CLASSIFICATION AND SAMPLING OF SHAPES THROUGH SEMIPARAMETRIC SKEW-SYMMETRIC SHAPE MODEL

Sajjad H. Baloch and Hamid Krim

Vision, Information and Statistical Signal Theories and Applications (VISSTA) group,
Department of Electrical and Computer Engineering,
North Carolina State University, Raleigh, NC 27695, USA.
email: {shbaloch, ahk}@ncsu.edu
web: www.vissta.ncsu.edu

ABSTRACT

We present a novel method for shape modeling using an extended class of semiparametric skew-symmetric (*SSS*) distributions. Given several realizations of a simple shape, the proposed method models it as a joint distribution of angle and distance from the centroid of all points on the boundary. The model, called "*Semiparametric Skew-Symmetric Shape Model*" (*SSSM*), is capable of capturing inherent variability of shapes provided the realization contours remain within a certain neighborhood range around a "mean" with high probability. In this paper, we will discuss *SSSM* learning, classification through *SSSM* and sampling shapes from the learned models.

1. INTRODUCTION

The goal of shape modeling is to find mathematical representations that capture the intrinsic morphologies of various shapes and simultaneously account for their variability. In recent years, various methods have been proposed to solve the problem including rigid models [3]. Although rigid models have been popular for many applications, it is their inability to reflect the inherent variability of shapes, e.g., anatomical shapes, that has prompted researchers to look for other more flexible approaches [7, 4, 5, 9, 6].

In contrast to these approaches, we view the variability of shape as one that allows realization contours to remain within a certain neighborhood range around a mean. This in turn suggests that for any given angle, a probability density function may be found to capture the corresponding potential excursion of the curves at that angle. We specifically exploit a class of semiparametric skew-symmetric distributions [8] because of potential skewness of data which may arise in practical problems. The method works equally well for non-skewed data since the distribution class includes the non-skewed distributions.

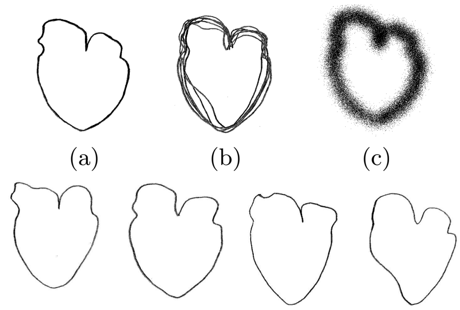
The paper is organized as follows. We start with a problem statement section given next. In Section 3, we describe the probabilistic model we wish to develop. Section 4 and 5 respectively describe classification using *SSSM* and shape sampling. In Section 6, we provide illustrating examples and finally give some conclusions in Section 7.

2. PROBLEM STATEMENT

Any shape S_i can be represented by a curve $C_{S_i}(t)$ in the parameterized form given below:

$$C_{S_i} : I \subseteq \mathbb{R}^+ \to \mathbb{R} \times [0, \pi] \tag{1}$$

This work was supported by AFOSR F49620-98-1-0190 and NSF CCR-9984067 grants.



(a) (b) (c)

(d)

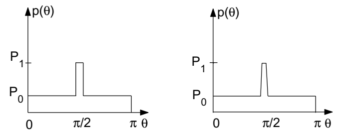
Figure 1: *(a) Heart shape (b) Some realizations superimposed on each other (c) Sampled superimposed realizations (d) Constituent realizations.*

and for convenience and clarity, we take $I \subset \mathbb{N}$, which corresponds to a sampled curve. Given a sampled curve, $\{C_{S_i}{}^j\}_{j=1,\ldots,N}$, we are asked to provide a probabilistic model for S_i in terms of its radius (from the centroid) and angle around. Alternatively, we may view $C_{S_i}(t)$ as a parametric representation $(x(t), y(t))$ or $(\sqrt{x^2(t)+y^2(t)}, \ \arctan[y(t)/x(t)])$.

To better illustrate the overall approach, we discuss a specific template of a heart. Given several realizations of a heart, we are asked to learn and to subsequently capture all objects having similar shape realizations. By learning, we mean to compute from training data, the relevant parameters of the underlying distribution of the shape. We assume that, after normalization, the boundaries of realizations can deviate from the *mean* shape only by some multiple of standard deviation σ and lie within a tubular region shown in Fig. 1(c) with probability close to 1. Combining all such realizations and then sampling them is tantamount to assembling a cloud of points in the neighborhood of the *mean* boundary as shown in Fig. 1(c). The points in the cloud may be assumed "i.i.d" or at most may be modeled as a first order Markov process. The boundary of any shape realization will be a subset of points within a tubular cloud, which is interpreted as a permissible shape space, as shown in Fig. 1(c).

$$S_i \sim \{(\sqrt{x^2+y^2}_i + n_i, \theta_i + n_{\theta_i})\} \tag{2}$$

Based on these realizations, the points in the cloud at a given angle may be distributed around the template boundary according to a skewed distribution $p(r|\theta)$. Note that the non-skewed densities are a particular case of skewed representation.



(a) (b)

Figure 2: *Prior distribution (a) Piecewise uniform (b) Piecewise tapered.*

3. SHAPE ANALYSIS

In this paper, we will be investigating a class of closed simple shapes similar to those given in Fig. 4. For convenience, we adopt a modified polar coordinate system (r,θ), where $\theta \in [0,\pi)$. In addition, we choose to translate the origin to shape centroid and scale the shape to a pre-specified area. We begin with sampling a shape at angles $\Theta \in [0,\pi)$ chosen randomly according to a prior distribution $p(\theta)$. For a given fixed $\Theta = \theta_i$, we identify all samples lying within an ε-neighborhood of θ_i. The closure of the shape allows us to associate two clusters of samples on either side of the centroid at θ_i, with a relative phase difference of π. The two clusters are distributed according to a bimodal conditional distribution (for fixed θ_i), $p(r|\theta_i)$. Slicing the image at a specified angle and combining the two clusters in this way, is advantageous since we need to learn distributions only for half the original angle space and that it also extends readily to complex multi-loop shapes.

We propose the following model to represent such a class of conditional distributions:

$$p(r|\theta) = 2\omega f\left(\frac{r-\xi(\theta)}{\sigma(\theta)}\right) H\left(P_K\left(\frac{r-\xi(\theta)}{\sigma(\theta)}\right)\right), \qquad (3)$$

where f is any symmetric pdf and H is any cumulative distribution function of a continuous random variable that is symmetric around zero and P_K is a K-order polynomial. ω is a parameter that makes $p(r|\theta)$ a valid density and depends on (ξ,σ) and the parameters of polynomial P_K. One can show that such a formulation affords much flexibility such as multimodality, skewness, symmetry, etc. [1].

Using a data sample of sufficient size, we may proceed to learn the density by standard techniques, for instance, maximization of likelihood function.

3.1 Prior Distribution

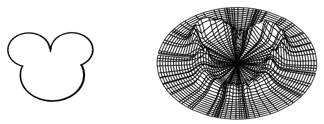
Because of the random nature of R and Θ, a complete shape representation requires assigning a prior on Θ, whose choice is influenced by the presence of shape singularities, for instance a cusp shown in Fig. 1(a). Provided that such events are present, their nonuniform occurrence throughout, suggests a piecewise uniform or piecewise uniform tapered (PUT) distribution for the prior as shown in Fig. 2.

3.2 Overall Shape Model

Using the prior, $p(\theta)$, together with a conditional density for r, $p(r|\theta)$, we can construct the overall density for a shape. Based on m angle samples, $\theta = (\theta_1,\ldots,\theta_m)^T$, drawn according to the prior, the overall shape has conditional likelihood:

$$p(Z_1,\ldots,Z_n|\theta) = \prod_{i=1}^{m} \prod_{\substack{j=1 \\ Z_j \in N_\varepsilon^{\theta_i}}}^{n} p(r_j|\theta_i)p(\tilde{\theta}_j), \qquad (4)$$

where $\{Z_j\}_{j=1,\ldots,n}$ is the point with radius r_j and angle $\tilde{\theta}_j$, $N_\varepsilon^{\theta_i}$ is the ε-neighborhood of $\theta = \theta_i$ and $Z_j = (r_j,\tilde{\theta}_j) \in N_\varepsilon^{\theta_i}$.



(a) (b)

Figure 3: *Illustration of using modes as boundary points: (a) Mickey face shape; (b) Bimodal distributions learned with 100 angles.*

3.2.1 Template Learning

We define a template as the best realization of the ideal shape S in *MLE* sense. In other words, the template, $R_{S\ \mathrm{template}}$ maximizes the conditional likelihood function (4) over a set of all possible realizations $\{R_{S_i}; i = 1,\ldots,l\}$:

$$R_{S\ \mathrm{template}} = \arg\max_{R_S} p(Z_1,\ldots,Z_n|\theta)\mid_{R_S}. \qquad (5)$$

This is equivalent to using modes of the bimodal distribution as the most likely representative for the boundary points. Hence, corresponding to each angle, we estimate the modes of the posterior, which will then be assumed to lie on the template boundary. In the limiting case, where the number of angle samples goes to infinity, the set of modes will constitute the closed contour of the learned template. This is illustrated for *Mickey* face in Fig. 3.

3.3 Prior performance assessment

In order to quantify the goodness of a learned template and the quality of a prior selection, we evaluate the cumulative deviation between an ideal shape and learned template with specific priors.

At a given angle, θ, the deviation of the ideal boundary points $r_1(\theta)$ and $r_2(\theta)$ from the estimated ones is given by:

$$dr_i(\theta) = r_i(\theta) - \hat{r}_i(\theta). \qquad (6)$$

As a performance measure, we may use l_2-norm of the difference between two shapes. Discretizing angle space and considering some ε-neighborhood of θ, the departure of learned template from the ideal shape is given by:

$$D = \sqrt{\sum_{\theta\in[0,\pi]} \sum_{\theta_i \in N_\varepsilon^\theta} \left(dr_{1,\theta_i}^2 + dr_{2,\theta_i}^2\right)}, \qquad (7)$$

where N_ε^θ is some ε-neighborhood around θ and dr_{j,θ_i} is the deviation for the j-th mode at a given angle $\theta_i \in N_\varepsilon^\theta$.

4. CLASSIFICATION OF SHAPES

With the probabilistic model (4) in hand, we may now employ maximum likelihood classifier. Suppose that we have learned models p_S for l shapes $\{S_i; i = 1,\ldots,l\}$. Given a test shape, S_0, we compute l likelihoods $\{p_{S_i}(s_i|S_0); i = 1,\ldots,l\}$ and assign S_0 to the class that achieves the largest likelihood.

5. SAMPLING FROM MODELS

Shapes can be sampled from the distribution given by (4). In order to sample a cloud of shapes, we need to learn the conditional distributions, $p(r|\theta)$, for m given angles $\Theta = \theta$ drawn according to a PUT distribution. Once the learning phase is complete, we proceed to generate $n >> m$ additional angle samples from the same PUT distribution. For each angle sample, the best approximation θ is selected and finally r is generated according to $p(r|\theta)$.

For sampling closed shapes from the model, we need to incorporate a first order Markov process, which tries to ensure a smooth boundary. Clearly, each value r assumed by R depends on its previous value r_0 along with the angle Θ.

$$\begin{aligned} p(r|\theta,r_0) &= p(r|\theta)p(r_0|r)/p(r_0) & (8) \\ &= p(r|\theta)p(r|r_0)/p(r), & (9) \end{aligned}$$

where $p(r|\theta)$ is given by (3). Hence, the modified shape model is given by:

$$p(Z_1,\ldots,Z_n|\theta) = \prod_{i=1}^{m} \prod_{\substack{j=1 \\ Z_j \in N_\varepsilon^{\theta_i}}}^{n} p(r_j|\theta_i, r_{j0})p(\tilde{\theta}_j). \quad (10)$$

$p(r|r_0)$ may be assumed to be Gaussian for each mode, with mean r_0. Its variance σ_0 may be treated as a smoothness parameter. However, Laplace distribution is more appropriate. Furthermore, a product of Laplace distribution and bimodal *SSS* distribution is yet a better choice for $p(r_0|r)$ [2].

In order to ensure closure of a shape, a current boundary point must be dependent on the starting point. This dependence is, however, weak, initially, but as a curve is traversed, the dependence gradually becomes stronger. Again Gaussian distribution is a good choice [2].

6. EXPERIMENTAL RESULTS

In this section, we present some results that demonstrate generality and effectiveness of the proposed method.

6.1 Model Learning

Using different priors, we learn models for three shapes, identified as star, brain, and heart, which are acquired from real images (Fig. 4). We used 20 angle samples in each case except for Star shape, which was learned with 10 samples. A third order polynomial was employed in (3) with skewed normal distribution. Learned templates are given in Fig. 5 and Fig. 6. Templates learned with 100 samples are shown in Fig. 7. Performance measures are tabulated in Table 1. It is clear, both visually and quantitatively by the performance measures, that the PUT prior gives better results than uniform prior.

Table 1: PERFORMANCE MEASURE D FOR CASE STUDIES.

Case Study	Prior for angle Θ	D ($\times 10^3$)
Star	Uniform	2.1
	PUT	1.6
	100 Samples	1.6
Brain	Uniform	0.5
	PUT	0.4
	100 Samples	0.4
Heart	Uniform	0.205
	PUT	0.211
	100 Samples	0.198

6.2 Classification

In this section, we present classification results that were obtained for a database of 143 car and 68 banana shapes, some of which are shown in Fig. 10 and Fig. 11 respectively. Templates learned from *SSSM* for the two shapes are presented in Fig. 10 and Fig. 11. Given an observation S_0, we test the following two hypotheses:

$$\begin{aligned} H_0 &: S_0 \text{ is a car.} & (11) \\ H_1 &: S_0 \text{ is a banana.} \end{aligned}$$

Likelihoods $p_{S_{\text{car}}}$ and $p_{S_{\text{banana}}}$ for 143 car and 68 banana shape realizations were computed. The likelihoods were then used to classify with 97.6% success rate.

6.3 Sampling from Model

A cloud of points simulated for *Mickey* face is shown in Fig. 8. Angle sample size was 100 while the number of points was 20000. Some sampled shapes are given in Fig. 9.

7. CONCLUSIONS

In this paper, we presented semiparametric model for shape representation, whose statistical nature accounts for variations present in different realizations. The method is sufficiently general to capture possible skewness that might appear in data. Computer simulations demonstrate that semiparametric skew-symmetric template learning is quite effective and robust for capturing variability inherent to shapes. It can capture shape singularities to some extent and may be applied to complex multi-loop templates using higher order polynomial P_K in (3). We also presented classification results and a way to sample shapes from the model. The method can also be extended to 3-D shapes, which we are currently investigating.

REFERENCES

[1] S. H. Baloch, H. Krim, and M. G. Genton, "Semiparametric skew symmetric distributions in shape representation", *Skew-Elliptical Distributions and their Applications: A Journey Beyond Normality*, M. G. Genton Editor, Chapman & Hall/CRC. In press.

[2] S. H. Baloch, and H. Krim, "Semiparametric Skew-Symmetric Shape Model for Classification and Sampling", In preparation.

[3] R. T. Chin, and C. R. Dyer, "Model-based recognition in robot vision", *Computing Surveys*, Vol. 18, No. 1, pp. 67-108, 1986.

[4] T. F. Cootes, C. J. Taylor, D. H. Cooper and J. Graham, "Active shape models – their training and application", *Computer Vision and Image Understanding*, Vol. 61, No. 1, pp. 38-59, 1995.

[5] U. Grenander, *"Elements of pattern theory"*, John Hopkins University Press, 1996.

[6] E. Klassen, A. Srivastava and W. Mio, "Analysis of Planar Shapes Using Geodesic Paths on Shape Spaces", *IEEE Trans. on Pattern Analysis and Machine Intelligence*, In press.

[7] M. Kass, A. Witkin and D. Terzopoulos, *Snakes: active contour models*, Proc. First IEEE Int. Conference on Computer Vision, pp. 259-268, 1987.

[8] Y. Ma, and M. G. Genton, "A semiparametric class of generalized skew-elliptical distributions", *Institute of Statist. Mimeo Series # 2541*, 2002.

[9] J. C. Ye, Y. Bresler, and P. Moulin, "Asymptotic global confidence regions in parametric shape estimation problems", *IEEE Trans. on Information Theory*, Vol. 46, No. 5, pp. 1881-1895, 2000.

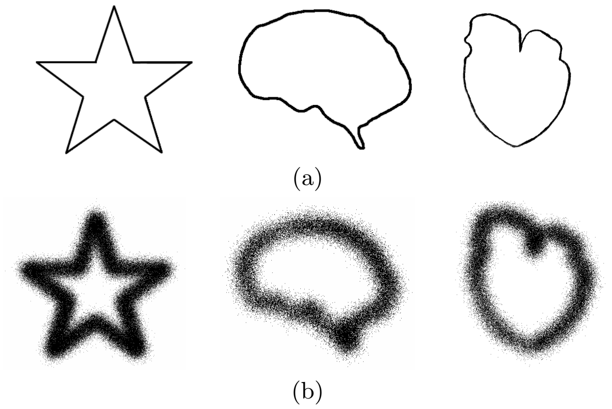


Figure 4: *(a) Actual shapes: star, brain, and heart; (b) Corresponding realizations.*

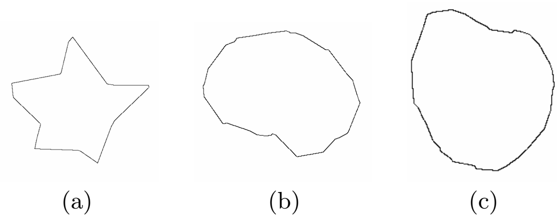


Figure 5: *Template learned using uniform prior: (a) Star; (b) Brain; (c) Heart.*

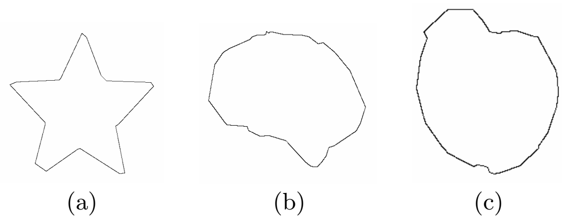


Figure 6: *Template learned using piecewise tapered uniform prior: (a) Star; (b) Brain; (c) Heart.*

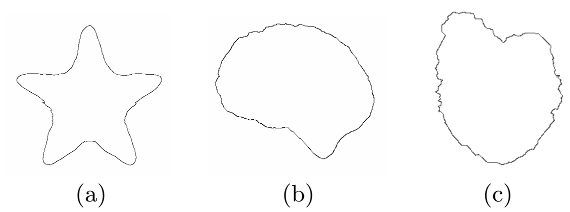


Figure 7: *Template learned with 100 angle samples: (a) Star; (b) Brain; (c) Heart.*

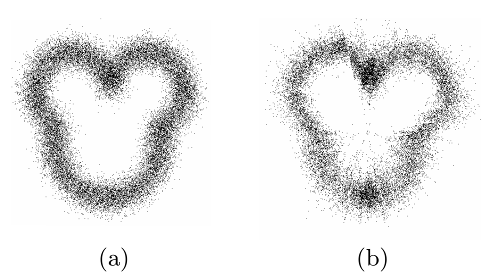


Figure 8: *Shape simulation of Mickey face according to Eq. 4: (a) Realizations; (b) Simulated realizations.*

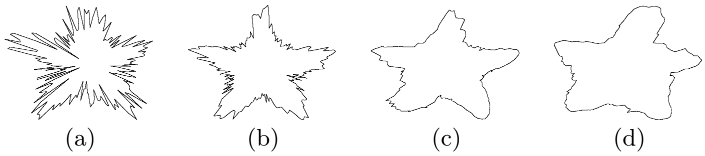


Figure 9: *Star shapes simulated according to Eq. 10: (a) Without Markov chain; (b)* $\sigma_0 = 1$*; (c)* $\sigma_0 = 0.09$*; (d)* $\sigma_0 = 0.04$.

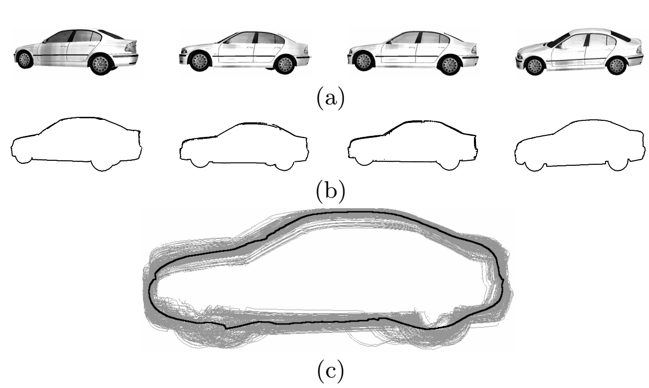


Figure 10: *Car Shape: (a) Some car images from different viewing angles; (b) Corresponding outer contours; (c) Learned template.*

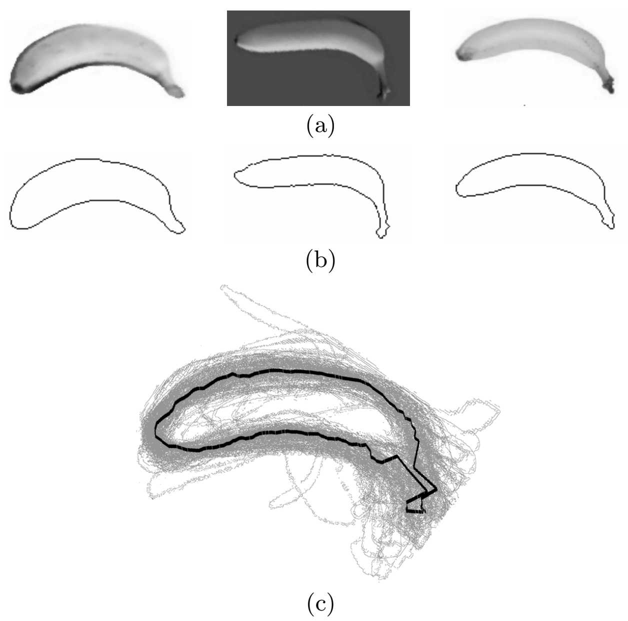


Figure 11: *Banana Shape: (a) Some sample bananas; (b) Corresponding outer contours; (c) Learned template.*