14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

OPTIMUM PERMUTATION AND RANK DETECTORS UNDER K-DISTRIBUTED CLUTTER IN RADAR APPLICATIONS

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ABSTRACT

In this paper, we realize a comparative performance analysis of nonparametric (permutation and rank) detectors against the parametric ones. The optimum permutation and rank tests are proposed for radar detection under K-distributed clutter, having considered an ideal case of independent and identically distributed (IID) clutter samples, and another more realistic case of spherically invariant random process (SIRP) clutter model. The detector performance analysis was realized for nonfluctuating and Swerling II target models by Monte-Carlo simulations, and the results are shown in curves of detection probability versus signal-to-clutter ratio.

1. INTRODUCTION AND PRELIMINARIES

Permutation Tests (PT) were proposed firstly by R.A. Fisher in 1935 [1], under the name of randomization test [2]. Other equivalent names used in the statistical literature are Fisher's exact tests, re-randomization tests, conditional tests, and permutation tests. The rank tests [3-5] can be considered as a particular case of the family of permutation tests [6-7], because the theory of permutation tests applied to a sample vector is also applied to its rank vector, as is shown in [7]. Therefore, the optimum permutation test [6] is more powerful than the optimum rank test, although the former has much more computational complexity than the latter. In the past, some rank tests have been applied to detection [8]; nevertheless, as long as the authors know, permutation tests have seldom been applied to radar detection [9].

A distribution-free statistic [3-7] over a family of distributions is a statistic whose distribution is independent of the particular distribution considered in such family. Note that a parametric family of distributions is defined by a finite number of parameters. On the contrary, a nonparametric family of distributions cannot be defined by a finite number of parameters (e. g. the family of all continuous and symmetric distributions). The importance of distribution-free statistics is apparent for constant false-alarm rate (CFAR) detectors, because if the test statistic is distribution-free under hypothesis H_0 (target absent), then the false-alarm probability is constant (e.g. $P_{fa}=10^{-6}$) for any distribution of H_0. Once the distribution-free statistic is defined, the following problem is to find an optimum test in the Neyman-Pearson sense.

We suppose that the signal comes from a two-dimensional pulsed-radar system. In order to test hypothesis H_0 (target absent) against hypothesis H_1 (target present) for each azimuth in a specific range cell, we take M reference samples corresponding to the range cells surrounding the cell under test. Also, we consider a block of N pulses for each azimuth (corresponding to the number of pulses per antenna beamwidth), then for each i^{th}-pulse we have the row vector of samples

$$\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{iM}, x_i), \quad i = 1, 2, \dots, N \qquad (1)$$

where the last component x_i of vector $\mathbf{x}_i$ is the sample of the range cell under test. Finally, we consider a non-coherent detection approach, i.e. we suppose that data are samples of the linear envelope (phase is discarded).

1.1 IID Clutter Model

In a first analysis, the two hypotheses H_0 (target absent) and H_1 (target present) are defined in terms of distribution functions, as they were defined in [8], which are as follows for nonfluctuating and Swerling II target models

$$H_0: \; F_{\mathbf{X}}(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N \,|H_0) = \prod_{i=1}^{N} F_{0i}(x_i) \prod_{j=1}^{M} F_{0i}(x_{ij}) \qquad (2)$$

$$H_1: \; F_{\mathbf{X}}(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N \,|H_1) = \prod_{i=1}^{N} F_{1i}(x_i) \prod_{j=1}^{M} F_{0i}(x_{ij}) \qquad (3)$$

where $F_{0i}(x)$ and $F_{1i}(x)$ are the cumulative distribution functions of the sample x_i under H_0 and H_1, respectively.

Note that under H_0, the components of the random vector $\mathbf{X}_i = (X_{i1}, X_{i2}, \dots, X_{iM}, X_i)$ are independent and identically distributed (IID) for each pulse i (i=1, 2, ..., N), i.e. H_0 of (2) is IID by blocks inside each random vector $\mathbf{X}_i$, $i = 1, 2, \dots, N$; then, the cumulative distribution function $F_{\mathbf{X}}(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N \,|H_0)$ satisfies the property of invariance under the permutation of components in each vector $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{iM}, x_i)$, $i = 1, 2, \dots, N$ and we can define distribution-free statistics inside each block of M+1 components, corresponding to vector $\mathbf{x}_i$. Under H_1, only the reference samples $X_{i1}, X_{i2}, \dots, X_{iM}$ are IID. Also, vectors $\mathbf{X}_i$, $i = 1, 2, \dots, N$, are independent each other.

1.2 SIRP Clutter Model

The Spherically Invariant Random Process (SIRP) clutter model [10-12] for the range cell under test and a block of N pulses can be represented by the compound model: $\tilde{\mathbf{X}} = \tilde{\mathbf{W}} \cdot Y$, where $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{W}}$ are N-dimensional complex random column vectors, and Y is a real random scalar (independent of $\tilde{\mathbf{W}}$). Also, $\tilde{\mathbf{W}} = (\tilde{W}_1, \tilde{W}_2, ..., \tilde{W}_N)^{\mathrm{T}}$ is a complex Gaussian random column vector, and the components $\tilde{W}_i$, $i = 1, 2, ..., N$, are mutually independent if the radar system uses frequency agility. If target is present, we have $\tilde{X} = \tilde{W} \cdot Y + B \cdot \exp(j\Phi)$, where $B \cdot \exp(j\Phi)$ is the target return: Φ is uniformly distributed in $[0, 2\pi)$, B is the amplitude of the return signal (B is a constant for nonfluctuating target model and B is Rayleigh distributed for Swerling II target model). Note that we are referring to the non-coherent detection, and if there is only clutter then $X = W \cdot Y$, where W is a Rayleigh random variable and Y is a positive real random scalar with a general probability density function.

Under H_0, if $X_i = W_i \cdot Y,\ \ X_{ij} = W_{ij} \cdot Y,\ \ j = 1, 2, ..., M$ and $i = 1, 2, ..., N$, then we can write $\mathbf{X}_i = \mathbf{W}_i \cdot Y$, being $\mathbf{W}_i = (W_{i1}, W_{i2}, ..., W_{iM}, W_i)$, $i = 1, 2, ..., N$. If the probability density function of the random variable Y is denoted as $f_Y(\cdot)$, the two hypotheses are expressed by

$$H_0 : F_{\mathbf{X}}(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N | H_0) = \int_0^\infty \prod_{i=1}^{N} F_{0i}(x_i | y) \prod_{j=1}^{M} F_{0i}(x_{ij} | y) f_Y(y) dy \quad (4)$$

$$H_1 : F_{\mathbf{X}}(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N | H_1) = \int_0^\infty \prod_{i=1}^{N} F_{1i}(x_i | y) \prod_{j=1}^{M} F_{0i}(x_{ij} | y) f_Y(y) dy \quad (5)$$

Although H_0 in (4) is not IID, $F_{\mathbf{X}}(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N | H_0)$ accomplishes the property of invariance under component permutations in each vector $\mathbf{x}_i = (x_{i1}, x_{i2}, ..., x_{iM}, x_i)$, $i = 1, 2, ..., N$, and we can define distribution-free statistics inside each block of M+1 components, corresponding to vector $\mathbf{X}_i$, $i = 1, 2, ..., N$, in a similar way as it was done in the IID case.

2. PERMUTATION AND RANK DETECTORS

The application of the Neyman-Pearson lemma to hypotheses (2) and (3), considering the log-likelihood ratio test, leads us to the following optimum parametric detector

$$T(\mathbf{x}) = \sum_{i=1}^{N} a_i(x_i) \underset{H_0}{\overset{H_1}{\gtrless}} T_0; \ \ a_i(x_i) = \log\left(\frac{f_{1i}(x_i)}{f_{0i}(x_i)} \right) \quad (6)$$

where T_0 is the detector threshold, $f_{1i}(x)$ and $f_{0i}(x)$ are the pdf's of the sample x_i under H_1 and H_0, respectively. Column vector $\mathbf{x} = (x_1, x_2, ..., x_N)^{\mathrm{T}}$ is composed of the N samples under test. Note that the reference samples x_{ij}, i=1, 2, ..., N; j=1, 2, ..., M, do not appear in the test statistic (6).

Also, we define the following permutation statistic [9]

$$T_{\mathbf{k}}(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N) = \log\left(\prod_{i=1}^{N} \frac{f_{1i}(x_{ik_i})}{f_{0i}(x_{ik_i})} \right) = \sum_{i=1}^{N} a_i(x_{ik_i}) \quad (7)$$

where $a_i(x)$ is defined in (6), $\mathbf{k} = (k_1, k_2, ..., k_N)$, k_i=1,2, ..., M, M+1, i=1,2, ..., N, and $T(\mathbf{x})$ is included in (7).

For a block of N pulses, we have $(M+1)^N$ possible values of $T_{\mathbf{k}}(\cdot)$ given by (7), which are equally likely under hypothesis H_0 defined in (2). Now, we order $T_{\mathbf{k}}(\cdot)$ from its smallest value to its greatest value, as follows

$$\begin{aligned} &\inf_{\mathbf{k}} \left(T_{\mathbf{k}}(\cdot) \right) = \left(T_{\mathbf{k}}(\cdot) \right)_{(M+1)^N} < \left(T_{\mathbf{k}}(\cdot) \right)_{(M+1)^N - 1} < \dots \\ &\dots < \left(T_{\mathbf{k}}(\cdot) \right)_K < \dots < \left(T_{\mathbf{k}}(\cdot) \right)_1 = \sup_{\mathbf{k}} \left(T_{\mathbf{k}}(\cdot) \right) \end{aligned} \quad (8)$$

Now, consider the threshold T_0 in (6) as given by

$$T_0 = \left(T_{\mathbf{k}}(\cdot) \right)_K \quad (9)$$

where K is the number of $T_{\mathbf{k}}(\cdot)$ greater than or equal to T_0, and it is easily obtained from (8) for low K-values. So the permutation detector is realized by

$$T(\mathbf{x}) \underset{H_0}{\overset{H_1}{\gtrless}} \left(T_{\mathbf{k}}(\cdot) \right)_K \quad (10)$$

where $T(\mathbf{x})$ is the test statistic of the samples under test, given by (6), and also included in (7). We have designed and implemented a new permutation test algorithm for computing (10) that can be implemented in workstations or in personal computers for any K-value and real-time applications.

The false-alarm probability (P_{fa}) of the permutation test (10) is given by

$$P_{fa} = \Pr\left\{ T(\mathbf{X}) \geq \left(T_{\mathbf{k}}(\cdot) \right)_K | H_0 \right\} = \frac{K}{(M+1)^N} \quad (11)$$

The detection probability (P_d) of the permutation test (10) is given by

$$P_d = \Pr\left\{ T(\mathbf{X}) \geq \left(T_{\mathbf{k}}(\cdot) \right)_K | H_1 \right\} = \int u\left[T(\mathbf{x}) - \left(T_{\mathbf{k}}(\cdot) \right)_K \right] f_{\mathbf{X}}\left(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N | H_1 \right) d\mathbf{x}_1 d\mathbf{x}_2 ... d\mathbf{x}_N \quad (12)$$

where $u[\cdot]$ is the unit-step function.

Note that the optimum statistic for the parametric detector is optimum also for the permutation detector, where we have considered "optimum" in the Neyman-Pearson sense. So, under K-distributed clutter, from [9] the clipping statistic is optimum for impulsive clutter (ν=0.5) and the linear (or quadratic) is optimum for Rayleigh clutter (ν=∞), where ν is the clutter shape parameter. In [9], results of the comparison between parametric tests and permutation tests were shown and discussed in some details.

Now, we summarize some theoretical results about the application of rank detectors to radar detection, more details are in [8]. We define the rank r_i of the sample under test x_i in the sample vector $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{iM}, x_i)$ as follows

$$r_i = \sum_{j=1}^{M} u(x_i - x_{ij}), \quad 0 \le r_i \le M, \quad i = 1, 2, \dots, N \tag{13}$$

where $u(\cdot)$ is the unit-step function.

Under hypothesis H_0, the rank probability is uniformly distributed, then the rank (or any rank statistic) is distribution-free (whenever IID conditions be satisfied) and any test based on ranks is nonparametric CFAR. Under hypothesis H_1, the rank probability is given by [8]

$$P\left(R_i = r_i \middle| H_1\right) = \binom{M}{r_i} \int_0^\infty \left[F_{oi}(x)\right]^{r_i} \left[1 - F_{oi}(x)\right]^{M - r_i} dF_{1i}(x)$$
$$r_i = 0, 1, 2, \dots, M \qquad (i = 1, 2, \dots, N) \tag{14}$$

where $F_{0i}(x)$ and $F_{1i}(x)$ are the distribution functions of the sample X_i under H_0 and H_1, respectively.

Considering $\mathbf{r} = (r_1, r_2, \dots, r_N)^{\mathrm{T}}$, the hypotheses in terms of rank probabilities are

$$H_0: \; P\left(\mathbf{R} = \mathbf{r} \middle| H_0\right) = \prod_{i=1}^{N} P\left(R_i = r_i \middle| H_0\right) = \left(\frac{1}{M+1}\right)^N \tag{15}$$

$$H_1: \; P\left(\mathbf{R} = \mathbf{r} \middle| H_1\right) = \prod_{i=1}^{N} P\left(R_i = r_i \middle| H_1\right) \tag{16}$$

where $P\left(R_i = r_i \middle| H_1\right)$ is giving by (14).

From hypotheses (15) and (16), we have the following optimum rank test

$$T(\mathbf{r}) = \sum_{i=1}^{N} a_i(r_i) \underset{H_0}{\overset{H_1}{\gtrless}} T_0;$$
$$a_i(r_i) = \log\left(\frac{P\left(R_i = r_i \middle| H_1\right)}{P\left(R_i = r_i \middle| H_0\right)}\right); \quad r_i = 0, 1, 2, \dots, M \tag{17}$$

where T_0 is the corresponding threshold.

Finally, the suboptimum detector structure for Gaussian noise is also suboptimum under the SIRP clutter considered in (4) and (5). The proof is as follows: for the same false-alarm probability (P_{fa}), both detectors satisfy (11) and the detection probability (P_d) given in (12) can be expressed by

$$P_d = \int_0^\infty \left(P_d(y)\right)_{Gauss} \cdot f_Y(y) dy \tag{18}$$

where $(P_d(y))_{\mathrm{Gauss}}$ is the detection probability (12) of the suboptimum detector for Gaussian noise with noise power y^2; as $(P_d(y))_{\mathrm{Gauss}}$ corresponds to the maximum values in the y-interval of interest, then P_d in (18) corresponds also to the maximum. This fact was confirmed by the simulation results, as we shall see in the next Section.

3. COMPUTER SIMULATION RESULTS

In this Section, we present results of detection probability (P_d) versus signal-to-clutter ratio (SCR) for optimum non-parametric (permutation and rank) detectors against optimum parametric detectors under K-distributed clutter and nonfluctuating (NF) and Swerling II (SWII) target models. The clutter shape parameter (ν) has been ν=0.5 for spiky clutter and $\nu=\infty$ for Rayleigh clutter. Also, optimum test statistics have been considered for each case. Some detector parameters are: the false-alarm probability $P_{fa}=10^{-3}$, 10^{-6} and 10^{-8}, the number of integrated pulses N = 8, 16 and 32, and the number of clutter reference samples M=7 and 15. Also, we have considered two clutter models: an ideal case of independent and identically distributed (IID) clutter samples, and another more realistic case of spherically invariant random process (SIRP) clutter model. Detector thresholds for each P_{fa} were obtained by computation of formulas or by Importance Sampling Techniques. The performance analysis was realized by Monte Carlo simulations on a personal computer.

Under IID case, Figures 1-3 show curves of P_d vs. SCR of optimal detectors: parametric, permutation and rank; the three cases in the same figure, in order to establish comparisons. In Figures 1 and 2, the parameters M=15 and $P_{fa}=10^{-6}$ are fixed. In Figure 3, N=16 and M=15 are fixed.

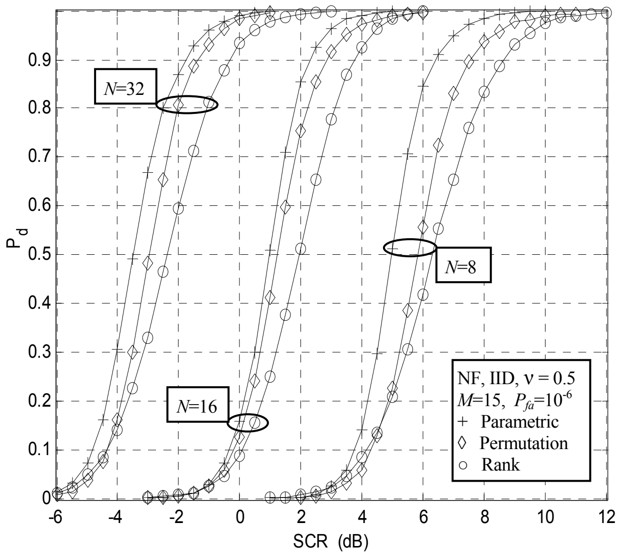

Figure 1. Detection probability (P_d) versus Signal-to-Clutter Ratio (SCR) for optimal detectors: parametric (+), permutation (◊) and rank (○), under nonfluctuating target model (NF) and spiky K-distributed IID clutter (ν=0.5). Parameters: M=15; N=8, 16 and 32; $P_{fa}=10^{-6}$.

In Figures 1 and 2, we can see the NF case with ν=0.5 (Figure 1) and $\nu=\infty$ (Figure 2). The best detector is the parametric one, followed by permutation and rank ones, respectively. If we consider P_d=0.8, permutation is about 1dB from parametric for N=8 (i. e. the loss $L\approx$1dB); for N=16 and 32, $L\approx$0.5dB. Also, if P_d=0.8, the rank has a loss $L\approx$2dB for N=8, and $L\approx$1dB for N=16 and 32. Also, from Figure 1 (spiky clutter), we can see that a duplication in N (from N=8 to 16, and from 16 to 32) implies a gain of 4dB approximately; however, from Figure 2 (Rayleigh clutter) a duplication in N implies a gain of 2dB approximately.

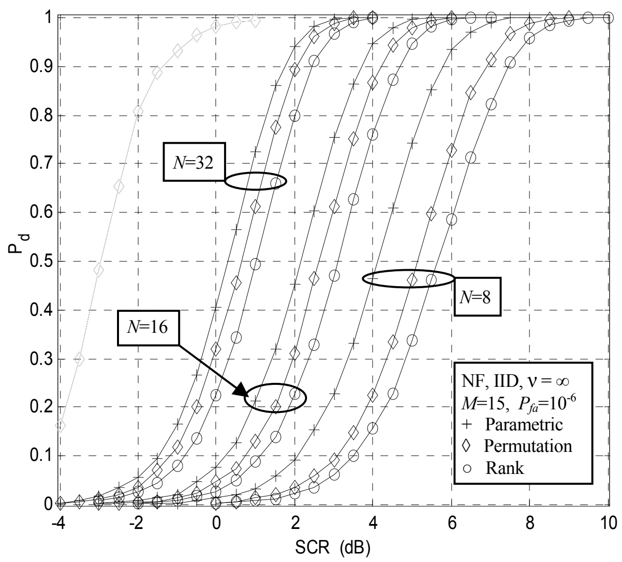

Figure 2. Detection probability (P_d) versus Signal-to-Clutter Ratio (SCR) for optimal detectors: parametric (+), permutation (◊) and rank (○), under nonfluctuating target model (NF) and IID Rayleigh clutter (ν= ∞). Parameters: M=15; N=8, 16 and 32; P_{fa}=10^{-6}.

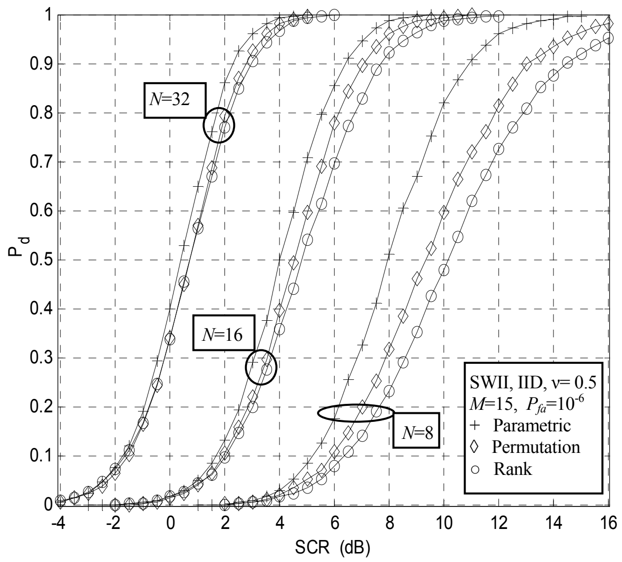

Figure 3. Detection probability (P_d) versus Signal-to-Clutter Ratio (SCR) for optimal detectors: parametric (+), permutation (◊) and rank (○), under Swerling II target model (SWII) and spiky K-distributed IID clutter (ν=0.5). Parameters: M=15; N=8, 16 and 32; P_{fa}=10^{-6}.

In Figure 3, we can see the SWII case. Similar tendencies are observed in this case when compared with the NF case (Figure 1), but the SCR required in SWII case for the same P_d is greater than that SCR required in the NF case. For $P_d\approx$0.8, the loss (L) of each detector with respect the optimum parametric one is approximately: under spiky clutter (ν=0.5) with permutation detector: $L\approx$2dB for N=8, and $L\approx$0.5dB for N=16 and 32, with rank detector: $L\approx$3dB for N=8, $L\approx$1dB for N=16, and $L\approx$0.5dB for N=32.

In Figure 4, we can see the NF case with ν=0.5, N=16, M=15, and P_{fa}=10^{-3}, 10^{-6} and 10^{-8}. The loss of each detector with respect to the optimum parametric detector is low sensitive with respect to P_{fa}. On the other hand, the detectability loss (L) of each detector when P_{fa} changes from 10^{-3} to 10^{-6} is about 5dB, and from P_{fa}=10^{-6} to 10^{-8}, L is about 2dB. For the SWII case, the detectability loss with respect to P_{fa} is similar to the NF case.

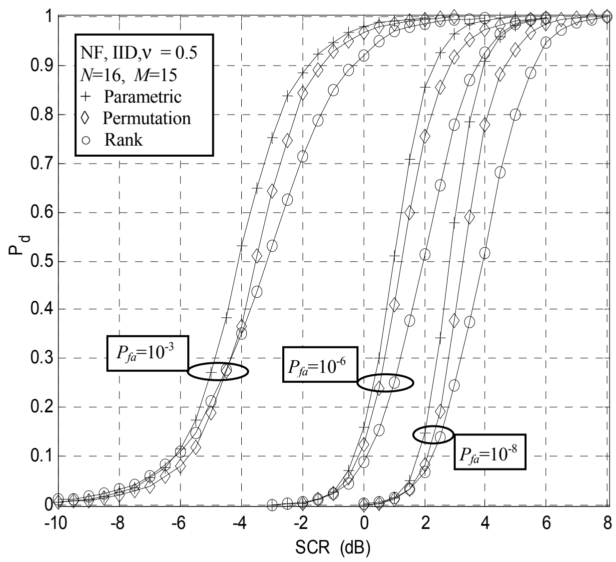

Figure 4. Detection probability (P_d) versus Signal-to-Clutter Ratio (SCR) for optimal detectors: parametric (+), permutation (◊) and rank (○), under nonfluctuating target model (NF) and spiky K-distributed IID clutter (ν=0.5). Parameters: N=16; M=15; P_{fa}=10^{-3}, 10^{-6} and 10^{-8}.

Under SIRP case, Figures 5 and 6 show results of P_d vs. SCR for optimal detectors: parametric, permutation and rank, under spiky and Rayleigh clutter, all in the same Figure for easy comparisons. We use the following symbols: continuous lines for denoting SIRP (K-distributed) clutter with ν=0.5, discontinuous (dotted) lines for denoting Rayleigh clutter; (lines with +) for the optimal parametric detector (ideal-CFAR detector [10,11]), (lines with ◊) for the optimal permutation detector, and (lines with o) for the optimal rank detector. As it is expected, under the same conditions, the best detector is the parametric, followed by the permutation and, then, the rank. Under Rayleigh clutter, P_d-curves of detectors increase rapidly from P_d=0.1 to P_d=0.95 as SCR increases (moreover, $P_d \approx P_{fa}$ as SCR < -20 dB); on the other hand, under SIRP (K-distributed) spiky clutter, P_d-curves of detectors increase slowly as SCR increases (now, $P_d \approx P_{fa}$ as SCR $< 20\log_{10}(P_{fa})$ dB). Also, it can be observed on both Figures that the separation between two curves under spiky clutter is approximately similar to the separation between the two corresponding curves under Rayleigh clutter.

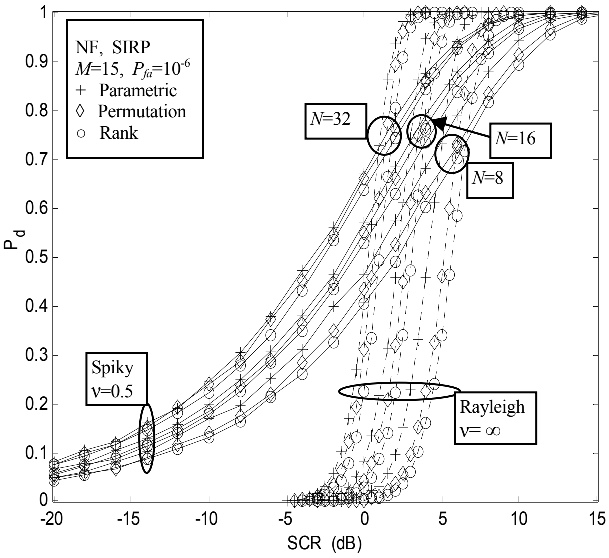

Figure 5. Detection probability (P_d) versus Signal-to-Clutter Ratio (SCR) for optimal detectors: parametric (+), permutation (◊) and rank (○), under nonfluctuating target model (NF), and SIRP K-distributed spiky clutter (ν=0.5, continuous lines) and Rayleigh clutter (ν= ∞, dotted lines). Parameters: M=15; N=8, 16 and 32; P_{fa}=10^{-6}.

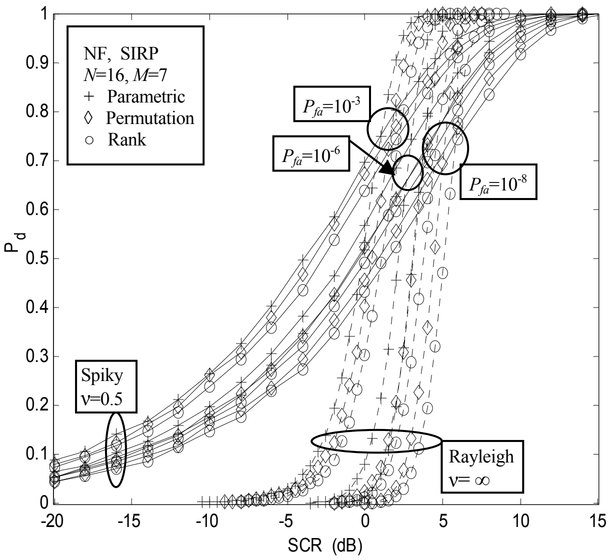

Figure 6. Detection probability (P_d) versus Signal-to-Clutter Ratio (SCR) for optimal detectors: parametric (+), permutation (◊) and rank (○), under nonfluctuating target model (NF), and SIRP K-distributed spiky clutter (ν=0.5, continuous lines) and Rayleigh clutter (ν= ∞, dotted lines). Parameters: N=16; M=7; P_{fa}=10^{-3}, 10^{-6} and 10^{-8}.

In Figure 5, we show curves of P_d vs. SCR of the optimal detectors with the parameters M=15 and P_{fa}=10^{-6}, and three N-values (8, 16 and 32) for easy comparisons. Note that the detectability gain of any detector from N =16 to N =32 is approximately 2dB (less than the 3dB of a coherent detector). In Figure 6, we show curves of P_d vs. SCR of the optimal detectors with the parameters N=16 and M=7, and three P_{fa}-values (10^{-3}, 10^{-6} and 10^{-8}). Note that the detectability loss (L) of any detector from P_{fa}=10^{-3} to P_{fa}=10^{-6} is $L \approx$3dB, and from P_{fa}=10^{-6} to P_{fa}=10^{-8} is $L \approx$1dB.

4. CONCLUSIONS

Nonparametric detectors maintain CFAR property under IID or SIRP clutter models, because the multivariate distribution of a sample block of these two clutter models are invariant under permutations of the clutter samples. Computer simulation results are shown in curves of P_d vs. SCR under K-distributed clutter and nonfluctuating and Swerling II target models, and the corresponding discussion was provided.

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