ENHANCED TIME RESOLUTION IN BAND LIMITTED COMMUNICATION SYSTEMS

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ABSTRACT

We consider the problem of providing highly accurate time delay or ranging estimates in bandwidth limited communication systems. Prior research has shown that the variance of the time delay estimation error in white Gaussian noise is inversely proportional to the time duration of a power limited probe signal and the square of its mean square bandwidth. To achieve higher resolution, we describe a new approach that synthetically generates a wider bandwidth probing signal using several signals with low bandwidth and processing their returns appropriately at the receiver. The lower bandwidth signals are the outputs of a full tree uniform multirate decomposition of a large bandwidth virtual probing signal. We show that by properly combining the returns corresponding to these low bandwidth signals, we effectively synthesize the return corresponding to the virtual large bandwidth signal. We also show that the approach is robust to oscillator phase mismatches that occur when switching from one sub-signal to another and yields superior performance when compared to traditional techniques.

1. INTRODUCTION

Signals that provide high time resolution are often used in communication systems for different purposes e.g. timing, synchronization, ranging and location awareness. Ranging or location awareness is a desirable feature in wireless networks, with applications ranging from military to safety, emergency (E911) and robotics [1]. Of particular interest to us in this paper, is designing high time resolution signals for wireless personal area (WPAN) networks within the emerging IEEE 802.15.3 standard and the 802.15.4a study group.

Range estimation has received considerable attention in radar and sonar applications (for example see [2,3] and references there in). Ranging is equivalent to estimating the time of arrival of a known signal with a narrow correlation function. It is well known that the time resolution, and therefore range estimation, in any system is inversely proportional to its bandwidth. Hence, the system bandwidth puts a limit on the accuracy of delay estimation. This limit is quantified by bounds on the variance of estimation error like Cramer-Rao lower bound (CRLB) [2] which shows that the error variance is inversely proportional to the duration of a finite power probing signal and the square of its bandwidth.

In some communication systems the desired accuracy cannot be achieved with a single transmission because of limited bandwidth and power. One example is the multi-band communication systems that recently have been proposed for ultra wideband (UWB) transmission. In particular, one of the leading proposals for IEEE 802.15.3a, the multi-band OFDM (MB-OFDM), is a multi-band system [4]. In multi-band systems, the whole bandwidth is divided into several sub-bands. In each time interval, a signal is transmitted over only one of the sub-bands. The system then switches to another sub-band to send another signal. Therefore, in any time interval we can send a signal with a bandwidth equal to that of a single sub-band. This would appear to decrease the achievable time resolution, which is normally touted as a desirable feature of UWB systems. The traditional technique in these cases is averaging multiple receptions from multiple transmissions to reduce the effect of noise, or equivalently lengthening the observation time. This technique provides an estimate error variance inversely proportional to the number of transmissions as noted above.

In this paper we describe a new scheme that uses multiple transmissions and leads to a time delay estimate with a variance inversely proportional to the *square* of the number of transmissions. To achieve this performance, we first properly design several sub-signals. Then, we transmit these signals in several time slots and combine the corresponding receptions at the receiver to generate a delayed version of a virtual large bandwidth signal. We then estimate the time delay from this large bandwidth signal. We show that the bandwidth of the virtual signal is equal to the number of original sub-signals multiplied by the bandwidth of each of them. Thus the variance of the delay estimate is inversely proportional to the square of the bandwidth of the combined signal or equivalently to the square of the number of transmissions.

The new scheme can be used to enhance the delay estimation accuracy both in single band and multi-band systems. The designed sub-signals can be transmitted in different time slots in single band systems and in different time-frequency slots in a multi-band system with possible band reuse. Note that another scheme for multi-band systems has been proposed in [5]. That scheme provides accuracy inversely proportional to the square of the number of sub-bands used for transmission as well. That scheme is based on sending the same signal in different bands and exploiting the relationship between the centre frequencies of different sub-bands to coherently combine and process the received signals. It can be used only in multi-band systems and with a number of transmission less than or equal to the number of sub-bands. It is also very sensitive to phase error when switching from one band to another and requires coherent band hopping. In contrast, we show here with simulations that the proposed scheme is much less sensitive to oscillator phase mismatch between transmissions.

2. DELAY ESTIMATION

2.1 Signal design for delay estimation

The maximum likelihood (ML) estimate of a delay is computed by correlating the delayed signal with a copy of the original signal and declaring the time at which the maximum output occurs as the estimate of the delay [2]. To achieve best resolution, the signal should have a narrow autocorrelation function. In digital communication systems, a pseudo random (PN) sequence is used for delay and range estimation to provide a narrow autocorrelation function. Obviously, the system bandwidth puts a limit on the resolution of the delay estimation as noted above. In the remainder of this paper, we will discuss time resolution in a discrete time setting. This is easily done by noting that in a system with a two-sided bandwidth *w*, the input and received signals can be sampled at Nyquist rate of *w* with no loss of information. Delay estimation can be performed in discrete time providing a resolution of one sample or equivalently *1/w* seconds in the continuous time domain.

2.2 Delay Estimation Accuracy

The accuracy of delay estimation in different situations was studied in late 70s and early 80s (see for example [2,6]). The most common expression of delay estimation accuracy is given by Cramer-Rao lower bound (CRLB) for the variance of the estimation error. For the simple case where the received signal can be modelled as:

$$r(t) = s(t-\tau) + n(t) \tag{1}$$

where $s(t)$ is a known signal and τ is unknown delay to be estimated, the CRLB is given by [2]:

$$\mathrm{var}(\hat{\tau}) \geq 1/\left[(E_s/2N_0).w^2\right] = 1/\left[(P_s T/2N_0).w^2\right]. \tag{2}$$

In the above equation, E_s, P_s and T are the energy, power and time duration of $s(t)$ respectively. The additive white Gaussian noise (AWGN), $n(t)$, has a density $N_0/2$. The variable w is its mean square bandwidth and is defined as [2]:

$$w^2 = \frac{\int \omega^2 \left|S(\omega)\right|^2 d\omega}{\int \left|S(\omega)\right|^2 d\omega} \tag{3}$$

where $S(\omega)$ is the Fourier transform of $s(t)$. This bound is achievable by the ML estimator only at high signal to noise ratios (SNR). For low or middle SNR regimes other bounds are tighter [6].

2.3 Delay estimation in Multi-band systems

Equation (2) shows that the variance of the estimation error is inversely proportional to the square of the bandwidth of the system *w.* In a multi-band system the whole available bandwidth *w* is divided into *N* sub-bands, each with a bandwidth of *w/N.* In a given time interval only a signal with bandwidth of *w/N* can be transmitted over one of the sub-bands. Then, for any signal transmission the time estimation error variance will be proportional to $1/(w/N)^2$. Furthermore, the resolution is equal to *N/w.* By transmitting *M* signals in a system that is power limited, or where the energy of any given transmission is limited, we can reduce the variance of the estimation error by a factor of *M* as indicated by (2). Specifically, if we denote by $w_s = w/N$, the CRLB in this case becomes

$$\mathrm{var}(\hat{\tau}_{ave}) = \mathrm{var}(\hat{\tau})/M \approx 1/\left[(E_s/2N_0).^2 w_s^2.M\right] \tag{4}$$

3. DELAY ESTIMATION WITH MULTIPLE TIME SLOTS

3.1 New scheme for better delay estimation with multiple time slots

Let us now describe a new scheme for delay estimation with multiple receptions that provides better performance than traditional techniques. Our goal is to design a scheme that uses *M* time-frequency slots and achieve accuracy proportional to $1/M^2$ rather than $1/M$ as in the traditional approach. In this scheme, we begin by properly designing *M* sub-signals, each with a bandwidth equal to the available bandwidth w_s. We then transmit these signals in *M* available time slots. At the receiver we combine the received signals before processing them to estimate the delay. In fact, we show that if the original sub-signals are designed properly and the channel does not change during the entire transmission, the combined virtual signal has an effective bandwidth of Mw_s and the CRLB for estimation error variance is then equal to:

$$\begin{aligned} \mathrm{var}(\hat{\tau}_{combined}) &\approx 1/\left[(E_s/2N_0).^2 \left(Mw_s\right)^2\right] \\ &= \mathrm{var}(\hat{\tau}_{ave})/M. \end{aligned} \tag{5}$$

In other words we can achieve a resolution of $1/(Mw_s)$.

3.2 Signal design for new scheme

To explain the concept behind our approach, consider *M* time-frequency slots each with bandwidth of w_s. To design the proper scheme we start with a traditional PN sequence

$c(n)$ with equivalent bandwidth of $M\,w_s$. Since the signal has a bandwidth of $M\,w_s$, then the resolution of the delay estimation will be $1/\,M\,w_s$. For simplicity of exposition, assume that the signal passes through a channel with delay equal to $\tau = n_0/(Mw_s)$. The digital equivalent of this system is presented in Fig. 1.

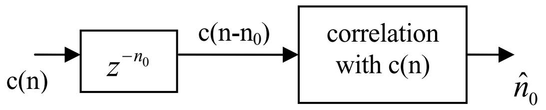

Figure 1: Simple delay estimation set up

The original signal $c(n)$ cannot be used directly in the system since in any time slot only a bandwidth of w_s is available. Now, suppose that we decompose $c(n)$ into M signals $c_i(n$, each with bandwidth of w_s , using a full tree uniform multirate decomposition filter band and follow the decomposition by a perfect reconstruction filter bank [7] as shown in Fig. 2. Obviously, the overall system remains unchanged, i.e., Figs. 1 and 2 are equivalent. Notice that the sub-signals $c_i(n)$, $i=0, M-1$ each have an equivalent bandwidth equal to w_s and can be transmitted in one of the available time slots.

Now observe that we can move the delay z^{-n_0} to the left of the upsamplers, as shown in Fig. 3. The resulting delays $z^{-n_0/M}$ are not realizable using simple shift registers and cannot be accurately estimated using the output of any single branch unless we use a very long observation time as discussed above. However, they do provide an exact discrete time model of the continuous time delay channel with delay τ at the sampling rate w_s [8, p. 100]. In other words, the discrete time equivalent model corresponding to a signal of bandwidth w_s going through a channel with a single delay τ is $z^{-n_0/M}$.

Assuming that the channel remains constant during the transmission time of the M signals $c_i(n)$, Fig. 3 therefore indicates that we can actually transmit the signals $c_i(n)$ in different time slots, collect the receptions corresponding to each signal, bring each to baseband, upsample each reception and pass it through the appropriate reconstruction filter and combine all the filterbank outputs to synthesize the reception due to the virtual signal $c(n)$. This is shown in Fig. 4. The synthesized signal can then be processed as usual to provide a fine time delay estimate with an error variance inversely proportional to M^2.

Note that in Fig. 4 the filters $G_i(z)$ represent the digital equivalent channels for each transmission and capture the effects of the transmitter pulse shaping filter, the actual channel and the receiver front end filters. In other word the impulse response of the equivalent channel is equal to:

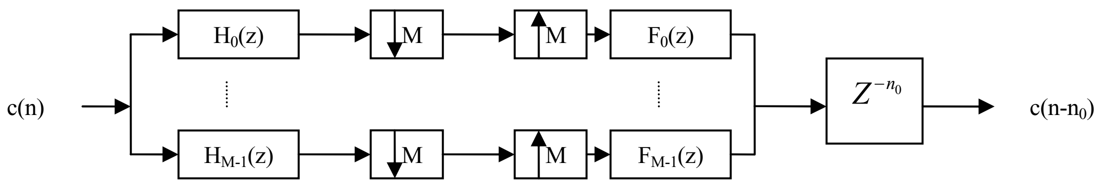

Figure 2: An equivalent to the system in Fig. 1

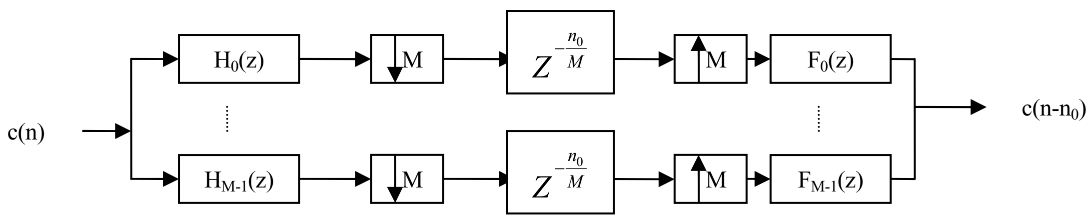

Figure 3: An equivalent to the system in Fig. 2

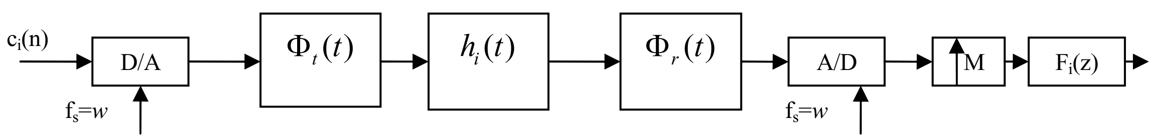

Figure 4: Block diagram of the sub-signal is transmitted in the *i*th time slot

$$g_i(n) = \left[\Phi_t^i(t) \otimes h_i(t) \otimes \Phi_r^i(t) \right]_{t=n/w} \quad (6)$$

where $\Phi_t^i(t)$ is the transmitter filter impulse response, $h_i(t)$ is the impulse response of the low-pass equivalent of actual channel in ith sub-band, $\Phi_r^i(t)$ is the receiver filter impulse response and $\otimes$ shows convolution operation. We assume that the shaping pulse is a Nyquist pulse or that the channel is equalized such that the equivalent digital channel can be modelled as a pure time delay as in Figs 2-3 above.

4. SIMULATION RESULTS

In this section we provide simulation results to compare the performance of proposed scheme with other schemes. In these simulations we used the PN sequence used as the preamble in the multi-band OFDM system for delay estimation [4]. In Fig. 5 we plotted the variance of normalized error versus signal to noise ratio (SNR) for both an averaging scheme that averages *M* receptions, and the new scheme presented in this paper. The number of transmissions is assumed to be *M=4*. This figures shows that in the high SNR regime, where the CRLB is a tight bound, the new scheme outperforms the averaging method as expected from equations (4) and (5). However, in the low SNR regime the CRLB is not valid and simulation results show that the averaging scheme has better performance.

We conclude by comparing the performance of the proposed scheme with that described in [5] in the presence of the phase mismatches that occur when the system switches from one sub-band to another. In the scheme of [5], the same signal is sent in each sub-band. After sampling the output of the matched filter at the receiver, the outputs of sub-bands are aligned and a fast Fourier Transform (FFT) is applied in each time bin along the sub-band index to get a time resolution proportional to the inverse of the total bandwidth. We compare the performance of the two systems. In Fig. 6 the error rate is plotted versus signal to noise ratio (SNR) assuming a random phase in each transition. The simulation shows that the scheme proposed in this paper is more robust to phase errors than the FFT method.

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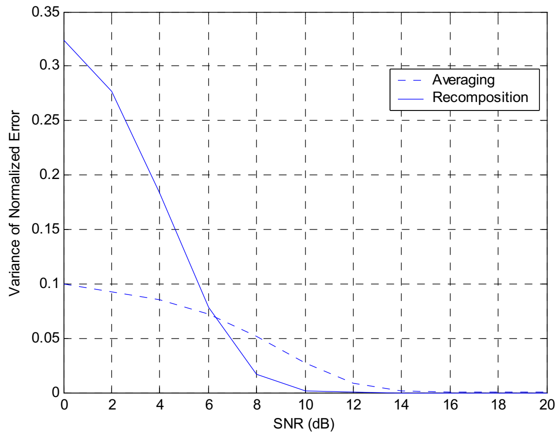

Figure 5: Variance of normalized error versus SNR for both averaging and recomposition schemes for M=4.

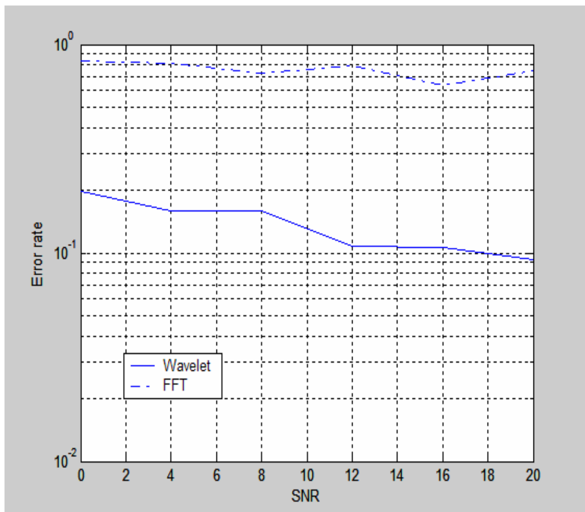

Figure 6: Delay estimation error rate for FFT and wavelet methods with random phase mismatch