# ADAPTIVE JOINT DETECTION FOR A RANDOM PERMUTATION-BASED MULTIPLE-ACCESS SYSTEM ON UNKNOWN TIME-VARYING FREQUENCY-SELECTIVE CHANNELS

Martial COULON and Daniel ROVIRAS

#### INP-ENSEEIHT / IRIT

2 Rue Camichel, 31071 Toulouse cedex 7, FRANCE Email: martial.coulon@enseeiht.fr, daniel.roviras@enseeiht.fr

#### **ABSTRACT**

This paper addresses the problem of joint detection for a spread-spectrum multiple-access system based on random permutations. The transmission channels are assumed to be frequency-selective and time-varying; moreover, these channels are unknown to the receiver. Consequently, the detection is achieved using an adaptive algorithm, whose objective consists of minimizing the minimum mean-square error. Alternatively, under the same hypotheses, an adaptive detector is also proposed for the DS-CDMA system, and compared with the previous detector. This comparison shows that, even if the detection is difficult in such a context for both detectors, the permutation-based method gives better performance.

# 1. INTRODUCTION

Spreading techniques are among the prominent multipleaccess schemes for the third-generation mobile cellular systems. Techniques based and derived from the CDMA paradigm are the most popular, and are already used in many wireless communication systems. Particularly, this success has given rise to many multi-user detectors, which are relevant in various situations (see for instance [6] and references therein).

Nevertheless, other multiple-access schemes have equivalent spreading capabilities. In this paper is investigated a particular spreading method, called Random Permutation Multiple Acces (RPMA), which can be regarded as a sub-class of linear periodic time-varying filters. This method consists of permuting the samples of an input data block in order to spread the spectrum of the data: this operation is a particular case of discrete periodic clock changes [4].

Other block-spreading techniques can be found in the literature (e.g., [5], [7]). However, it is important to note that the method proposed in this paper differs from the Chip-Interleaved Block-Spread CDMA method developed for instance in [7]: indeed, the interleaver used here is a random interleaver, whose main property consists of whitening the part of the received multi-user signal which is not synchronized with the part of signal of interest [4]. At the contrary, the interleaver used in [7] is a matrix (row-column) interleaver, which does not present the same properties: therefore, it does not work equally, and generally assumes a synchronous or quasi-synchronous transmission.

In this paper, the case of asynchronous transmissions on time-varying frequency-selective channels is considered. Previous works using RPMA have already been proposed on this topic (e.g. [1], [2]), where it is compared to the wellknown DS-CDMA system. In [2], it has been shown that RPMA performs better than DS-CDMA for fast fading channels. However, this previous study assumed perfect knowledge of the channels at the receiver, which is obviously not realistic, in particular when the channel variations are fast. In this paper, one assumes a partial knowledge of the channels, i.e. only the tap delays are known, whereas the fading coefficients are unknown at the receiver. Therefore, one resorts in this paper to an adaptive algorithm, and more specifically to the Least-Mean-Squares (LMS) detector, which adaptively minimizes the linear Mean Square Error (MSE). Obviously, the originality of this paper does not hold in the use of the LMS algorithm, which is a classical adaptive algorithm. Instead, the originality relies on the fact that a new spreadspectrum technique can be used in the context of unknown time- and frequency-selective channels, and that this method gives better results than a similar detector derived for a "classical" spread-spectrum method (here, the DS-CDMA system).

Section 2 presents the random permutation technique, and gives the expressions of the transmitted and received continuous/discrete signals. The detection is investigated in section 3 for the RPMA system; an equivalent detector for the DS-CDMA system is also presented. Simulation results are provided in section 4, along with a comparison between RPMA and DS-CDMA systems.

## 2. PROBLEM FORMULATION

## 2.1 The permutation process

Let  $(b_n)_{n\in\mathbb{Z}}$   $(b_n\in\{-1;+1\})$  be a sequence of equiprobable bits. This sequence is modulated by an antipodal baseband code with duration T and waveform pattern m(t). The modulated process Z(t), defined by  $Z(t)\triangleq\sum_{n\in\mathbb{Z}}b_nm(t-nT)$ , is sampled with period  $T_s$ , such that  $N_s\triangleq T/T_s$  is an integer number (i.e.,  $N_s$  is the number of samples per bit). Let  $(Z_n)_{n\in\mathbb{Z}}$  denote this sampled sequence. A new sequence  $(U_n)_{n\in\mathbb{Z}}$  is formed from  $(Z_n)_{n\in\mathbb{Z}}$  as follows: considering blocks of  $N_b$  consecutive bits, the  $N_sN_b$  samples of  $(Z_n)_{n\in\mathbb{Z}}$  corresponding to a given block are permuted using an uniformly distributed permutation of the set  $\{1,\ldots,N_sN_b\}$  (this permutation is the same for all blocks). The sequence  $(U_n)_{n\in\mathbb{Z}}$  is defined as the resulting sequence of this block permutation. One can then show that the power spectral density of  $(U_n)_{n\in\mathbb{Z}}$  is spreaded by a factor  $N_s$  with respect to the one of  $(Z_n)_{n\in\mathbb{Z}}$  [4]. Consequently, this permutation procedure is

<sup>&</sup>lt;sup>1</sup>Actually, the sequence  $U_n$  is not stationary in general; however, the sequence  $V_n$  defined by  $V_n \triangleq U_{n+\Phi}$  where  $\Phi$  is uniformly distributed on the

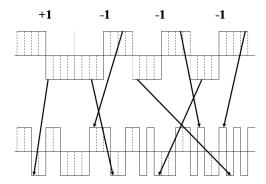


Figure 1: Example of permutations for one block, using biphase signaling,  $N_b = 4$ , and  $N_s = 8$ .

a particular spread-spectrum technique. An example of this permutation procedure is given in fig. 1, for one block of  $N_b = 4$  bits and  $N_s = 8$  samples.

Denote  $\mathbf{b}_r = \begin{bmatrix} b_{(r-1)N_b+1}, \dots, b_{rN_b} \end{bmatrix}^T$  as the rth block of bits, and  $\mathbf{m} \triangleq \begin{bmatrix} m_1, \dots, m_{N_s} \end{bmatrix}^T$  as the result of the sampling of the waveform pattern m(t) with  $N_s$  samples. Then, the rth block of the sequence  $(Z_n)_{n \in \mathbb{Z}}$ , i.e. the vector  $\mathbf{Z}_r = \begin{bmatrix} Z_{(r-1)N_sN_b+1}, \dots, Z_{rN_sN_b} \end{bmatrix}^T$ , can be expressed as:  $\mathbf{Z}_r = M^T \mathbf{b}_r$ , where  $M = \mathbf{m}^T \otimes \mathbf{I}_{N_b}$  is the  $N_b \times (N_sN_b)$  matrix defined by

$$M = \begin{bmatrix} \mathbf{m}^T & 0 & \cdots \\ 0 & \ddots & \ddots \\ & \cdots & \mathbf{m}^T \end{bmatrix}$$

( $\otimes$  is the Kronecker product, and  $\mathbf{I}_n$  is the identity matrix of order n). Let P denote the  $(N_sN_b) \times (N_sN_b)$  matrix corresponding to this permutation (i.e.  $P_{i,j} = 1$  iff the integer j is transformed into the integer i by this permutation, and  $P_{i,j} = 0$  otherwise). The rth block of the sequence  $(U_n)_{n \in \mathbb{Z}}$ , defined by  $\mathbf{U}_r \triangleq [U_{(r-1)N_sN_b+1}, \dots, U_{rN_sN_b}]^T$ , can be expressed as

$$\mathbf{U}_r = P\mathbf{Z}_r = PM^T\mathbf{b}_r.$$

The continuous-time process x(t) obtained from  $(U_n)_{n\in\mathbb{Z}}$  using a rectangular waveform signaling is then given by

$$x(t) = \sum_{r \in \mathbb{Z}} \sum_{i=1}^{N_s N_b} (PM^T \mathbf{b}_r)_j \rho (t - jT_s - rN_b T)$$

where  $(v)_j$  denotes the *j*th component of any vector v, and  $\rho(t)$  is the indicator function on  $[0; T_s]$ .

# 2.2 The multi-user signal

Consider the asynchronous transmission of K users using the spread-spectrum technique presented above. Let  $\mathbf{b}_{r,k}$ ,  $P_k$  and  $x_k(t)$  denote respectively the rth block of bits, the permutation matrix, and the continuous-time process corresponding to user k. The channel associated to the kth user is a

time-varying frequency-selective channel, whose impulse response at time t is given by  $^2$ :

$$c_k(t, au) = \sum_{l=1}^L c_{k,l}(t) \delta\left( au - au_{k,l}
ight),$$

where  $c_{k,l}(t)$  is the time-varying (complex) gain of the lth path of the kth user,  $\tau_{k,l}$  is the propagation delay, and  $\delta$  is the Dirac function. If each user transmits B blocks of bits, the received signal is expressed as:

$$y(t) = \sum_{r=1}^{B} \sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{j=1}^{N_{s}N_{b}} c_{k,l}(t) (P_{k}M^{T} \mathbf{b}_{r,k})_{j} \times \rho (t - jT_{s} - rN_{b}T - \tau_{k,l}) + n(t)$$

where n(t) is an additive white Gaussian noise (AWGN) with variance  $\sigma^2$ , independent of the transmitted signals.

At the receiver, this continuous signal is sampled by passing it through an appropriate filter bank. Referring to [2], the optimal filtering, i.e. which yields to a sufficient statistic, is defined by

$$\int_{jT_s+rN_bT+\tau_{k,l}}^{(j+1)T_s+rN_bT+\tau_{k,l}} c_{k,l}^*(t)y(t)\rho\left(t-jT_s-rN_bT-\tau_{k,l}\right)dt,$$
(1)

for  $k=1,\ldots,K, l=1,\ldots,L, j=1,\ldots,N_sN_b$ , and  $r=1,\ldots,B$ . Now, it is assumed in this paper that the complex gains  $c_{k,l}(t)$  are unknown to the receiver (whereas the delays  $\tau_{k,l}$  are known). Consequently, this filtering can not be achieved in such a context, and one must resort to a sub-optimal filtering. One proposes here to use the filter bank defined in [1], suited for constant channels, i.e. to define variables  $y_{k,l,r}(j)$  as

$$\int_{jT_s+rN_bT+\tau_{k,l}}^{(j+1)T_s+rN_bT+\tau_{k,l}} y(t)\rho\left(t-jT_s-rN_bT-\tau_{k,l}\right)dt,$$

for k = 1, ..., K, l = 1, ..., L,  $j = 1, ..., N_s N_b$ , and r = 1, ..., B. For  $T_s$  small enough (or equivalently,  $N_s$  large enough),  $T_s$  is smaller than the coherence time of the channel, and  $c_{k,l}(t)$  can be supposed constant in the integral interval in (1). Therefore,  $y_{k,l,r}(j)$  is quasi-equal to the integral expression (1), up to the scalar constant  $c_{k,l}(jT_s + rN_bT + \tau_{k,l})$ , and both filter banks performs similarly. Define now vectors

$$\mathbf{y}_{k,l,r} \triangleq [y_{k,l,r}(1), \dots, y_{k,l,r}(N_sN_b)]^T,$$

$$\mathbf{y}_{l,r} \triangleq [\mathbf{y}_{1,l,r}^T, \dots, \mathbf{y}_{K,l,r}^T]^T,$$

$$\mathbf{y}_r \triangleq [\mathbf{y}_{1,r}^T, \dots, \mathbf{y}_{L,r}^T]^T$$

$$\mathbf{b}_{k,r} \triangleq [b_{k,r}(1), \dots, b_{k,r}(N_b)]^T,$$

$$\mathbf{b}_r \triangleq [b_{1,r}^T, \dots, b_{K,r}^T]^T$$

Thus, the vectors  $\mathbf{y}_r$  and  $\mathbf{b}_r$  are the concatenation of all correlations, and of all bits, respectively. If  $T_s$  is assumed smaller than the coherence time, as mentioned above, one can show that  $\mathbf{y}_r$  can be written as:

$$\mathbf{y}_r = \mathbf{\Lambda}_r \mathbf{\Pi} \mathbf{b}_r + \mathbf{n}_r \tag{2}$$

where:

set  $\{1, \dots, N_s N_b\}$  is stationary and ergodic, and the autocorrelation function and power spectral density refer rather to this stationarized sequence.

<sup>&</sup>lt;sup>2</sup>Actually, the number of paths is generally not equal for all users. However, one can consider L as the maximum of the path lengths, nulling gains  $c_{k,l}(t)$  if necessary.

•  $\Lambda_r$  is the (Hermitian) symmetric matrix whose element in row  $(l-1)N_sKN_b + (k-1)N_sN_b + j$  and column  $(l'-1)N_sKN_b + (k'-1)N_sN_b + j'$  is

$$\int c_{k,l}(t)c_{k',l'}^*(t)\rho(t-jT_s-\tau_{k,l})\rho(t-j'T_s-\tau_{k',l'})dt$$

- $\Pi = \mathbf{1}_L \otimes \Pi_d$ , with  $\mathbf{1}_L = [1, \dots, 1]^T$  (with length L), and  $\Pi_d$  is the block-diagonal matrix whose kth diagonal block is the matrix  $P_k M^T$ .
- $\mathbf{n}_r$  is a zero-mean Gaussian vector with covariance matrix  $\sigma^2 \mathbf{\Lambda}_r$ .

Note that matrices  $\Lambda_r$  and  $\Pi$  can be huge, since they have dimensions  $(N_sKN_bL) \times (N_sKN_bL)$  and  $(N_sKN_bL) \times (KN_b)$ , respectively. However, the memory and computationnal costs can be drammatically reduced given the fact that these matrices are sparse. For instance, using parameters used for fig. 2 (see section 4), matrix  $\Lambda_r$  contains only 8.8% of nonzero elements (and this percentage decreases when the dimension parameters increase). Also, the matrix  $\Pi$  has only  $KN_sN_bL$  non-zero elements.

In order to retrieve the users'bits with minimum error rate, the receiver must severely reduce the multiple-access interference, the inter-symbol interference, and the additive noise. In previous works ([1], [2]), Linear Minimum MSE (LMMSE) detectors have been studied, assuming perfect knowledge of the channels. In this paper, due to the assumption of unknown channel coefficients, one proposes to resort to the adaptive version of the LMMSE, i.e. the LMS algorithm.

It must also be noted that, ideally, all vectors  $(\mathbf{y}_r)_r$  are required for the detection of one single bit of one single user, as indicated by the form of the sufficient statistic. This is due to the inter-symbol interference (for each user) and to the asynchronism between users. Obviously, this cannot be done for an adaptive algorithm such as the LMS algorithm, since the number of coefficients would be far too large. We propose here to base the on-line detection of the block of bits  $\mathbf{b}_r$  on the use of the unique vector of matched-filter outputs  $\mathbf{y}_r$ . However, the algorithm can easily be generalized to the used of a set of vectors of the form  $(\mathbf{y}_{r'})_{r' \in \Theta_r}$ , where  $\Theta_r$  is a set of block indexes including r.

### 3. LMS DETECTOR

# 3.1 The RPMA case

For perfectly known channel coefficients, the problem addressed in [2] consists of detecting all bits of all users by considering simultaneously all transmitted blocks. Now, it is assumed in this paper that the channel coefficients are unknown. One then resorts to an adaptive algorithm, and the detection is performed block after block, i.e. the objective is to minimize for each block r the MSE

$$f(\mathbf{H}) \triangleq E\left[\|\mathbf{b}_r - \mathbf{H}\mathbf{y}_r\|^2\right] \tag{3}$$

with respect to the  $(KN_b) \times (N_s K N_b L)$  matrix **H** (the matrix norm is defined by  $||A|| \triangleq (trace(AA^H))^{1/2})$ . Thus, one searches to adaptively derive at each block time r the optimal matrix, which minimizes this error. Denote **h** as the vector obtained by columnwise reshaping the matrix **H** in (3). Let R denote the transformation such that H = R(h). The function f in (3) can be expressed as a function of **h** as

$$f(\mathbf{h}) \triangleq E[\Phi(\mathbf{h}; \mathbf{b}_r)]$$

with

$$\Phi(\mathbf{h}; \mathbf{b}) = \|\mathbf{b} - R(\mathbf{h})\mathbf{y}_r\|^2$$

The LMS algorithm is then performed on vector  $\mathbf{h}$ . The initialization is defined by  $\mathbf{h}_0 = 0$ . For the learning sequence, the bit vector  $\mathbf{b}_r$  is known by the receiver, and the up-date equation is given by

$$\mathbf{h}_{r+1} = \mathbf{h}_r - \mu \nabla \Phi_{\mathbf{h}}(\mathbf{h}_r; \mathbf{b}_r) \tag{4}$$

where  $\mu$  is the step-size of the algorithm, and  $\nabla \Phi_{\mathbf{x}}(\mathbf{x}; \mathbf{b})$  denotes the gradient of  $\Phi$  at point  $(\mathbf{x}; \mathbf{b})$  with respect to  $\mathbf{x}$ . It can be shown that

$$\nabla \Phi_{\mathbf{h}}(\mathbf{h}_r; \mathbf{b}_r) = 2(\mathbf{Y}_r \mathbf{h}_r^* - R(\mathbf{y}_r \mathbf{b}_r^H)),$$

where  $\mathbf{Y}_r \triangleq I_{KN_b} \otimes (\mathbf{y}_r \mathbf{y}_r^H)$ . For the decision-directed sequence,  $\mathbf{b}_r$  is estimated: equation (4) then becomes

$$\widehat{\mathbf{b}}(r) = sign(R(\mathbf{h}_r)\mathbf{y_r}) 
\mathbf{h}_{r+1} = \mathbf{h}_r - \mu \nabla \Phi_{\mathbf{h}}(\mathbf{h}_r; \widehat{\mathbf{b}}_r)$$

For appropriate  $\mu$ , and after a long enough learning sequence, the algorithm converges, i.e. the vector  $\mathbf{h}_r$  is close to the optimal vector at time r. This convergence is assured if the function f is strictly convex. Now, one can show that this assumption is equivalent to have  $\Omega_r \triangleq \sigma^2 \Lambda_r + \mathbf{W}_r \mathbf{W}_r^H$  positive definite, where  $\mathbf{W}_r \triangleq \mathbf{\Lambda}_r \mathbf{\Pi}$ . However, due the particular structure of  $\Lambda_r$ ,  $\Omega_r$  is only positive semi-definite, which may cause poor convergence, or convergence to a non-optimal solution. A means to overcome this inconvenience consists of slightly modifying  $\Omega_r$  by reinforcing its diagonal, i.e. by changing  $\Omega_r$  into  $\Omega_r + \varepsilon I_{N_s K N_b L}$  with small  $\varepsilon$ . Obviously, this cannot be achieved on the theoretical matrix  $\Omega_r$ , which is not available for the receiver, but can be obtained from the received data  $\mathbf{y}_r$  by adding a zero-mean Gaussian vector with covariance matrix  $\varepsilon I_{N_sKN_bL}$ . Note that this is not equivalent to add an AWGN with variance  $\varepsilon$  to the continuous received signal y(t), since, in that case,  $\Omega_r$  would simply be transformed into  $(\sigma^2 + \varepsilon)\Lambda_r + \mathbf{W}_r\mathbf{W}_r^H$ , which would also be semi-definite. Thus, this slight correction must not be regarded as an increase of the noise level, but as a modification of the noise structure.

#### 3.2 The DS-CDMA case

Consider now the DS-CDMA system. For such a system, the bits  $(b_k(j))_{j\in\mathbb{Z}}$  of the kth user are modulated by a signature waveform (code)  $s_k(t)$ , which is assumed to be zero outside the interval [0,T]. The signal transmitted by user k is then

$$\sum_{j\in\mathbb{Z}}b_k(j)s_k(t-jT),$$

and the received signal, when B bits are transmitted per user, can be expressed as:

$$\widetilde{y}(t) = \sum_{l=1}^{L} \sum_{k=1}^{K} \sum_{j=1}^{B} c_{k,l}(t) b_k(j) s_k(t - jT - au_{k,l}) + n(t)$$

Given codes  $(s_k(t))_k$  and delays  $(\tau_{k,l})_{k,l}$ ,  $\widetilde{y}(t)$  can be sampled in a similar way as for y(t) in section 2.2, yielding vector  $\widetilde{\mathbf{y}}_r$  such that

$$\widetilde{\mathbf{y}}_r = \widetilde{\mathbf{\Lambda}}_r \widetilde{\mathbf{U}} \mathbf{b}_r + \widetilde{\mathbf{n}}_r$$

where  $\widetilde{\Lambda}_r$  is the matrix formed by the correlations

$$\int c_{k,l}(t)c_{k',l'}^*(t)s_k(t-jT-\tau_{k,l})s_{k'}(t-j'T-\tau_{k',l'})dt,$$

 $\widetilde{\mathbf{U}} = \mathbf{1}_L \otimes \mathbf{I}_{KN_b}$ , and  $\widetilde{\mathbf{n}}$  is a zero-mean Gaussian vector with covariance matrix  $\sigma^2 \widetilde{\mathbf{\Lambda}}_r$ . The structure of  $\widetilde{\mathbf{y}}_r$  is then similar to that of  $\mathbf{y}_r$  in (2). Thus, an equivalent LMS algorithm can be performed (note that the convexity problem mentioned in the RPMA case still occurs in that case, and can be solved in a similar way).

#### 4. SIMULATION RESULTS

This section presents some simulation results. The objective here is: i) to observe the convergence of the channel coefficient vectors  $\mathbf{h}_r$ , and: ii) to evaluate the performance of the RPMA system, by computing the bit error rate (BER), and compare the results to those of the DS-CDMA. For these simulations, one defines the users' characteristics (i.e., parameters K,  $N_b$ ,  $N_s$ , and the permutation matrices), which fixes the matrix  $\Pi$ . The channels are independent Rayleigh-fading time-varying channels generated according to the Jake's model, which enables to compute the matrices  $\Lambda_r$ . An important parameter for the simulations is the signal-to-noise ratio (SNR), which fixes the variance  $\sigma^2$  to be used for the additive Gaussian noise. More precisely, for non-random time-varying channel coefficients, the signal is non-stationary, and one must resort to a mean SNR (denoted  $\overline{SNR}$ ), defined by

$$\overline{SNR} = \frac{\sum_{i} E\left[\left|\mathbf{x}_{r}(i)\right|^{2}\right]}{\sum_{i} E\left[\left|\mathbf{n}_{r}(i)\right|^{2}\right]},$$
(5)

where  $\mathbf{x}_r \triangleq \mathbf{y}_r - \mathbf{n}_r$  (i.e.,  $\mathbf{x}_r$  is the unnoisy received data). Now,  $\sum_i E\left[\left|\mathbf{x}_r(i)\right|^2\right] = trace(Cov(\mathbf{x}_r)) = trace(E[\mathbf{\Lambda}_r \mathbf{\Pi} \mathbf{b}_r \mathbf{b}_r^T \mathbf{\Pi}^T \mathbf{\Lambda}_r^H]) = trace(\mathbf{W}_r \mathbf{W}_r^H)$ , and  $\sum_i E\left[\left|\mathbf{n}_r(i)\right|^2\right] = trace(Cov(\mathbf{n}_r)) = \sigma^2 trace(\mathbf{\Lambda}_r)$ . Finally, the variance  $\sigma^2$  is obtained by:

$$\sigma^{2} = \frac{trace(\mathbf{W}_{r}\mathbf{W}_{r}^{H})}{trace(\mathbf{\Lambda}_{r})} 10^{-\overline{SNR}/10}.$$
 (6)

In fig. 2 is plotted the evolution of the coefficient vectors  $\mathbf{h}_r$  obtained first with non-varying channels. The simulated channels are Rural-Area channels. The number of users is K=2, and the number of paths per user is L=2. The spreading factor is  $N_s = 8$ , and the number of bits per block is  $N_b = 2$ . The mean powers are -0.97dB and -4.58dB for the two paths of user 1, and -0.61dB and -4.76dB for the two paths of user 2 (the powers of both users are then similar); moreover, the delays are:  $\tau_{1,1} = 4.7e - 6s$ ,  $\tau_{1,2} = 13.3e - 6s$ ,  $\tau_{2,1} = 1.1e - 6s$ , and  $\tau_{2,2} = 12.4e - 6s$ . The other parameters are:  $T_s = 4.88e - 7s$ ,  $N_s = 8$ ,  $N_b = 2$ , and  $\overline{SNR} = 10$ . For clarity, only 10 out of 256 coefficients are presented. Moreover, the optimal coefficients are also plotted, for comparison. It can be shown that for such channels, the optimal matrix  $\mathbf{H}_r^{\varepsilon}$ of these coefficients (including the correction of the covariance matrix diagonal mentioned in section 3.1) is given by

$$\mathbf{H}_r^{\varepsilon} = \mathbf{\Pi}^T \mathbf{\Lambda}_r^H \left( \sigma^2 \mathbf{\Lambda}_r + \varepsilon \mathbf{I}_{N_s K N_b L} + \mathbf{\Lambda}_r \mathbf{\Pi} \mathbf{\Pi}^T \mathbf{\Lambda}_r^H \right)^{-1} \tag{7}$$

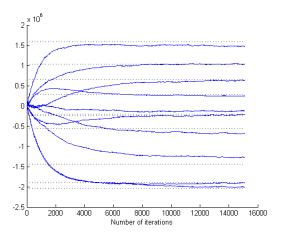


Figure 2: Evolution of the coefficients for constant channels for RPMA. solid: estimated - dotted: optimal.

where  $\Lambda_r$  remains constant during the simulation (since the channels are static in that case). One can see on these figures that the LMS algorithm converged towards the optimal after approximately 2.9ms, which corresponds to 6e3 iterations (or blocks).

Fig. 3 presents equivalent curves, obtained with the same channels as previously, but with a vehicle speed equal to 30km/h, which leads to a coherence time equal to 7.2ms. 5e3 blocks have been used for the learning sequence, and 15e3 for the decision-directed sequence. The optimal coefficients are also plotted: these coefficients are obtained from (7), where the matrix  $\Lambda_r$  is now varying at each iteration, according to the channel coefficient variations. One can see that the algorithm converges after about 10<sup>3</sup> blocks, and succeeds in tracking the time-variations of the optimal coefficients, as long as these variations are not too fast (for the first 15e3 blocks). Now, when these variations accelerate (from the 15.e3th block), this tracking is more difficult. Fig. 4 presents the results obtained for the same channels with the equivalent LMS algorithm derived for the DS-CDMA system. This latter uses Gold codes, with 7 chips, which corresponds to the number of "chips"  $(N_s = 8)$  used for the RPMA system. The learning sequence uses 10<sup>4</sup> bits, which is equivalent to the  $5.10^3$  blocks of  $N_b = 2$  bits used in RPMA. The variance is there obtained from the mean SNR as in (6) by replacing  $\mathbf{W}_r$  and  $\mathbf{\Lambda}_r$  by  $\mathbf{W}_r$  and  $\mathbf{\Lambda}_r$ , respectively (with  $\mathbf{W}_r \triangleq \mathbf{\Lambda}_r \mathbf{U}$ ). Also, the optimal coefficient matrix is obtained from (7) by replacing  $\Pi$  and  $\Lambda_r$  by  $\mathbf{1}_L \otimes \mathbf{I}_{KN_h}$  and  $\Lambda_r$ . The conclusion is quite similar to that of the previous figure.

Fig. 5 presents the BER (computed on the decision-directed sequence) as a function of  $\overline{SNR}$  for the previous channels, for both the RPMA and the DS-CDMA systems. For the former, the figure shows the results of the two bits of each block (since  $N_b = 2$ ). Clearly, the RPMA system gives better performance than the DS-CDMA system. This can be explained by the fact that the RPMA introduces some time-diversity in the bit sequence due to the permutation-based interleaving. This time-diversity provides more robustness vis--vis the channel fades (in the time domain).

#### 5. CONCLUSION

In this paper is developed an adaptive algorithm suited for a spread-spectrum system based on random permutations, named RPMA, in the case of unknown selective channels. An equivalent algorithm is also studied for the DS-CDMA system. The simulations have shown that the convergence behaviors of both algorithms are similar under equivalent transmission characteristics. In particular, the algorithms are able to track the optimal coefficients, as long as the timevariations are not too fast. However, in any case, the RPMA performs better. Now, theoretically (i.e. with known channels), the RPMA performs better than to the DS-CDMA system especially in presence of fast fading [2], and when the time-spreading (i.e. the parameter  $N_b$ ) is high. But, in this context (which implies fast optimal coefficient variations and larger vector dimensions), the tracking is difficult, and the behavior of the LMS algorithm remains far from that of the optimal LMMSE detector. To overcome this problem, greater values of the algorithm step-size have been tested, but the tracking becomes quite "noisy" and divergence may even occur. Also, a normalized LMS algorithm [3] has been considered, without satisfying results. A possible way to improve the tracking would consist of reducing the complexity of the algorithm, i.e. of considering coefficient vectors with smaller dimensions. This could be achieved by adapting the proposed algorithm for parallel detection, according to the principle of Parallel-Interference-Cancelation (PIC) techniques. Moreover, it is still assumed here that the channel delays are known to the receiver. If this assumption does not hold, a possible approach would consist of considering the delays as multiple of the "chip" period (which is roughly the inverse of the signal bandwidth), up to a maximum given by the maximum delay spread. These two issues are currently under study.

#### REFERENCES

- [1] M. Coulon and D. Roviras, "An LMMSE Detector for a Spread-Spectrum System based on Random Permutations over Frequency-Selective Fading Channels", Proc. of ICASSP'2005, Philadelphia, March 2005.
- [2] M. Coulon and D. Roviras, "Joint Detection for a Random Permutations-Based Spread-Spectrum System based over Frequency-Selective Time-Varying Channels", Proc. of GLOBECOM'2005, Saint-Louis, USA, 28 Nov. - 2Dec. 2005.
- [3] S. Haykin, *Adaptive Filter Theory*, third edition, Prentice Hall, Upper Saddle River, NJ, 1996.
- [4] B. Lacaze and D. Roviras, "Effect of random permutations applied to random sequences and related applications", Signal Processing, 82 (2002) pp. 821-831.
- [5] G. Leus and M. Moonen, "MUI-free receiver for a synchronous DS-CDMA system based on block spreading in the presence of frequency-selective fading", IEEE Trans. on Signal Processing, Vol. 48, pp. 3175-3188, Nov. 2000.
- [6] S. Verdu, *Multiuser Detection*, Cambridge University Press, Cambridge, 1998.
- [7] S. Zhou, G.B. Giannakis and C. Le Martret, "Chip-Interleaved Block-Spread Code Division Multiple Access", IEEE Trans. on Communications, Vol. 50, No. 2, February 2002.

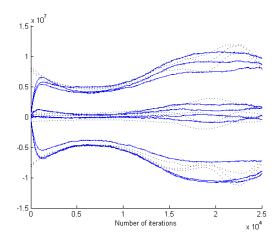


Figure 3: Evolution of the coefficients for time-varying channels for RPMA. solid: estimated - dotted: optimal.

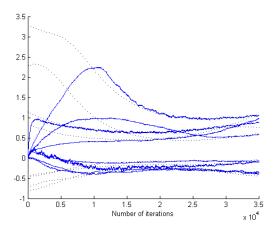


Figure 4: Evolution of the coefficients for time-varying channels for DS-CDMA. solid: estimated - dotted: optimal.

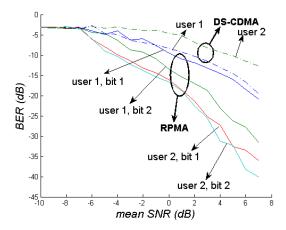


Figure 5: BERs for RPMA (solid) and DS-CDMA (dashdot).