MODIFIED HERMITE FUNCTIONS FOR DESIGNING NEW OPTIMAL UWB PULSE-SHAPERS

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ABSTRACT

In this paper, we propose a new technique to design time-limited pulse shapes for ultra wideband (UWB) communication systems. The technique is based on searching the optimal pulse on a sub-field spanned by set of orthogonal functions built from the modified Hermite function. The design problem can be formulated as a linearly constrained convex problem. Simulations results, show that the pulses designed using the proposed technique, conform to the spectral emission constraints. More over, it out preforms other pulses design with other approaches, in terms of normalized efficient signal power.

1. INTRODUCTION

Ultra-Wideband (UWB) technology is gaining increasing interest for its potential application to short-range indoor wireless communications. Utilizing ultra-short pulses, UWB base band transmissions enable rich multi-path diversity, and can be demodulated with low complexity receivers[1] . For spectrum overlay consideration, FFC regulations[2] imposed a spectral mask that strictly limits the power spectrum of UWB signal to be well below the noise floor. On the other hand the transmission quality of the UWB system is determined by the received signal-to-noise ratio (SNR). Given the stringent transmit power limitations, maximization of the received SNR requires efficient utilization of the bandwidth and power allowed by the FFC spectral mask. The goal is to design the underlying UWB pulse shape so as to optimize the spectral shape of the transmitted signal. Unfortunately, the widely adopted Gaussian monocycle exhibits a poor fit to the FFC spectral mask and it is not desirable for practical usage. Recently, several design methods were introduced[3] [4] [5] [6] [7] . In [3] new pulse is designed corresponding to the dominant eigenvector of a channel matrix that is constructed by sampling the spectral mask. Although the generated pulse [3] conforms to the spectral mask, it does not achieve the most efficient spectral utilization, and requires a high sampling rate ($64GHz$) that could lead to implementation difficulties. In [4], design method, based on semidefinite programming (SDP), is introduced to achieve optimal spectral utilization at a relatively low sampling rate. It consists on transforming the design problem to a linearly constrained convex optimization problem that can be efficiently solved for globally optimum solution.

In this paper, a new design method is introduced to achieve optimal spectral power allowed by the FCC spectral mask at a relatively low sampling rate. The approach consists on building a sub-space by a number of orthogonal modified Hermite function and find the optimal pulse on this subspace which satisfies the FFC. The new design problem becomes linearly constrained convex optimization problem.

The paper is organized as follows: in section 2 we review the Hermite function[8] . In section 3 we establish the problem statement. In section 4 we propose a new method to design optimal UWB pulse shaper. finally in section 5 simulation results are presented to support the new design approach.

2. THE HERMITE FUNCTION FAMILY

Hermite function, and indeed Hermite polynomial[9] are not new. Hermite transform have been used to shed light on space-temporal relationships in image processing. In this section, we begin with defining these function. The Hermite polynomials are defined as follows [9]:

$$h_n(t) = (-1)^n e^{t^2} \frac{d^n}{dt^n} \left(e^{-t^2} \right), \text{ where } n = 1, 2, \cdots \quad (1)$$

$$\text{and } -\infty < t < +\infty \quad (2)$$

These polynomials form a set of orthogonal polynomials with respect to the Gaussian weighting function $m(t) = e^{-t^2}$:

$$\int_{-\infty}^{+\infty} h_n(t) h_m(t) e^{-t^2} dt = 2^n n! \sqrt{\pi} \delta_{mn} \quad (3)$$

From these polynomials, we can construct the Hermite orthonormal function $d_n(t)$, which are related to the Hermite polynomials $h_n(t)$ by:

$$d_n(t) = \frac{h_n(t) e^{-\frac{t^2}{2}}}{\sqrt{2^n n! \sqrt{\pi}}}. \quad (4)$$

These functions are orthogonal, and can be used as orthogonal basis of a some sub-space. The Fourier transform $D_n(\omega)$ of the Hermite function $d_n(t)$ is given by:[10]

$$D_n(\omega) = TF\left(d_n(t)\right) = (-j)^n \sqrt{2\pi} \times d_n(\omega) \quad (5)$$

We note that the Fourier transform of the Hermite function is also a Hermite function apart from a multiplication factor that is dependent on the order of the Hermite function[10]. These properties will be used in the next section to design the new optimal UWB pulse.

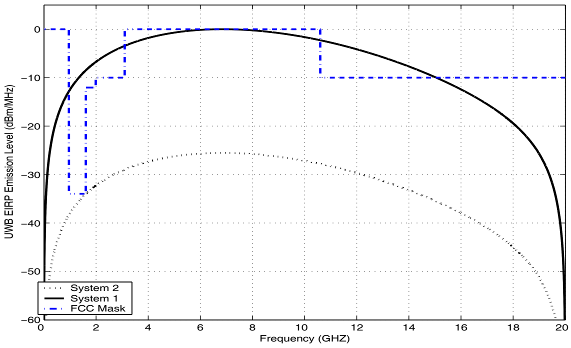

Figure 1: The power spectral density of the Gaussian monocycle pulse for the tow peak amplitude A_1 and A_2 and the FFC indoor spectral emission mask

3. PROBLEM STATEMENT

Based on the FCC spectral mask $S(f)$ illustrated in figure.1, we observe that most of the UWB signal power should be allocated to the 3.1 - 10.6GHz band, while considerable attenuation is imposed in other regions of the spectrum, especially for frequencies up to 3.1GHz. These constraints are designed to avoid interference to legacy narrowband systems. Accordingly, we define $F_p := \{f \backslash f \in [3.1, 10.6]GHz\}$ as the UWB passband [4]. Before introducing the problem statement, let us first consider the Gaussian monocycle pulse, which has been widely adopted by UWB radar and communication systems. The Gaussian monocycle can be expressed as

$$g(t) = 2\sqrt{e}A\frac{t}{\tau_g}e^{-2(\frac{t}{\tau_g})^2}, \tag{6}$$

where τ_g is the time duration between its minimum and maximum values and A represents its peak amplitude. The pulse duration is approximately equal to $T_g = 4\tau_g$. Accordingly, the Fourier Transform (FT) of $g(t)$ is

$$G(f) = \frac{1}{2}\sqrt{\frac{2e}{\pi}}\frac{A}{f_g^2}e^{-\frac{1}{2}\left(\frac{f}{f_g}\right)^2} \tag{7}$$

where $f_g := 1/(\pi\tau_g)$ is the frequency where $|G(f)|$ is maximum[11].

In figure.1, we have plotted the FFC indoor spectral emission mask and frequency response $G(f)$ for the two amplitudes A_1 (system 1) and A_2 (system 2). Trying to maximize transmission power, system 1 violates the FFC spectrum mask; whereas trying to respect the FFC mask, system 2 does not exploit the FFC mask in a power efficient manner. Consequently, the Gaussian monocycle does not lead to optimal utilization of the spectrum assigned by FFC.To maximize the received SNR, the spectral shape of UWB pulse $p(t)$ should optimally utilize the allowed bandwidth and power spectra, while at the same time respecting the spectral mask $S(f)$. The spectrum utilization efficiency can be measured by the normalized effective signal power (NESP) ψ[4] :

$$\psi = \frac{\int_{F_p}|P(f)|^2}{\int_{F_p}|S(f)|^2} \times 100, \tag{8}$$

where $P(f)$ is the FT of $p(t)$.

The objective of our pulse design problem is to find $p(t)$ that maximizes ψ under the spectral mask constraint. this can be mathematically formulated as follows:

$$\max_{p(t)} \psi \quad \text{subject to } |P(f)| \leq S(f), \ \forall f \tag{9}$$

4. NEW DESIGN APPROACH

In this section, we exploit the property of Hermite function to design time-limited pulse shape for UWB systems , with given frequency magnitude characteristic. To control pulse width of the Hermite functions, we propose to use a scaling factor α, for each Hermite function $d_n(t)$ (If the α increases, the bandwidth of the pulse becomes narrower). Consequently, the bandwidth can be fitted into the bounds of the spectral mask.

From the modified Hermite function, we construct a set of even functions defined by:

$$g_{i,\alpha}(t) = \begin{cases} d_i(\alpha t) \times \cos(2\pi t f_c) & \text{if } i \text{ is even} \\ d_i(\alpha t) \times \sin(2\pi t f_c) & \text{if } i \text{ is odd} \end{cases} \tag{10}$$

where f_c is the center frequency of the FFC mask.

It is worth noting, that these $g_{i,\alpha}(t)$ forms a set of orthogonal function.

The Fourier transformation $G_{i,\alpha}(f) = FT(g_{i,\alpha}(t))$ is given by:

$$G_{i,\alpha}(f) = \begin{cases} \text{if } i = 2n \\ \frac{(-1)^n\sqrt{2\pi}}{2\alpha}\left[d_i\left(2\pi(\frac{f}{\alpha} - f_c)\right) + d_i\left(2\pi(\frac{f}{\alpha} + f_c)\right)\right] \\ \text{if } i = 2n+1 \\ \frac{\sqrt{2\pi}(-1)^{n+1}}{2\alpha}\left[d_i\left(2\pi(\frac{f}{\alpha} - f_c)\right) - d_i\left(2\pi(\frac{f}{\alpha} + f_c)\right)\right] \end{cases} \tag{11}$$

For an appropriate α, the two terms $d_i\left(2\pi(\frac{f}{\alpha} - f_c)\right)$ and $d_i\left(2\pi(\frac{f}{\alpha} + f_c)\right)$ will not interfere, so we can approximate $G_{i,\alpha}(f)$ for $f \in [0, +\infty[$ as follows:

$$G_{i,\alpha}(f) \approx \begin{cases} \frac{(-1)^n\sqrt{2\pi}}{2\alpha}d_i\left(2\pi(\frac{f}{\alpha} - f_c)\right) & \text{if } i = 2n \\ \frac{\sqrt{2\pi}(-1)^{n+1}}{2\alpha}d_i\left(2\pi(\frac{f}{\alpha} - f_c)\right) & \text{if } i = 2n+1 \end{cases}.$$

For a fixed scaling factor α,we build a sub-field $E_{L,\alpha}$ spanned by L orthogonal functions $\left\{g_{0,\alpha}, g_{1,\alpha}, \cdots, g_{L-1,\alpha}\right\}$. We restrict, the optimization problem, by finding an optimal pulse $p_{L,\alpha}(t)$ in the sub-field $E_{L,\alpha}$. Thus $p_{L,\alpha}(t)$ will be expressed as follows:

$$p_{L,\alpha}(t) = \sum_{i=0}^{L-1} x_i \, g_{i,\alpha}(t). \tag{12}$$

The power spectrum $S_p(f) := \left|P_{L,\alpha}(f)\right|^2$ of $p_{L,\alpha}(t)$ is given by

$$S_p(f) = \left| \sum_{i=0}^{L-1} x_i G_{i,\alpha}(f) \right|^2 \tag{13}$$
$$= \left(\sum_{i=0}^{L-1} x_i G_{i,\alpha}(f) \right)^2 .$$

For the chosen pulse model 12 and an appropriate α, the NESP can be approximated as follows:

$$\psi = \frac{\int_{F_p} \left(\left(\sum_{i=0}^{L-1} x_i G_{i,\alpha}(f) \right) \right)^2 df}{\int_{F_p} S(f)\, df} \times 100 \tag{14}$$

$$\approx \frac{\int_0^{+\infty} \left(\left(\sum_{i=0}^{L-1} x_i G_{i,\alpha}(f) \right) \right)^2 df}{\int_0^{+\infty} S(f)\, df} \times 100$$

Using the orthogonality property of function $g_{i,\alpha}(t)$, the NESP can be approximated by:

$$\psi \approx \frac{\frac{1}{2\alpha} \sum_{i=0}^{L-1} x_i^2}{\int_0^{+\infty} S(f)\, df} \times 100.$$

Due to the fact that $G_{i,\alpha}(f)$ is real, the Fourier transformation $P_{L,\alpha}(f)$ of our designed pulse $p_{L,\alpha}(t)$ will be also real, so the mask constraint $\left| P_{L,\alpha}(f) \right|^2 \leq S(f)$ can also be equivalently written as:

$$-\sqrt{S(f)} \leq P_{L,\alpha}(f) \leq \sqrt{S(f)}. \tag{15}$$

In the frequency domain, the mask was represented with infinite number of linear constraints. We relax this constraint by a discretization method, in witch the entire UWB passband is sampled in an ascending order at a fine frequency resolution to form a frequency set $\{f_0, f_1, \cdots, f_{M-1}\}$. Now our pulse design problem defined over the continuous frequency variable f can then be reformulated as a convex quadratic function of $X = [x_0, x_1, \cdots, x_{L-1}]^T$ with finite number of linear constraints formulated as follows:

$$\min_{\vec{x}} \left(\frac{-100}{2\alpha \int_0^{+\infty} S(f)\, df} \right) \sum_{i=0}^{L-1} x_i^2 \tag{16}$$

$$\begin{bmatrix} H_{L,\alpha} \\ -H_{L,\alpha} \end{bmatrix} X \leq \begin{bmatrix} S(f_0) \\ S(f_1) \\ \vdots \\ S(f_{M-1}) \end{bmatrix}.$$

where $H_{L,\alpha}$ is the matrix defined as follows:

$$H_{L,\alpha} = \begin{pmatrix} G_{0,\alpha}(f_0) & \cdots & G_{L-1,\alpha}(f_0) \\ G_{0,\alpha}(f_1) & \cdots & G_{L-1,\alpha}(f_1) \\ \vdots & \cdots & \vdots \\ G_{0,\alpha}(f_{M-1}) & \cdots & G_{L-1,\alpha}(f_{M-1}) \end{pmatrix}.$$

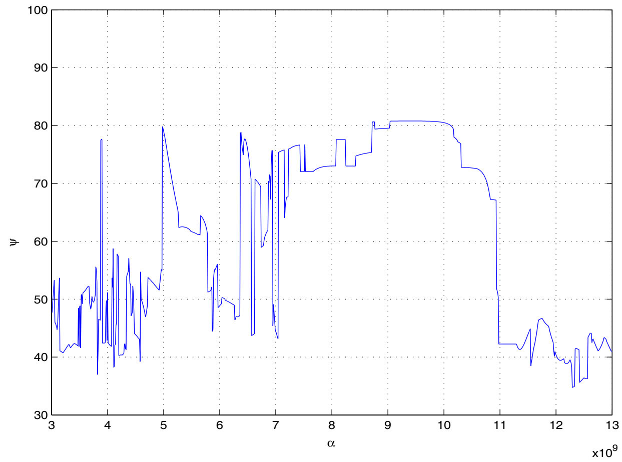

Figure 2: Evolution of the normalized effective signal power versus the scaling factors α, for $L = 10$ and $N = 21$.

Our global design problem can now be written as the following optimization problem:

$$\min_{\alpha} \left(\min_{X} -\psi(X, \alpha) \right) \tag{17a}$$
$$s.t. \begin{bmatrix} H_{L,\alpha} \\ -H_{L,\alpha} \end{bmatrix} X \leq M$$

To relax the optimization problem 17, we split it into tow steps. First, for a fixed α, we solve the convex problem described by equation 16, and compute the optimum NESP denoted $\psi_L(\alpha)$, In the second step, we analyze the evolution of $\psi_L(\alpha)$ Versus α. Then we choose the optimum α, α_{opt}, which maximizes $\psi_L(\alpha)$.

5. DESIGN EXAMPLES

In this section, we apply our approach design method, presented in the previous section, to design optimal ultra-wadeband pulse shaper. To lessen the hardware implementation difficulty, we set the sampling frequency to be relatively low value of $25GHz$.

We denote N, the number of taps for the pulse $p_{L,\alpha}(t)$. So,N will fix the pulse duration.

In the first case, we solve the optimization problem for $L = 10$, and $N = 21$, using the optimization package of Matlab. In figure 2, we report the evolution of $\psi_{10}(\alpha)$ versus α. We deduce that the optimum α is equal to $\alpha_{opt} = 9.32\ 10^9$. The optimal generated pulse and its spectrum is presented in figure 3.Compliant to the FFC mask, the synthesized pulse achieves a maximum NESP of $\psi = 80.7\%$.

In a second case, we have find that for $N = 11$ the optimal factor α_{opt} is equal to $12.26\ 10^9$, where the synthesized pulse achieves a maximum NESP of $\psi = 68.7\%$. The optimal generated pulse and its FT are plotted in figure 4.

In figure 5 we have compared the evolution of the optimal normalized effective signal power achieved by the pulse shapers designed in [4] with our new design pulse shaper versus the pulse duration. It is clear that our design utilizes the FFC spectral mask most efficiently. As example, for $N = 10$, the pulse in [4] has $\psi \approx 43\%$ however our new pulse has $\psi \approx 68.7\%$

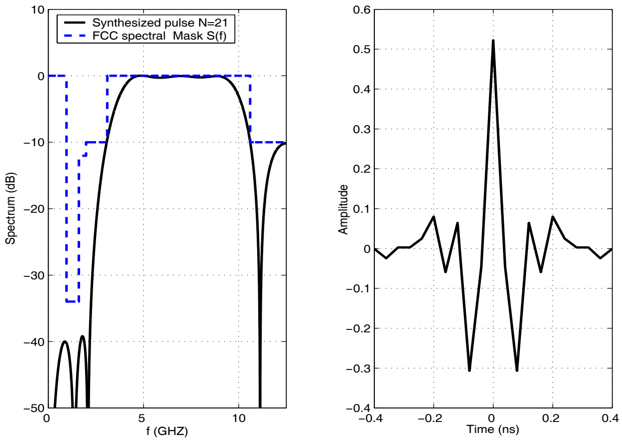

Figure 3: For $N = 21$, optimally designed pulse shaper and its FT for $\alpha = 9.32\ 10^9$and $L = 10$.

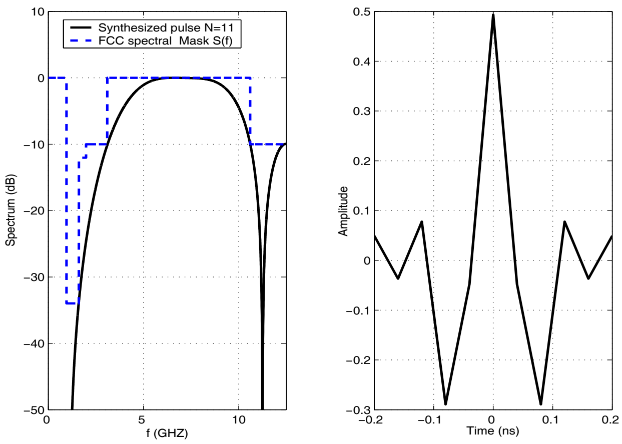

Figure 4: For $N = 11$, optimally designed pulse shaper and its FT for $\alpha = 12.26\ 10^9$ and $L = 10$.

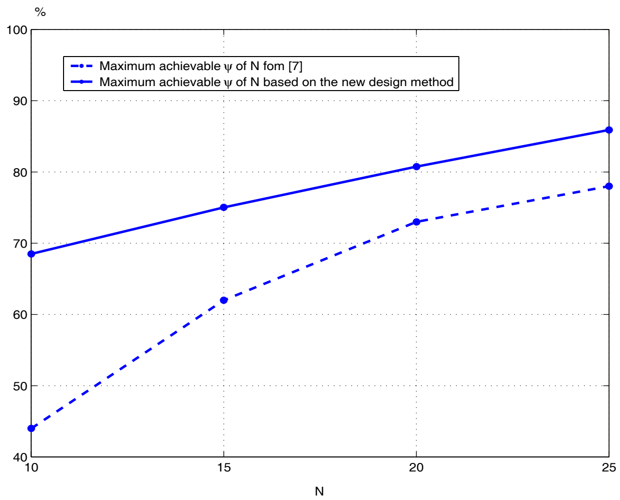

Figure 5: Evolution of the maximum normalized effective signal power versus the pulse duration.

6. CONCLUSION

In this paper, we have proposed a new pulse design approach. The approach consists on searching the optimal pulse in a sub-field spanned by a set of orthogonal functions, built from the modified Hermite functions. The problem becomes a linearly constrained convex problem. The obtained results, show that the generated pulses meet the FCC mask and out perform other design approach.

REFERENCES

[1] M. Z. Win and R. A. Scholtz, "Impulse radio: How it works", IEEE Commun. Lett., vol. 2, pp. 3638, Febuary 1998.

[2] H.Sheng, P.Orlik, "On the Spectral and Power Requirements for Ultra-Wideband Transmission", Communications, 2003, ICC03, IEEE International Comm. Conference,Volume:1,2003 pp:738-742

[3] Parr, B.; ByungLok Cho; Wallace, K.; Zhi Ding, "A novel ultra-wideband pulse design algorithm", Communications Letters , IEEE, vol. 7, no. 5, pp. 219-221, May 2003.

[4] X. Wu, Z. Tian, T. N. Davidson, G. B. Giannakis, "Optimal Waveform Design for UWB Radios", Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP'2004), Montreal, QC, vol. 4, pp. 521-524, May 17-21, 2004.

[5] N.C. Beaulieu and B. Hu, "A Novel Pulse Design Algorithm for Ultra-wideband Communications", IEEE GLOBECOM, Dallas, Texas, Nov. 29-Dec. 3, 2004.

[6] X. Wu, Z. Tian, T. N. Davidson, G. B. Giannakis, "Orthogonal Waveform Design for UWB Radios", Proceedings of IEEE Signal Processing Workshop on Advances in Wireless Communications (SPAWC2004), Lisbon, July 11-14, 2004.

[7] Zeng, D., A. Annamalai Jr., A.I. Zaghloul," Pulse Shaping Filter Design in UWB System", IEEE Conference on Ultra Wideband Systems and Technologies, pp. 6670, November 2003.

[8] M. Ouertani, H. Besbes, A. Bouallegue, "A new design approach for ODMA systems", 2004 First International Symposium on Control, Communications and Signal Processing, ISCCSP 2004, Tunisia 2004.

[9] G. Andrews, R. Askey, R. Roy, Special functions, edited by Cambridge University Press, Cambridge, UK, 1999.

[10] Ghavami M., L. B. Michael and R. Kohno, "Hermite Function Based Orthogonal Pulses for Ultra Wideband Communication", in Proc. International Symposium on Wireless Personal Multimedia Communications (WPMC), Aalborg, Denmark, pp. 437440, Sep. 2001.

[11] X. Luo, L. Yang, and G. B. Giannakis, "Designing Optimal Pulse-Shapers for Ultra-Wideband Radios," IEEE Journal on Communications and Networks, vol. 5, no. 4, pp. 344-353, December 2003.