TWO CROSS COUPLED KALMAN FILTERS FOR JOINT ESTIMATION OF MC-DS-CDMA FADING CHANNELS AND THEIR CORRESPONDING AUTOREGRESSIVE PARAMETERS

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ABSTRACT

This paper deals with the estimation and equalization of time-varying Rayleigh fading channels for synchronous Multi-Carrier Direct-Sequence Code Division Multiple Access (MC-DS-CDMA) systems. The approach we propose operates in three steps. Firstly, decorrelating filtering is carried out along each carrier to eliminate the Multiple Access Interference (MAI). Secondly, the fading channel responses, modeled by autoregressive (AR) processes, are estimated by using Kalman filtering. Nevertheless, since the AR parameters are also unknown, one has to estimate both the channels and the corresponding AR parameters. Avoiding a non linear approach such as the Extended Kalman Filter (EKF), the method we present is based on two cross coupled Kalman filters. Thirdly, the channel estimates are fed into a frequency diversity Maximal Ratio Combining (MRC) rule for channel equalization and symbol detection.

The performances of the proposed approach are investigated under realistic Jakes' fading model with slow and fast fading scenarios.

1. INTRODUCTION

A great deal of interest has been paid to the combination of Direct-Sequence Code Division Multiple Access (DS-CDMA) with Multi-Carrier (MC) modulation for the last years [1]. Indeed, this approach has several advantages such as high bandwidth efficiency, fading resilience and interference suppression capability, which are three crucial issues for future broadband wireless communications. In this paper, we will focus our attention on the MC-DS-CDMA scheme proposed in [2] and that has been adopted as an option in the CDMA2000 third generation standard [3].

The conventional MC-DS-CDMA receiver is based on a correlator for each carrier followed by a Maximal Ratio Combiner (MRC) to counteract fading and narrow band interference [2]. However, this approach cannot eliminate the Multiple Access Interference (MAI). Another approach that avoids this drawback consists in designing a Minimum-Mean-Square-Error (MMSE) receiver. In [4], Miller et al. have investigated two structures: on the one hand, a Wiener filter is carried out along each carrier. On the other hand, a joint structure is considered and involves a single optimal filter, concatenating the filter weights dedicated to each carrier. In both structures, the approach requires the fading channel coefficients which are explicitly estimated using a Least Square solution. In [5], the authors have investigated MMSE detectors in the framework of MC-CDMA for rapidly fading Rayleigh channels:

- Firstly, they present the so-called "MMSE per carrier" detector which needs the explicit estimations of the channel coefficients obtained by using the Least Mean Square (LMS) or Recursive Least Square (RLS) for instance. However, as these adaptive methods do not take into account channel features such as the Doppler frequency, Kalofonos et al. [5] examine the relevance of Kalman filtering, by modeling the channels as first order autoregressive (AR) processes. Nevertheless, they do not investigate higher order AR models for fast fading channels.
- Secondly, the so-called "MMSE per user" detector is implemented by means of LMS or RLS algorithms, without any explicit channel estimation. In [6], we have carried out a comparative study between methods using Normalized LMS, Affine Projection Algorithm (APA) or RLS, for MC-DS-CDMA systems. Theses techniques provide significant results at static or slow fading, but this is no longer the case for fast fading channels.

There are other families of receivers combining for instance multi-user detection techniques and explicit channel estimation. Thus, in the field of Single-Carrier DS-CDMA systems, Wu et al. [7] compare the performances of Kalman filter based channel estimator combined with various multi-user detectors, such as the decorrelator detector, the decision-feedback detector, etc. According to the authors, the decorrelator is the most robust detector.

So, designing receivers often requires the explicit channel estimations. When dealing with Kalman based approach, the channels can be modeled by AR processes. However, choosing the AR model order may be questionable and depends on the nature of the fading. In addition, several authors express the AR parameters from the Jakes model [8], but this requires the knowledge of the Doppler frequency. An alternative approach consists in carrying out the joint estimations of the channel coefficients and their corresponding AR parameters. Some authors have already addressed this problem, but for digital transmission in single-user systems. Thus, Tsatsanis et al. [9] focus on the estimation and the equalization of time-varying frequency selective fading channels. The estimations of the channel coefficients are obtained by using Kalman filtering whereas the AR parameters are estimated by using a Higher Order Statistics block processing based method. In [10], the Kalman filter is cross coupled with a RLS estimator.

In this paper, we propose a receiver structure that combines decorrelation multiuser detection, channel estimation and equalization for synchronous MC-DS-CDMA systems, in time-varying Rayleigh fading channels. For this purpose, we propose to use two cross coupled Kalman filters for the joint estimation of the channel coefficients and their corresponding AR parameters.

The remainder of the paper is organized as follows: Section 2 presents a mathematical model and a description for synchronous MC-DS-CDMA system in time-varying Rayleigh fading channel. In section 3, the proposed receiver structure is presented. Kalman channel estimation is introduced in section 4. Second order AR parameter estimation is outlined in section 5. Simulation results are reported in section 6.

2. SYSTEM MODEL

In the following, let us consider the synchronous transmission in MC-DS-CDMA systems with K active users and M carriers. The transmitted signal at the m^{th} carrier can be expressed in the following manner:

$$s_m(t) = \mathrm{Re}\left[\sum_{n=-\infty}^{+\infty}\sum_{k=1}^{K}\sqrt{E_{b_k}}\, b_k(n) c_k(t-nT_b) e^{j2\pi f_m t}\right] \tag{1}$$

where E_{b_k} is the bit energy of the k^{th} user, $b_k(n)\in\{-1,1\}$ is the n^{th} data bit of the k^{th} user, T_b is the bit duration and f_m is the m^{th} carrier frequency. In addition, the spreading waveform of the k^{th} user is given by:

$$c_k(t) = \sum_{i=0}^{N-1} s_k(i)\psi(t-iT_c) \tag{2}$$

where T_c is the chip duration, $N = T_b/T_c$ is the processing gain, $s_k(i)\in\{\pm 1/\sqrt{N}\},\ i=0,1,...,N-1$ is the normalized spreading sequence and $\psi(t)$ is the chip pulse shape, assigned to 1 over the interval $[0,T_c]$ and 0 otherwise.

Firstly, the transmitted MC signal goes through a frequency selective fading channel. Providing the number M of carriers, carrier spacing and the bandwidth of the chip pulse shape $\psi(t)$ are suitably chosen [2], each carrier can be assumed to undergo independent frequency non-selective flat fading. Therefore, the system will have a frequency diversity gain equal to the number of carriers.

In addition, the transmitted signal at the m^{th} carrier is corrupted by an independent zero mean additive white Gaussian noise (AWGN) process $\eta_m(t)$. The noise processes $\{\eta_m(t)\}_{m=1,\cdots,M}$ are assumed to be independent with equal variance σ_η^2. Hence, the continuous time received signal at the m^{th} carrier in its complex analytic form is given by:

$$r_m(t) = \sum_{n=-\infty}^{+\infty}\sum_{k=1}^{K}\sqrt{E_{b_k}}\, b_k(n) c_k(t-nT_b) h_m(n) e^{j2\pi f_m t} + \eta_m(t) \tag{3}$$

where $\{h_m(n)\}_{m=1,\cdots,M}$ are mutually independent and identically distributed zero-mean complex Gaussian channels coefficients with Rayleigh distributed envelops. Their statistical properties are defined by the autocorrelation function $r_h(\tau)$ governed by the Doppler rate $f_d T_b$ according to the Jakes' model [8]:

$$r_h(\tau) = E[h_m(n)h_m^*(n-\tau)] = \sigma_h^2 J_o(2\pi f_d T_b \tau) \tag{4}$$

where $J_o(.)$ denotes the zero-order Bessel function of the first kind and σ_h^2 is the channel coefficient variance. The corresponding power spectrum density is given by [8]:

$$\Psi_h(f) = \begin{cases} \dfrac{\sigma_h^2}{\pi f_d\sqrt{1-(f/f_d)^2}}, & |f| \le f_d \\ 0, & else\, where \end{cases} \tag{5}$$

Given (5), the fading channel along the m^{th} carrier can be described with satisfactory accuracy [11] by a second order autoregressive process, AR(2), as follows:

$$h_m(n) = -a_1 h_m(n-1) - a_2 h_m(n-2) + u_m(n) \tag{6}$$

where $u_m(n)$ is a zero-mean complex white Gaussian noise driving process with variance σ_u^2, assumed to be equal over all carriers. From (4) and (6), it can be shown that [7]:

$$a_1 = -2r_d\cos\left(\frac{2\pi f_d T_b}{\sqrt{2}}\right),\quad a_2 = r_d^2 \text{ with } r_d \in [0.9, 0.999] \tag{7}$$

$$\sigma_u^2 = \frac{[(1+a_2)^2 - a_1^2](1-a_2)\sigma_h^2}{(1+a_2)} \tag{8}$$

Equations (7) and (8) can be used providing f_d is available. In the sequel, our purpose is to find an alternative solution by estimating both the channel coefficients $\{h_m(n)\}_{m=1,\cdots,M}$ and their corresponding AR parameters. This will make it possible to retrieve the desired data sequence of the first user $b_1(n)$, from the received signal.

3. PROPOSED RECEIVER STRUCTURE

The receiver we propose in this paper operates as follows: First, the demodulated signal over the m^{th} carrier $x_m(t) = r_m(t)e^{-j2\pi f_m t}$ is processed with a chip-matched filter, which consists of an integrator with duration T_c. The samples are then stored during a one bit interval, resulting in the following $N\times 1$ vector:

$$\mathbf{x}_m(n) = \sum_{k=1}^{K}\sqrt{E_{b_k}}\, b_k(n) h_m(n)\mathbf{s}_k + \boldsymbol{\eta}_m(n) \tag{9}$$

where $\mathbf{s}_k = [s_k(0)\ \ s_k(1)\ \ \cdots\ \ s_k(N-1)]^T$ is the normalized spreading vector of the k^{th} user and $\boldsymbol{\eta}_m(n)$ is a vector of AWGN samples.

The decorrelating filter [12] is designed to completely eliminate the MAI caused by other users. In a de-centralized receiver, this filter has the following form for user 1:

$$\mathbf{w}_1=\sum_{k=1}^{K}[\mathbf{R}^{-1}]_{1k}\mathbf{s}_k \tag{10}$$

where $\mathbf{R}=[\mathbf{s}_1 \ \ \mathbf{s}_2 \ \ \cdots \ \ \mathbf{s}_K]^T[\mathbf{s}_1 \ \ \mathbf{s}_2 \ \ \cdots \ \ \mathbf{s}_K]$ is the normalized cross correlation matrix of the spreading vectors and $[\mathbf{R}^{-1}]_{ij}$ denotes the $(i,j)^{th}$ element of the inverse of the matrix $\mathbf{R}$.

The received vector at the m^{th} carrier is processed by the decorrelating filter to obtain the following observation:

$$y_m(n)=\mathbf{w}_\mathbf{1}^T\mathbf{x}_\mathbf{m}(n)=\sqrt{E_{b_1}}\,b_1(n)h_m(n)+v_m(n) \tag{11}$$

where $v_m(n)$ is a zero-mean Gaussian noise with variance $\sigma_v^2=\sigma_\eta^2[\mathbf{R}^{-1}]_{11}$.

Finally, MRC results in the estimated data symbol of the desired user as follows:

$$\hat{b}_1(n)=\operatorname{sgn}\left(\operatorname{Re}\left(\sum_{m=1}^{M}\hat{h}_m^*(n)y_m(n)\right)\right) \tag{12}$$

where $\{\hat{h}_m(n)\}_{m=1,\cdots,M}$ are the estimated channel coefficients obtained by the Kalman channel predictor introduced in the next section.

4. KALMAN CHANNEL ESTIMATION

Given the system (6) and (11), the estimation of the channel coefficient $h_m(n)$ can be recursively obtained by means of a Kalman filter, providing the model parameters are known at the receiver.

To this end, let us define $\mathbf{h}(n)=[h(n) \ \ h(n-1)]^T$. Note that, for the sake of simplicity and clarity of presentation, we drop the carrier subscript. Then, equation (6) can be written in the following state space form:

$$\mathbf{h}(n+1)=\mathbf{\Phi}(n)\mathbf{h}(n)+\mathbf{g}u(n) \tag{13}$$

where:

$$\mathbf{\Phi}(n)=\begin{bmatrix}-a_1 & -a_2\\ 1 & 0\end{bmatrix} \quad \text{and} \quad \mathbf{g}=[1 \quad 0]^T \tag{14}$$

In addition, given (11) and (13), one has:

$$y(n)=\mathbf{b}^T(n)\mathbf{h}(n)+v(n) \tag{15}$$

where $\mathbf{b}(n)=[\sqrt{E_{b_1}}\,b_1(n) \quad 0]^T$ in the training mode and $\mathbf{b}(n)=[\sqrt{E_{b_1}}\,\hat{b}_1(n) \quad 0]^T$ in the decision directed mode.

Hence, equations (13) and (15) define the state space representation of the fading channel system (6) and (11). A standard complex Kalman filter can then be carried out to recursively compute the state vector $\mathbf{h}(n+1)$ as listed bellow:

Obtaining the innovation process:

$$\alpha(n)=y(n)-\mathbf{b}^T(n)\hat{\mathbf{h}}(n/n-1) \tag{16}$$

Defining its variance:

$$C(n)=E[\alpha(n)\alpha^*(n)]=\mathbf{b}^T(n)\mathbf{P}(n/n-1)\mathbf{b}(n)+\sigma_v^2 \tag{17}$$

Calculating the Kalman gain:

$$\mathbf{K}(n)=\mathbf{\Phi}(n)\mathbf{P}(n/n-1)\mathbf{b}(n)C^{-1}(n) \tag{18}$$

Obtaining the estimation of the state vector and the channel:

$$\hat{\mathbf{h}}(n+1/n)=\mathbf{\Phi}(n)\hat{\mathbf{h}}(n/n-1)+\mathbf{K}(n)\alpha(n) \tag{19}$$

$$\hat{h}(n+1/n)=\mathbf{g}^T\hat{\mathbf{h}}(n+1/n) \tag{20}$$

Updating the error covariance matrix:

$$\mathbf{P}(n/n)=\mathbf{P}(n/n-1)-\mathbf{P}(n/n-1)\mathbf{b}(n)C^{-1}(n)\mathbf{b}^T(n)\mathbf{P}(n/n-1) \tag{21}$$

$$\mathbf{P}(n+1/n)=\mathbf{\Phi}(n)\mathbf{P}(n/n)\mathbf{\Phi}^H(n)+\mathbf{g}\sigma_u^2(n)\mathbf{g}^T \tag{22}$$

This approach can be carried out providing the AR(2) parameters that are involved in the transition matrix $\mathbf{\Phi}$ and the driving noise process variance σ_u^2 are available. They will be estimated following the method given in the next section.

5. AR(2) PARAMETER ESTIMATION FROM THE ESTIMATED CHANNEL COEFFICIENTS

Equations (19) and (20) are firstly combined to express the estimated channel coefficient $\hat{h}(n+1/n)$ as a function of the AR(2) parameters:

$$\begin{aligned}\hat{h}(n+1/n)&=\mathbf{g}^T\mathbf{\Phi}(n)\hat{\mathbf{h}}(n/n-1)+\mathbf{g}^T\mathbf{K}(n)\alpha(n)\\ &=-\hat{\mathbf{h}}^T(n/n-1)\mathbf{a}(n)+\upsilon(n)\end{aligned} \tag{23}$$

where $\mathbf{a}(n)=[a_1(n) \ \ a_2(n)]^T$ is a vector of the AR(2) parameters and $\upsilon(n)=\mathbf{g}^T\mathbf{K}(n)\alpha(n)$ with variance equal to $\sigma_\upsilon^2=\mathbf{g}^T\,\mathbf{K}(n)C(n)\,\mathbf{K}^H(n)\mathbf{g}$.

If the AR(2) parameters are time invariant, one has:

$$\mathbf{a}(n+1)=\mathbf{a}(n) \tag{24}$$

Equations (23) and (24) hence define a state space representation for the estimation of the AR(2) parameters. A second Kalman filter is then used to recursively estimate $\mathbf{a}(n+1)$.

Besides, since the AR(2) parameters must be initially estimated, one cannot apply equation (8) to calculate the driving noise variance $\sigma_u^2(n)$. As an alternative, it can be estimated from the Kalman algorithm (16)-(22) as follows:

$$\begin{aligned}&\hat{\sigma}_u^2(n+1)=\frac{n-1}{n}\hat{\sigma}_u^2(n)+\\ &\frac{1}{n}\mathbf{f}[\mathbf{P}(n+1/n)-\mathbf{\Phi}(n)\mathbf{P}(n/n-1)\mathbf{\Phi}^H(n)+\mathbf{K}(n)|\alpha(n)|^2\mathbf{K}^H(n)]\mathbf{f}^T\end{aligned} \tag{25}$$

where $\mathbf{f}=[\mathbf{g}^T\mathbf{g}]^{-1}\mathbf{g}^T=[1 \quad 0]$.

6. SIMULATION RESULTS

6.1 Simulation protocols

First, we carry out a comparative study on channel estimation between the proposed Kalman based estimator and the LMS based one [5], for various fading rates scenarios.

We consider a system of K=10 multiple-access active users each using a gold code of length N=31 to spread his information. Moderate values for the number of carriers (i.e. M=1 and 3) are used as adopted in [3]. The channel coefficients $\{h_m(n)\}_{m=1,\cdots,M}$ are generated according to the modified Jakes' model [13] with 24 distinct oscillators where Walsh-Hadamard code words are used to insure that the channel coefficients along the M carriers are uncorrelated. The channel coefficients are normalized to have a unit variance, i.e. $\sigma_h^2 = 1$. The average Signal-to-Noise Ratio (SNR) per carrier is hence defined as $10\log_{10}(E_{b_1}/\sigma_\eta^2)$.

6.2 Results and comments

The effects of different fading rates on the Bit Error Rate (BER) performance is illustrated in Fig. 1 for M=1, 3. Comparable performances for the Kalman and LMS channel estimators can be noticed at low Doppler rates ($f_d T_b < 0.01$). However, for moderate and high Doppler rates ($f_d T_b > 0.01$), the Kalman estimator performs much better than the LMS estimator.

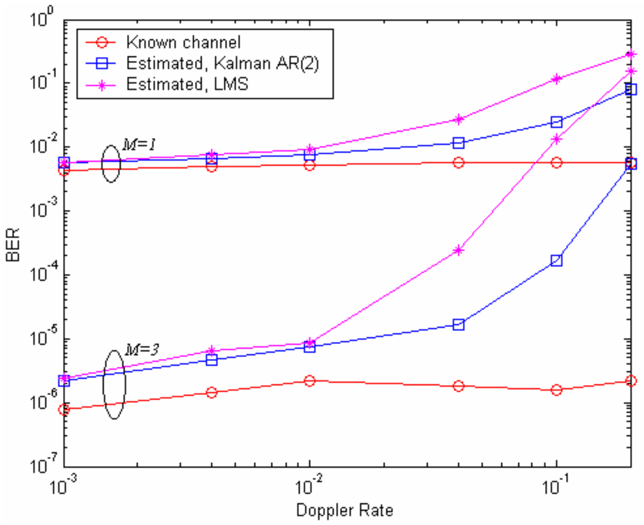

Fig.1: BER versus Doppler rate for the various channel estimators compared with the perfectly known channel case. SNR=15dB.

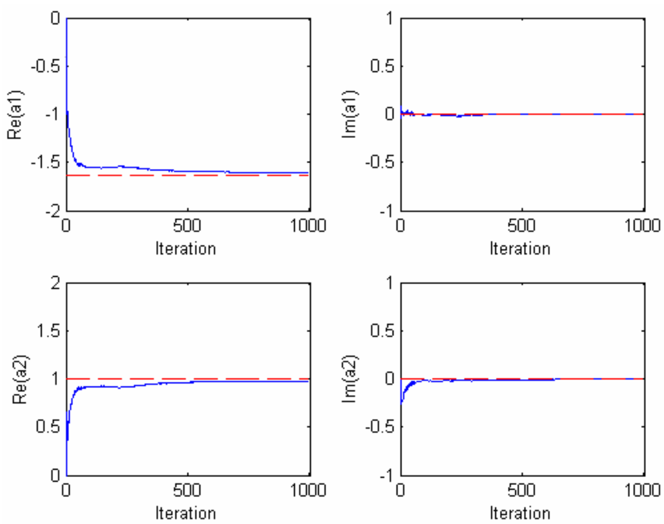

Fig.2: Real and imaginary parts of the estimated AR(2) parameters. Actual AR(2) parameter values (dashed lines) are $a_1 = -1.6279$ and $a_2 = 0.9960$. $f_d = 500Hz$, $1/T_b = 3.6\,Kbit/s$ and SNR=25dB.

In addition, increasing the Doppler rate increases the estimation error and consequently the BER. Furthermore, a significant frequency diversity gain is obtained by increasing the number of carriers from M=1 to M=3.
According to Fig.2, the estimated AR(2) parameters converge to the true values after approximately 500 symbols.

7. CONCLUSION

A receiver structure that combines decorrelation multiuser detection, channel estimation and equalization is proposed for synchronous MC-DS-CDMA systems, in time-varying Rayleigh fading channels. Two cross coupled Kalman filters are used for the joint estimation of channel coefficients and their corresponding AR parameters. According to our simulations, this approach provides significant results and can be used efficiently in high Doppler rate environments.

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