LOW-COMPLEXITY EQUIVALENT REALIZATIONS OF NONLINEAR MULTIRATE FILTERS

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ABSTRACT

In this paper low-complexity equivalent realizations for multirate Volterra systems are presented. All analysed filter configurations contain in addition to the nonlinear filter both an upsampler and a downsampler. Using Volterra filters a wide class of nonlinear systems can be approximated with arbitrary precision. Also special cases, like cascades of linear filters and memoryless nonlinearities (LNL filters, Hammerstein model , Wiener model) are considered. To derive the new realizations linear and nonlinear polyphase decompositions are employed. All operations are performed at the lowest possible sampling rate. Furthermore, some coefficients disappear in the polyphase representation and therefore, the computational complexity can be reduced significantly.

1. INTRODUCTION

In the following sections we consider the basic scheme in Fig. 1 consisting of an upsampler, a general nonlinear operator H and a downsampler. The goal is to minimize the high computational complexity (CC) caused by the nonlinear filter operating at the highest sampling rate. As known from linear multirate systems ([1],[2]) we should move the upsampler as far as possible to the right and the downsampler as far as possible to the left. Generally, this is not practicable because a nonlinear filter and an up- or downsampler do not commute. Therefore, we focus on Volterra systems [3] which offer polyphase implementations operating at the low sampling rate [4]. Depending on the up- and downsampling factor L and M different cases are analysed.

Such configurations typically occur in the field of nonlinear echo cancellation for asymmetric data transmission systems. E.g. Fig. 2 depicts a simplified ADSL-CO (central office) application [5, 6] involving oversampling AD and DA conversion. The downstream signal has a sampling rate of 2.208 MHz, the upstream signal is processed at 276 kHz. Note that only the echo relevant parts are shown: upsampler, interpolation filter, ΣΔ-DAC, transmit filter, linedriver, hybrid, receive filter, ΣΔ-ADC, decimation filter and downsampler. In this example the ADC and DAC are clocked both with 26.496 MHz and mainly the linedriver could exhibit a nonlinear behaviour limiting the performance of a linear echo canceller. Therefore, a nonlinear echo canceller should be employed. To model the nonlinearity a Volterra filter could be used. If all the linear filters in the echo path are included in an extended Volterra filter we get a scheme as shown in Fig. 1 with $L = 12$ and $M = 96$. In fhe following we focus on low-complexity realizations for the the nonlinear part of the configuration in Fig. 1.

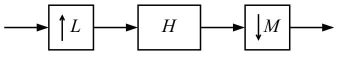

Figure 1: Nonlinear multirate system consisting of an upsampler, a nonlinear filter and a downsampler.

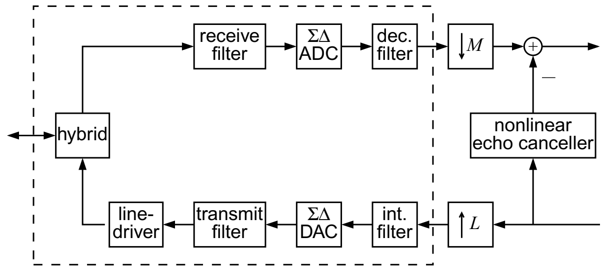

Figure 2: Echo path in a full-duplex communication system.

2. IDENTITIES FOR LINEAR AND NONLINEAR MULTIRATE FILTERS

In this section we present selected identities known from linear and nonlinear multirate systems. Using these identities a simplification of the scheme in Fig. 1 in terms of computational complexity (CC) will be possible. In the following we select a homogenous second-order Volterra kernel [3] with impulse response $h_2[m_1,m_2]$ for the nonlinear operator H in Fig. 1.

Fig. 3 and Fig. 4 show the polyphase implementation of a homogenous second-order Volterra kernel [4]. $H_2^{ij}(z_1,z_2)$ is the z-transform of one single polyphase component operating at the low rate:

$$H_2^{ij}(z_1,z_2) = \sum_{m_1=0}^{N} \sum_{m_2=m_1}^{N} h_2[i+Lm_1, j+Lm_2] z_1^{-m_1} z_2^{-m_2} \qquad (1)$$

In Fig. 4(b) the blocks with two inputs correspond to bilinear Volterra systems (see [7]) with the input-output relation:

$$y[n] = H_2^{ij}\{u_1,u_2\} = \sum_{m_1=0}^{N} \sum_{m_2=m_1}^{N} h_2[i+Lm_1, j+Lm_2] \cdot u_1[n-m_1]u_2[n-m_2]. \qquad (2)$$

This system behaves like a linear filter (FIR) if one of the inputs is held constant. The corresponding two-dimensional z-transform is the same as in Eq. (1).

In Fig. 3(c) a more compact drawing is shown. We use the notation $\overline{\mathbf{H}}_2(z_1,z_2)$ with bold font to emphasize that $\overline{\mathbf{H}}_2(z_1,z_2)$ is a SIMO system, i.e. it has a single input and L outputs. The overline should point out the fact, that only the L (out of L^2) components $H_2^{ii}(z_1,z_2)$, $i \in \{0,1,\ldots,L-1\}$, appear in the polyphase implementation. In Fig. 4(c) all the M^2 polyphase components occur in the MISO realization block and, therefore, we use the notation $\mathbf{H}_2(z_1,z_2)$ without overline.

Note that in the upsampling case the CC can be reduced by a factor of about L^2 (all operations are performed at the low rate and in addition, not all polyphase components appear in the final polyphase representation [4]!), in the downsampling only by a factor of about M (all operations are performed at the low rate).

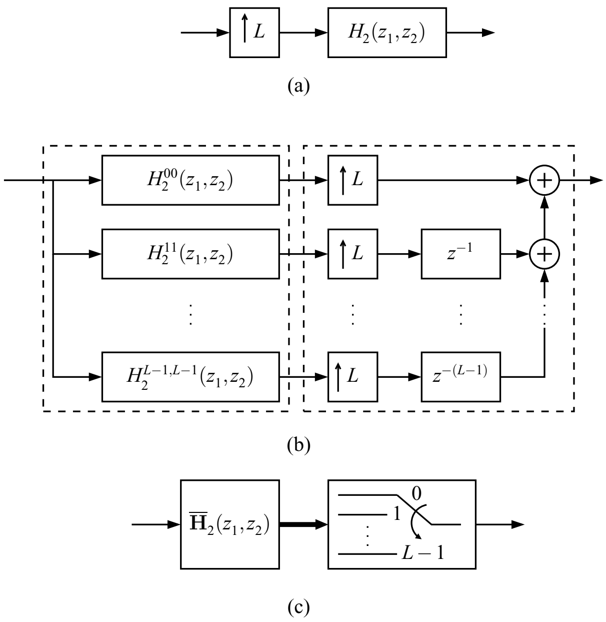

Figure 3: Polyphase implementation of a homogenous second-order Volterra kernel (upsampling case). (a) starting scheme. (b) corresponding polyphase implementation. (c) compact drawing for the two dashed boxes in (b).

Fig. 5 depicts interconnections of upsamplers with downsamplers. Depending on the value of L and M different simplifications can be performed. Note that the commutativity identity in Fig. 5(c) is valid if and only if L and M are coprime, i.e. their greatest common divisor (GCD) is 1. A proof can be found in [2].

Further identities are shown in Fig. 6. The parts (a), (b), (c) and (d) can be easily proven. The identity in (e) can be shown via the Bezout identity [8]. This identity guarantees that there exist integers k_1 and k_2 satisfying the equation

$$k_1 L + k_2 M = GCD(L,M) \tag{3}$$

or as needed in (d) the equation

$$-k_1 Li - k_2 Mi = -i\mathrm{GCD}(L,M) = -i. \tag{4}$$

See [9] and references cited therein for a comprehensive set of rules for multirate signal processing including also the multidimensional case.

3. SIMPLIFICATIONS FOR A HOMOGENOUS SECOND-ORDER VOLTERRA KERNEL

Generally, there are two ways to reduce the CC of the system given in Fig. 1. Either we exchange first the ordering of the upsampler and the nonlinear filter or we move first the downsampler to the left. Then, further simplifications become possible. In the following we consider homogenous second-order Volterra kernels. Note that based on the same principles a generalization for a Volterra kernel of higher order is possible, too.

Depending on L and M different cases are distinguished:

Case 1: L is a multiple of M ($L = L'M$)

In this case the output rate is higher than the input rate. Therefore it is advantageous to interchange the upsampler with the nonlinear filter so that the filter is operating at the low input sampling

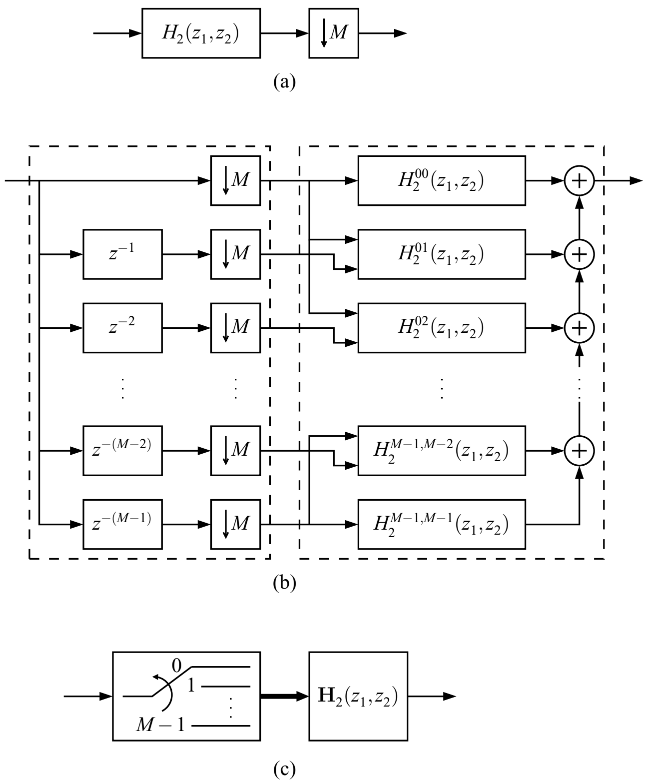

Figure 4: Polyphase implementation of a homogenous second-order Volterra kernel (downsampling case). (a) starting scheme. (b) corresponding polyphase implementation. (c) compact drawing for the two dashed boxes in (b).

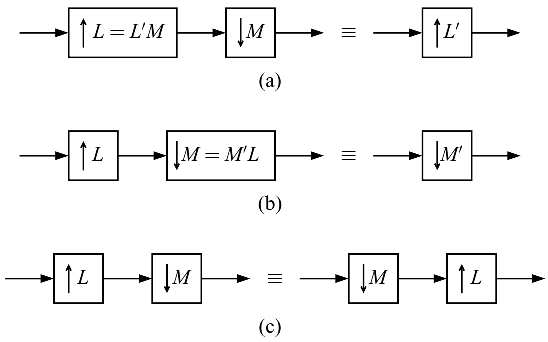

Figure 5: Series connection of an upsampler and a downsampler. (a) L is a multiple of M. (b) M is a multiple of L. (c) L and M are coprime.

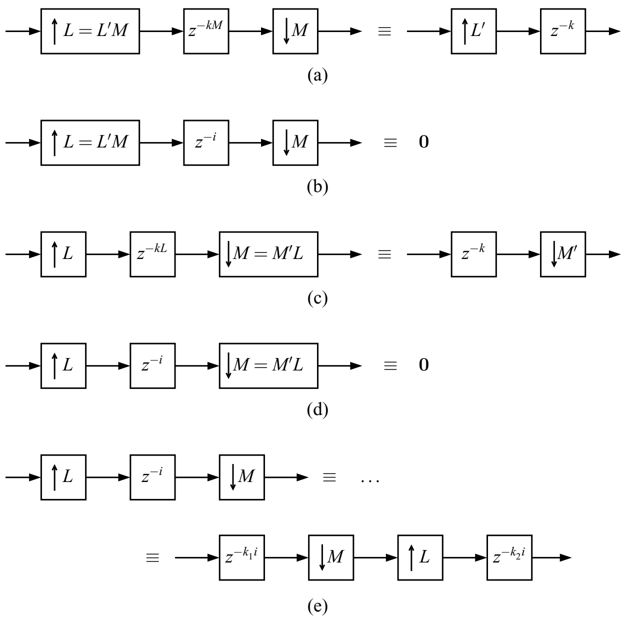

Figure 6: Series connection of an upsampler, a delay and a downsampler. (a) L is a multiple of M, $k \in \mathbf{Z}$. (b) L is a multiple of M, $i \neq kM$, $k \in \mathbf{Z}$. (c) M is a multiple of L, $k \in \mathbf{Z}$. (d) M is a multiple of L, $i \neq kL$, $k \in \mathbf{Z}$. (e) L and M are coprime, $i \neq kL$, $i \neq kM$, $k \in \mathbf{Z}$, k_1 and $k_2 \in \mathbf{Z}$.

rate. Applying the polyphase decomposition shown in Fig. 3 and the identities of Fig. 5(a) and Fig. 6(b) a much more efficient realization can be derived. Fig. 7 illustrates this procedure. Note that in the final realization (see Fig. 7(b)) only L' polyphase components occur. Furthermore, for the simple case where $L = M$ ($L' = 1$) only the first term $H_2^{00}(z_1, z_2)$ contributes to the output.

Case 2: M is a multiple of L ($M = M'L$)
Also in this case, for the derivation of a polyphase implementation of the scheme in Fig. 1 it is advantageous to interchange the upsampler with the nonlinear filter. This result has been shown before in Fig. 7(a). Now, we can use the identity in Fig. 6(d): All the branches including a delay can be cancelled because these delays are not mulitples of L. In the remaining first branch with the term $H_2^{00}(z_1, z_2)$ the up- and downsampler can be merged to one single downsampler with decimation factor M' (see Fig. 8(a)) using the identity in Fig. 5(b). Next, we can apply the identity of Fig. 4 to exchange the ordering of the downsampler and the nonlinear subfilter $H_2^{00}(z_1, z_2)$. Fig. 8(b) shows the final result. It should be noted, that all arithmetic operations are performed at the low output rate ($F_s^{out} = F_s^{in}/M'$).

Case 3: L and M are coprime and $L > M$ (fractional interpolation)
In this case, the simplifications necessary to minimize the CC are illustrated by the following example.

Example 1 *We consider a multirate system involving a homogenous second-order Volterra kernel as depicted in Fig. 9(a). The interpolation- and decimationfactors are $L = 3$ and $M = 2$, which correponds to a fractional interpolation. To minimize the computational efficiency 3 steps are applied:*

1. *Exchange the ordering of the upsampler and the nonlinear filter using the identity shown in Fig. 3. See Fig. 9(b) for the result after this step.*
2. *Move the downsampler into all parallel branches and apply the identities of Fig. 5(c) and Fig. 6(e). The result after this step is depicted in Fig. 9(c).*

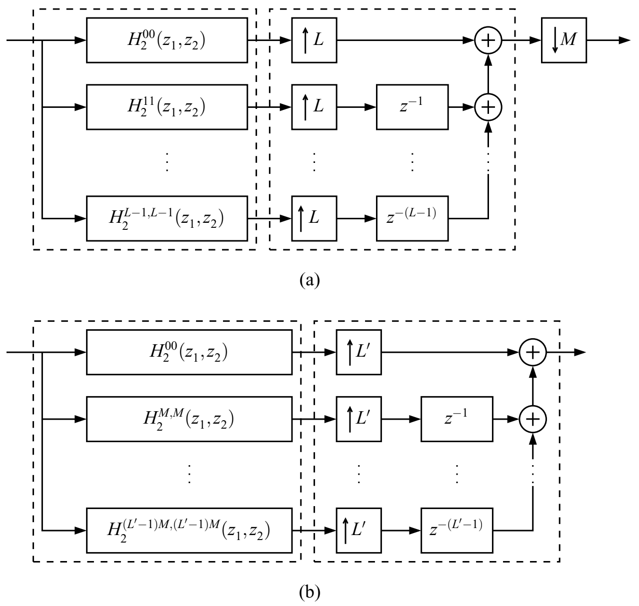

Figure 7: Polyphase realization for the multirate circuit shown in Fig. 1, L is a multiple of M. (a) intermediate result. (b) final result.

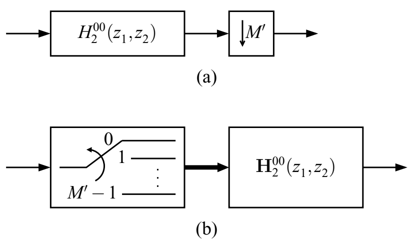

Figure 8: Polyphase realization for the multirate circuit shown in Fig. 1, M is a multiple of L. (a) intermediate result. (b) final result.

3. *Exchange the ordering of the downsampler and the nonlinear subfilters H_2^{ii} using the identity shown in Fig. 4. The Fig. 9(d) depicts the final result.*

Note that the system is not causal due to the z^1 and z^2. To avoid this causality problem the output must be delayed by 2 samples.

The fractional decimation case (L and M are coprime, $L < M$) can be treated in the same way. Therefore it will be omitted here.

4. SIMPLIFICATIONS FOR AN LNL FILTER

We consider now an LNL cascade between an upsampler and a downsampler as shown in Fig. 10. The LNL cascade consists of a linear filter, a memoryless nonlinerity and another linear filter. $H_l(z)$ and $H_r(z)$ are the corresponding system functions of the left and right linear filter, respectively. Such a configuration could be derived from Fig. 2 if we assume that the nonlinar behaviour of the linedriver can be modelled via a memoryless nonlinearity.

To reduce the CC of this filter structure we have to apply a polyphase representation to the linear filters. Considering that a memoryless nonlinearity and an upsampler/downsampler commute, further simplifications can be performed. The next example should illustrate this methology.

Example 2 *In Fig. 10 we select $L = 2$ and $M = 4$. The final representation with reduced CC can be derived in 3 steps and is shown in Fig. 11:*

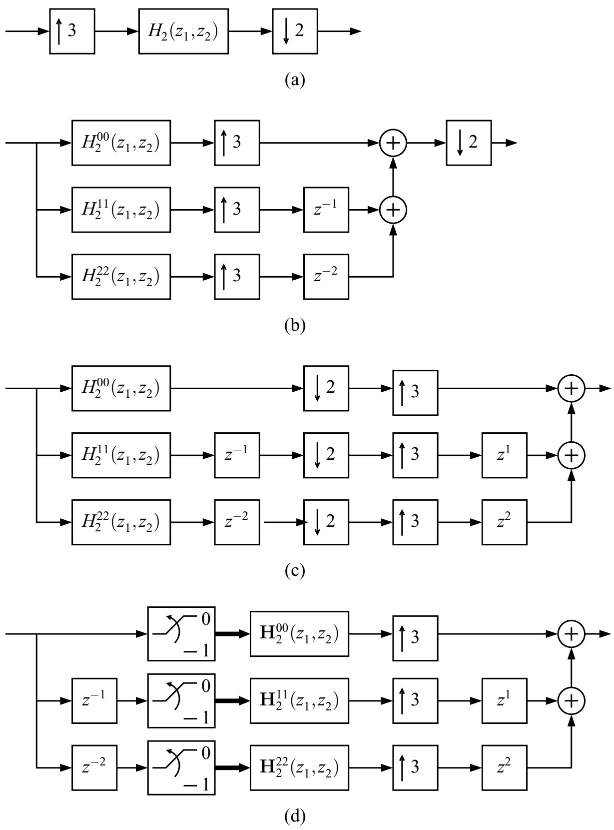

Figure 9: Polyphase realization of the multirate circuit of example 1. (a) starting scheme. (b)-(d) intermediate results. (d) final result.

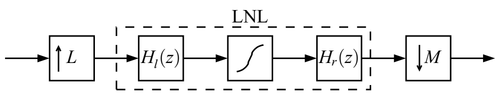

Figure 10: Nonlinear multirate system consisting of an upsampler, an LNL cascade and a downsampler.

1. *Apply a polyphase representation to the linear filters*
2. *Interchange the ordering of the memoryless nonlinearity and the upsampler*
3. *Combine up- and downsampler in proper branches using the identies in Fig. 5(b), Fig. 6(c) and Fig. 6(d)*

Note that the CC of the memoryless nonlinearity is not reduced. Instead of one linearity at the high rate $2F_s^{in}$ we have to compute the same nonlinearities twice at the input rate F_s^{in}. But the overall CC could be lowered due to the polyphase implementation of the linear filters.

Since the Hammerstein and Wiener model are special cases of the LNL cascade presented above, the corresponding polyphase representation can be easily derived if one of the linear filters is replaced by a through-connection.

5. CONCLUSION

Using linear and nonlinear polyphase decompositions new low-complexity equivalent representations for a series connection of an upsampler, a nonlinear filter and a downsampler are presented. For the derivation of these representations a homogenous second-order Volterra filter has been selected as an example but the results can be extended to higher-order kernels. Depending on the upsampling and downsampling factor different cases including the fractional decimation/interpolation case are discussed. Such configurations where the input sampling rate differs from the output sampling rate occur typically in echo paths of asymmetric data transmission systems. Further applications will be addressed in future investigations.

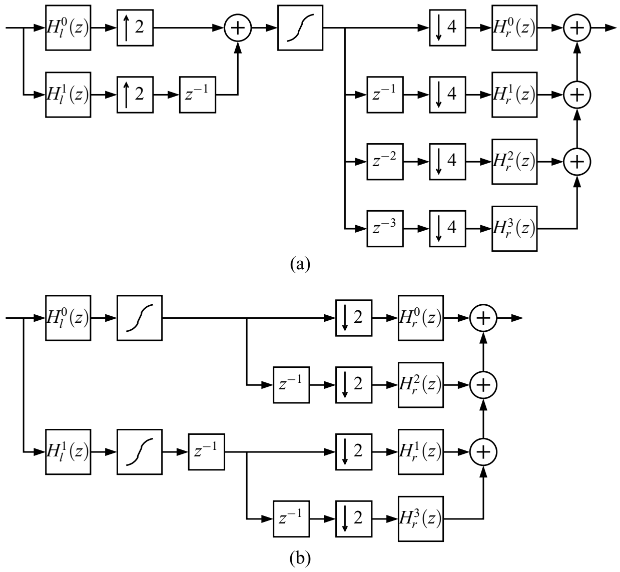

Figure 11: Polyphase realization for the multirate circuit shown in Fig. 10, $L = 2$, $M = 4$. (a) intermediate result (b) final result

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