) [1-4]. Although the approach has been shown to achieve good results at low SNRs, there is no guarantee for the case of nonstationary random noise because the approach was developed under the assumption that the corrupting noise is stationary.

Time-delay estimation is one of important issues in signal processing. Most of conventional methods have been developed by assuming the corrupting noise is stationary with time-invariant power spectrum (e.g., [5]). As well-known, there exist several tools for analyzing signals in time-frequency domain such as short-time Fourier transform, wavelet and Wigner distribution. However, these may be effective for signal detection, but the usefulness for estimating parameters attributed to the signal to be detected is still uncertain because these do not use positively any information about the corrupting nonstationary random noise.

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In this paper, a method of estimating time-delay of the signal which is corrupted by a nonstationary random noise is proposed. The principal line of attack of the approach is to convert the nonstationary observation process to a stationary one, and then to apply the WD-based parameter estimation method developed by the authors to the signal detection.

2. PROBLEM STATEMENT

Let $s(t)$ be a scalar (real) signal transmitted actively to a target or emanated from a remote source. When the signal is received at a receiver, it is delayed in time, attenuated, and contaminated by random noise. Then the observation data is obtained in the following manner:

$$y(t) = as(t-D) + n(t), \quad t \geq 0 \tag{1}$$

where a and D are the attenuation coefficient and time-delay, respectively; and $n(t)$ is the additive random noise. The form of signal $s(t)$ is assumed to be known, but its duration is local in time. The additive noise $n(t)$ is assumed to be nonstationary and given as an output of the process described by the Itô stochastic differential equations:

$$dn(t) = -\beta(t)n(t)dt + \alpha(t)dw(t), \quad n(0) = n_0, \tag{2}$$

where $w(t)$ is a (scalar) standard Wiener process; $\alpha(t)$, $\beta(t)$ are slowly and smoothly varying positive but unknown functions; and the initial value n_0 is a Gaussian random variable with zero-mean and unit variance. Since (unknown) parameters $\alpha(t)$ and $\beta(t)$ are time-varying, the noise $n(t)$ and also the observation process $y(t)$ become inevitavely nonstationary.

Then, our purpose is to propose a method of estimating the time-delay D from the nonstationary observation data $\{y(t)\}$. It should be emphasized here that the existing approaches can not be applied directly without any modification for such estimation problem of the time-delay because the additive noise $n(t)$ is nonstationary. Most of all existing approaches are developed for stationary noise processes. From this standpoint, the approach taken in this paper is as follows:

(i) First, the unknown coefficient functions $\alpha(t)$ and $\beta(t)$ in the noise model (2) are estimated using observation data $\{y(t)\}$.

(ii) Then, using the estimates for $\alpha(t)$ and $\beta(t)$, the nonstationary observation data $\{y(t)\}$ are modified to be stationary ones. Similar procedure has recently attempted to the detection problem of signals corrupted by nonstationary noise [6].

(iii) Based on the stationarized observation data in the procedure (ii), the estimation of D is achieved via the WD-based maximum likelihood estimation method developed by the authors [1-4].

Since the attenuation parameter a is linearly related to the signal, the procedure of its estimation will be decoupled [7]. So, for simplicity, it is assumed in the sequel that a is known.

3. STATIONARIZATION OF NONSTATIONARY OBSERVATION DATA

A. Estimation of Unknown Coefficients

First, the unknown coefficient functions of time, $\alpha(t)$ and $\beta(t)$, in the noise model (2) are identified. To do this, recalling that the duration of the signal to be detected is very local, let us consider the signal-free case, neglecting the signal's existence,

$$y(t) = n(t), \tag{3}$$

which has the stochastic differential [8,9],

$$\begin{aligned} dy(t) &= dn(t) \\ &= -\beta(t)y(t)dt + \alpha(t)dw(t). \end{aligned} \tag{4}$$

We have assumed that the coefficient functions $\alpha(t)$ and $\beta(t)$ change slowly and smoothly. More concretely speaking, in an interval I_t around the current time t, they are assumed to behave approximately like constant, i.e.,

$$\alpha(t) = \alpha_t, \quad \beta(t) = \beta_t \quad \text{for } t \in I_t. \tag{5}$$

Recalling that the power spectral density of the process (2) with constant parameters α_0 and β_0 is given by $S(\lambda) = \alpha_0^2/(\lambda^2 + \beta_0^2)$, the power spectral density of the $n(t)$-process is approximately evaluated around the current time as

$$S_t(\lambda) = \frac{{\alpha_t}^2}{\lambda^2 + {\beta_t}^2} \tag{6}$$

under the local stationarity assumption (5). The suffix in the notation $S_t(\lambda)$ stands for the dependence on the current time t. In this sense, $S_t(\lambda)$ may be interpreted as the time-varying spectral density or evolutionary spectral density in the sense of Priestley [10].

With the help of Priestley's method for the estimation of evolutionary (i.e., time-varying) spectral density, the density $S_t(\lambda)$ can be estimated from the observation data $\{y(t)\}$ (provided that no signal exists in the data). Let it denote by $\hat{S}_t(\lambda)$.

From the relation (6), we have

$$\frac{1}{S_t(\lambda)} = \left(\frac{1}{{\alpha_t}^2}\right)\lambda^2 + \left(\frac{{\beta_t}^2}{{\alpha_t}^2}\right). \tag{7}$$

Based on this version, the coefficients $1/{\alpha_t}^2$ and ${\beta_t}^2/{\alpha_t}^2$ are determined by minimizing the square-error, $|1/S_t(\lambda) - 1/\hat{S}_t(\lambda)|^2$ with respect to these coefficients. However, only the first one is accepted as a least-squares estimate to obtain $\hat{\alpha}_t = \sqrt{\hat{\alpha}_t^2}$ because the estimate for β_t obtained by this can not be recommended from the viewpoint of accuracy. Instead, this is estimated from the standpoint of energy of the noise process. It is calculated in the neighborhood of t as

$$v(t) = \int_{-\infty}^{\infty} S_t(\lambda)d\lambda = \frac{{\alpha_t}^2}{\beta_t}\pi. \tag{8}$$

From this, we have the estimate $\hat{\beta}_t$ by

$$\hat{\beta}_t = \frac{\hat{\alpha}_t^2}{\hat{v}_t}\pi, \tag{9}$$

where $\hat{v}_t = \int_{-\infty}^{\infty} \hat{S}_t(\lambda)\,d\lambda$.

B. Stationarization of the Observation Data

By assuming still the signal-free case, the observation process (4) can be approximated locally using the estimated coefficients,

$$dy(t) = -\hat{\beta}_t\, y(t)dt + \hat{\alpha}_t\, dw(t). \tag{10}$$

This is expressed in the discretized version as follows:

$$\delta y_t = -\hat{\beta}_t\, y_t\, \delta t + \hat{\alpha}_t\, \delta w_t\,, \tag{11}$$

where δy_t $(= y(t+\delta t) - y(t) + o(\delta t))$ and δw_t $(= w(t+\delta t) - w(t) + o(\delta t))$ are small increments of $y(t)$ and $w(t)$, respectively. Dividing both sides by $\hat{\alpha}_t\,\delta t$ provided that $\hat{\alpha}_t \neq 0$, we have

$$\frac{\delta y_t + \hat{\beta}_t\, y_t\, \delta t}{\hat{\alpha}_t\, \delta t} = \frac{\delta w_t}{\delta t}. \tag{12}$$

Here, it should be noted that the right-hand side of (12) can be regarded as a stationary white Gaussian noise sequence with zero-mean and unit power spectral density. Keeping this fact in mind, let us define for each t a sequence $\hat{y}_t$ by

$$\hat{y}_t = \frac{\delta y_t + \hat{\beta}_t\, y_t\, \delta t}{\hat{\alpha}_t\, \delta t}\,. \tag{13}$$

Hence, the sequence $\hat{y}_t$ can be regarded as a (discrete-time) stationarized version (for signal-free case) of the observation process $y(t)$.

Same argument can be possible for the case when the signal exists. Indeed, the observation process (1) is expressed in the stochastic differential form, using the estimated coefficients, as

$$\begin{aligned} dy(t) &= a\dot{s}(t-D)dt + dn(t) \\ &= [\, a\dot{s}(t-D) - \hat{\beta}_t\, n(t)\,]dt + \hat{\alpha}_t\, dw(t) \\ &= a\{\dot{s}(t-D) + \hat{\beta}_t\, s(t-D)\}dt \\ &\qquad - \hat{\beta}_t\, y(t)dt + \hat{\alpha}_t\, dw(t). \end{aligned} \tag{14}$$

From this we have the discretized version,

$$\delta y_t + \hat{\beta}_t\, y_t\, \delta t = a\{\dot{s}(t-D) + \hat{\beta}_t\, s(t-D)\}\, \delta t + \hat{\alpha}_t\, \delta w_t\,. \tag{15}$$

Dividing both sides again by $\hat{\alpha}_t \delta t$, we have the expression,

$$\hat{y}_t = a\hat{s}_t(D) + \gamma_t, \tag{16}$$

where

$$\hat{s}_t(D) = \frac{1}{\hat{\alpha}_t}\{\dot{s}(t-D) + \hat{\beta}_t\, s(t-D)\} \tag{17}$$

and $\gamma_t = \delta w_t/\delta t$ is the white Gaussian noise sequence. The expression (16) is familiar to us as the mathematical model for the detection problem of signals in the *stationary* random noise [11].

4. WIGNER DISTRIBUTION-BASED ESTIMATION OF TIME-DELAY

The signal detection will be possible if it is done based on (16). Of course, there are so many approaches such as the binary test based on likelihood-ratio, WD for the observation data, and so on. The estimation of the time-delay D is achieved here by incorporating the WD with the idea of the maximum likelihood function. Hereafter, write $\hat{y}_t$ as $\hat{y}_n$ for $t = n\Delta t$ (Δt: small increment of the time partition). Then, given the observation data $\{\hat{y}_t\}$, or $\{\hat{y}_n\}_{n=0,1,2,\cdots}$ generated by the discrete-time process (16), let $W_{\hat{y}}(n,k;D)$ be the discrete WD represented in the following form [12]:

$$W_{\hat{y}}(n,k;D) = \sum_{m=0}^{N-1} \hat{y}_{n+m}\, \hat{y}_{n-m} \cos(4\pi mk/N). \tag{18}$$

Equation (18) can be expressed as the sum

$$W_{\hat{y}}(n,k;D) = W_{\hat{s}\hat{s}}(n,k;D) + 2W_{\hat{s}\gamma}(n,k;D) + W_{\gamma\gamma}(n,k), \tag{19}$$

where

$$W_{\alpha\beta}(n,k;D) = \sum_{m=0}^{N-1} \alpha_{n+m}\, \beta_{n-m} \cos(4\pi mk/N) \quad (\alpha, \beta = \hat{s} \text{ or } \gamma). \tag{20}$$

It should be noted that in (19) there appear two terms due to the random noise, $2W_{\hat{s}\gamma}(n,k;D)$ and $W_{\gamma\gamma}(n,k)$, and that these two interfere in the legitimate auto-component $W_{\hat{s}\hat{s}}(n,k;D)$ to detect the signal by observing the spectrum of $W_{\hat{y}}(n,k;D)$ over the time-frequency domain. This situation leads us to formulate the binary test:

$$\left.\begin{array}{l} H^1\colon W_{\hat{y}}(n,k;D) \;\text{ described by (19)} \\ H^0\colon W_{\hat{y}}(n,k;D) = W_{\gamma\gamma}(n,k). \end{array}\right\} \tag{21}$$

In order to perform the hypothesis-testing (21), consider the probability density function for $W_{\hat{y}}(n,k;D)$.

For fixed D, $W_{\hat{y}}(n,k;\cdot)$ computed by (18) is a realization on the grid (n,k), so that the set $\mathcal{W} = \{W_{\hat{y}}(n,k;\cdot)\}$ constitutes a random field over time-frequency domain.

Assuming $1 \le n \le N$ and $1 \le k \le M$, let w be the MN-dimensional vector consisting of realizations $\{W_{\hat{y}}\}$ arranged in lexicographic order. Recall that the value of WD $W_{\hat{y}}$ can be treated asymptotically as the Gaussian random field [3]. Then, the likelihood function $p\{w|D\}$ can be written as the joint probability density function of w:

$$p\{w|D\} = (2\pi)^{-\frac{MN}{2}} |R(D)|^{-\frac{1}{2}} \cdot \exp\left\{-\frac{1}{2}\,[w - m(D)]^T R^{-1}(D)[w-m(D)]\right\}, \tag{22}$$

where $m(D)$ and $R(D)$ are the mean vector and covariance matrix, respectively. Furthermore, let us introduce another likelihood function $p_0\{w\}$ under the hypothesis H^0 which says equivalently that the observation data comes from $\hat{y}_t = \gamma_t$:

$$p_0\{w\} = (2\pi)^{-\frac{MN}{2}} |R_0|^{-\frac{1}{2}} \cdot \exp\left\{-\frac{1}{2}(w-m_0)^T R_0^{-1}(w-m_0)\right\}, \tag{23}$$

where m_0 and R_0 are the mean and covariance under H^0. The introduction of $p_0\{w\}$ is only for technical reason to derive the following log-likelihood ratio function:

$$L(w;D) = -\frac{1}{2}\{\ln|R(D)| - \ln|R_0| + [w-m(D)]^T R^{-1}(D)[w-m(D)] - (w-m_0)^T R_0^{-1}(w-m_0)\}. \tag{24}$$

The estimation of the unknown parameter D can be obtained by maximizing this function with respect to D over its possible region $\mathcal{D}$:

$$\hat{D} = \arg\left\{\max_{D\in\mathcal{D}} L(w;D)\right\}. \tag{25}$$

5. SIMULATION STUDIES

To confirm the proposed method several simulation experiments were performed. The observation data $y(t)$ is generated by the discretized version for (1) with the time step size $\Delta t = 1$ sec, the time-varying coefficients of the nonstationary noise process $n(t)$ are set as

$$\alpha(t) = 1 - 0.8\cos\left(\frac{2\pi(t-500)}{1024}\right)$$

$$\beta(t) = 0.5 - 0.45\cos\left(\frac{2\pi(t-700)}{1024}\right),$$

and a was set as $a = 1$.

A. Sinusoidal Signal

The transmitted signal $s(t)$ is assumed to have the one-cycle sinusoidal waveform,

$$s(t) = \begin{cases} 0.8 \sin \dfrac{2\pi t}{6} & \text{for } 0 < t \leq 6 \\ 0 & \text{otherwise.} \end{cases} \tag{26}$$

The top figure in Fig. 1 depicts the observation data during the time interval [0, 1000] sec. The signal to be detected is illustrated on the bottom, and this signal is embedded in the observation process with true parameter $D^* = 660$ sec. The estimated coefficients $\hat{\alpha}_t$ and $\hat{\beta}_t$ are illustrated in Fig. 2. Comparing them with their true values, we may say that the estimates are fairly well performed. The top in Fig. 3 depicts the modified (stationarized) observation data $\hat{y}_t$. By comparing this with the original nonstationary process $y(t)$ shown in Fig. 1 it can be said that the sequence $\hat{y}_t$ depicted in Fig. 3 is well stationarized.

The log-likelihood ratio function $L(w; D)$ is shown also in Fig. 3. It is apparent from this figure that $L(w; D)$ takes a distinctively large peak around $D = 660$ sec. From this we may see that the proposed method has an excellent performance.

B. Pulse Signal

The proposed estimator is also effective to estimate the pulse signal. We verify this by numerical simulations. The transmitted signal $s(t)$ is assumed to be

$$s(t) = \begin{cases} 0.78 & 0 \leq t \leq 11 \\ 0 & \text{otherwise.} \end{cases} \tag{27}$$

We set the true parameter as $D^* = 70$ sec. The observation data and the signal to be detected are shown in Fig. 4. The estimated coefficients $\hat{\alpha}_t$ and $\hat{\beta}_t$ are illustrated in Fig. 5. Figure 6 shows the modified observation data $\hat{y}_t$ and the result of function $L(w; D)$. We can see that $L(w; D)$ takes a conspicuous peak around $D = 70$ sec, clearly.

6. CONCLUSION

It seems that almost all conventional methods for estimating the time-delay of the signal to be detected are proposed under the assumption of stationary random noise. On the contrary to such situation, an approach to the estimation problem of time-delay has been proposed in this paper under the situation of nonstationary random noise. The usefulness of the approach has been verified affirmatively by simulation studies.

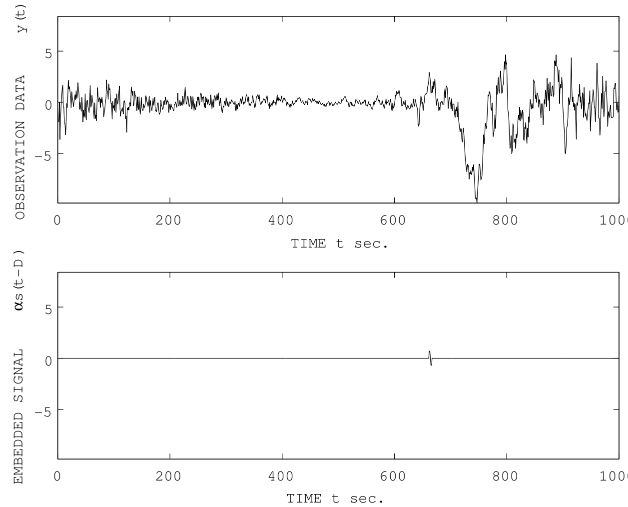

Fig. 1. A sample path of the observation data $y(t)$ (top) and the embedded one-cycle sinusoidal signal $s(t)$ (bottom).

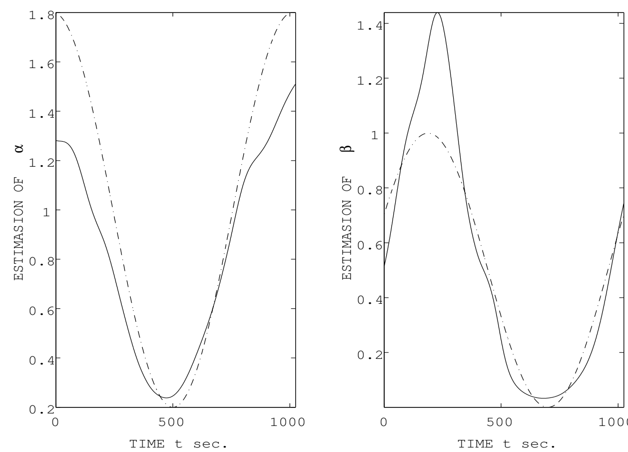

Fig. 2. Estimations of $\alpha(t)$ (left) and $\beta(t)$ (right) (where the dash-dot line indicates their true values).

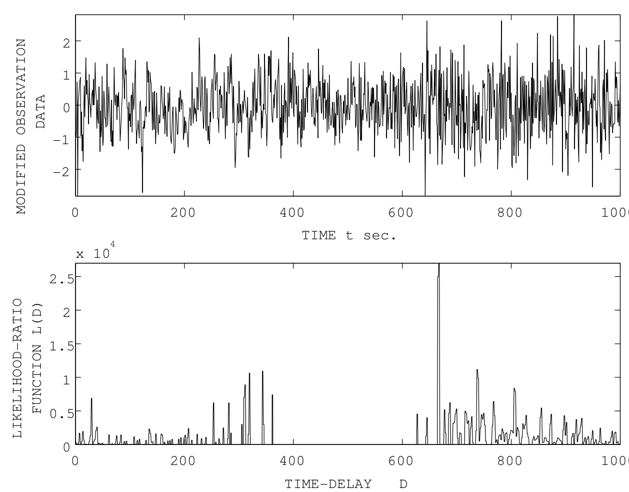

Fig. 3. The stationarized observation data $\hat{y}_t$ (top) and the log-likelihood ratio function $L(w; D)$ (bottom).

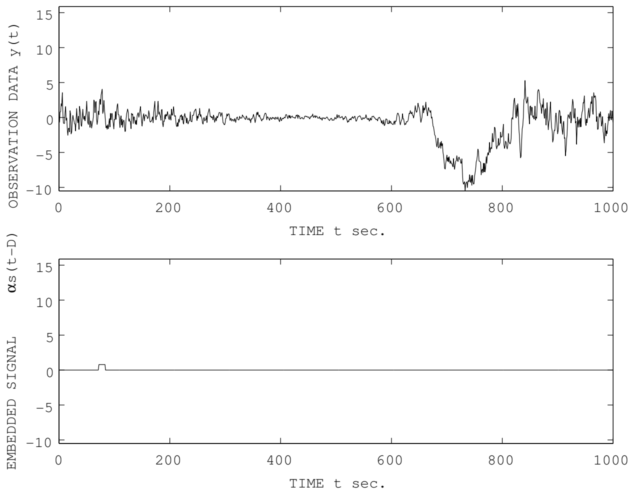

Fig. 4. A sample path of the observation data $y(t)$ (top) and the embedded pulse signal $s(t)$ (bottom).

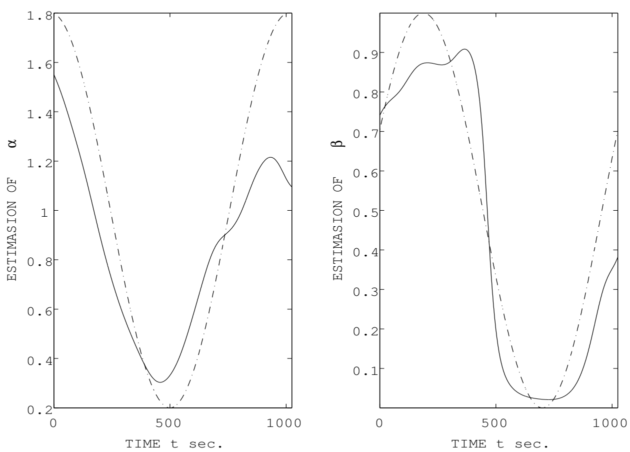

Fig. 5. Estimations of $\alpha(t)$ (left) and $\beta(t)$ (right).

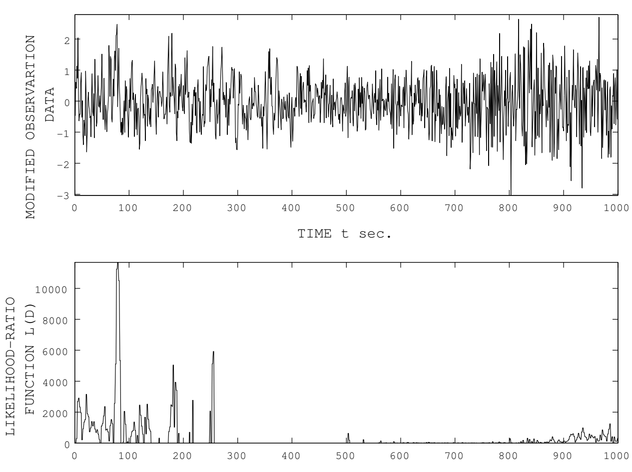

Fig. 6. The stationarized observation data $\hat{y}_t$ (top) and the log-likelihood ratio function $L(w; D)$ (bottom).

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