

# ALGEBRAIC TECHNIQUES FOR THE BLIND DECONVOLUTION OF CONSTANT MODULUS SIGNALS

Lieven De Lathauwer

ETIS, UMR 8051 (CNRS, ENSEA, UCP)  
6, avenue du Ponceau, BP 44, F 95014 Cergy-Pontoise Cedex, France  
email: delathau@ensea.fr

## ABSTRACT

In this paper we derive algebraic algorithms for the blind extraction of Constant Modulus source signals from convolutive mixtures. In the single-channel case the equalizer follows from the best rank-1 approximation of a fourth-order tensor. In the multi-channel case we consider the case of paraunitary mixtures. The solution can then be obtained by means of a simultaneous matrix decomposition.

## 1. INTRODUCTION

In this paper we derive deterministic algorithms for the Single-Input Single-Output (SISO) and Multiple-Input Multiple-Output (MIMO) blind deconvolution of Constant Modulus (CM) signals. These techniques extend the Analytic Constant Modulus Algorithm (ACMA) [15], which applies to instantaneous mixtures.

The results are obtained from combining the derivation of ACMA with principles of subspace processing [11] and concepts of (multi)linear algebra [2, 5, 6, 7, 13].

We consider the following data model:

$$\tilde{Y}(n) = \sum_{l=0}^{L_s-1} \mathbf{A}(l)X(n-l),$$

in which  $X(n) \in \mathbb{C}^{K_1}$  are the unknown CM source signals,  $\tilde{Y}(n) \in \mathbb{C}^{K_2}$ , with  $K_2 \geq K_1$ , are the observed outputs and  $\mathbf{A}(l) \in \mathbb{C}^{K_2 \times K_1}$ ,  $l = 0, \dots, L_s - 1$  contain the unknown channel coefficients. We assume that it is possible to equalize the channel by means of a Finite Impulse Response (FIR) filter  $\mathbf{B}(z)$ :

$$\hat{X}(n) = \sum_{l=0}^{L_e-1} \mathbf{B}(l)\tilde{Y}(n-l), \quad (1)$$

in which  $\mathbf{B}(l) \in \mathbb{C}^{K_1 \times K_2}$ ,  $l = 0, \dots, L_e - 1$ . It is well-known that sources may only be recovered up to a permutation, phase shift and time delay.

In Section 2 we assume that channel and equalizer are paraunitary. This is the case when first a prewhitening has been carried out. In Section 3 we consider a SISO channel and equalizer. Section 4 is a small note on blind deconvolution in the case of oversampled data. In Section 5 the performance of our algorithms is illustrated by means of some simulations. Section 6 is the conclusion.

On-line algorithms for blind deconvolution in digital communications often need long data blocks to converge (typically from 10,000 to 100,000 symbols). Off-line algorithms exhibit much shorter convergence times. However, techniques that exploit the statistical independence of the source signals may still require substantial sample sizes for the estimation of (higher-order) statistics.

The author holds a permanent research position with the French Centre National de la Recherche Scientifique; he also holds a honorary research position with the K.U.Leuven, Belgium. Part of this research was supported by the Research Council K.U.Leuven (GOA-MEFISTO-666), the Flemish Government (F.W.O. project G.0240.99, F.W.O. Research Communities IC-CoS and ANMMM) and the Belgian Federal Government (IUAP V-22). The scientific responsibility is assumed by the author.

The algorithm presented in Section 2 only presupposes a whitening. The equalization of the paraunitary system is merely based on the algebraic structure induced by the CM property; higher-order statistics are not required. The SISO algorithm and the technique proposed in Section 4 are purely deterministic. The transmitted sequences do not have to be mutually statistically independent, nor independent identically distributed (i.i.d.). As a result, the algorithms can be used for small sample sizes (e.g. in the case of slow channel fading).

We use the following notation.  $\cdot^T$  denotes the transpose,  $\cdot^H$  the complex conjugate transpose,  $\cdot^*$  the complex conjugate.  $\otimes$  denotes the Kronecker product. Scalars are denoted by lower-case letters ( $a, b, \dots$ ), vectors are written as capitals ( $A, B, \dots$ ) (italic shaped), matrices correspond to bold-face capitals ( $\mathbf{A}, \mathbf{B}, \dots$ ) and higher-order tensors (multi-way arrays) are written as calligraphic letters ( $\mathcal{A}, \mathcal{B}, \dots$ ).  $K, L$  and  $N$  are reserved to denote the upper bounds of indices  $k, l$  and  $n$ .

## 2. PARAUNITARY FILTERS

In this section we assume that channel and equalizer are paraunitary due to a prewhitening. The channel transfer function can then be factorized as [14]

$$\mathbf{A}[z] = \mathbf{Q}_{L-1} \cdot \mathbf{Z}[z] \cdot \mathbf{Q}_{L-2} \cdot \dots \cdot \mathbf{Z}[z] \cdot \mathbf{Q}_0, \quad (2)$$

where  $\mathbf{Q}_l \in \mathbb{C}^{K \times K}$ ,  $l = 0, \dots, L-1$  are unitary and  $\mathbf{Z}[z]$  is  $(K \times K)$  diagonal

$$\mathbf{Z}[z] = \begin{pmatrix} \mathbf{I}_{K-1} & 0 \\ 0 & z^{-1} \end{pmatrix},$$

with  $\mathbf{I}_{K-1}$  the  $((K-1) \times (K-1))$  identity matrix.

A special property of paraunitary filters is that they can be equalized by an FIR filter of the same length. The equalizing filter is also paraunitary. For  $\mathbf{A}[z]$  given by Eq. (2), we obtain the equalizer

$$\mathbf{B}[z] = \mathbf{Q}_0^H \cdot \tilde{\mathbf{Z}}[z] \cdot \dots \cdot \mathbf{Q}_{L-2}^H \cdot \tilde{\mathbf{Z}}[z] \cdot \mathbf{Q}_{L-1}^H, \quad (3)$$

with

$$\tilde{\mathbf{Z}}[z] = \begin{pmatrix} z^{-1} \mathbf{I}_{K-1} & 0 \\ 0 & 1 \end{pmatrix}.$$

This equalizer is usually estimated from the higher-order statistics of  $\tilde{Y}(n)$ , by claiming that the source signals are mutually statistically independent [3, 4, 12]. Here we will derive a new technique based on the CM property of the sources.

Let  $\mathbf{B} \in \mathbb{C}^{K \times LK}$  be the concatenation of the matrices  $\{\mathbf{B}(l)\}$ . Due to the paraunitary constraint,  $\mathbf{B}$  is row-wise orthonormal [3]. Let us stack the vectors  $\tilde{Y}(n), \tilde{Y}(n-1), \dots, \tilde{Y}(n-L+1)$  in one big vector  $Y(n) \in \mathbb{C}^{LK}$ . Then Eq. (1) can be rewritten as

$$\hat{X}(n) = \mathbf{B}Y(n). \quad (4)$$

$\mathbf{B}$  should be determined in such a way that  $\hat{X}(n)$  has unit-modulus entries.

Initially, we may work as in [15]. Let  $B \in \mathbb{C}^{1 \times LK}$  be a row of  $\mathbf{B}$ . The signal

$$\hat{x}(n) = BY(n) \quad (5)$$

is unit-modulus if and only if

$$\begin{aligned} |\hat{x}(1)|^2 &= BY(1)Y(1)^H B^H = 1 \\ &\vdots \\ |\hat{x}(N)|^2 &= BY(N)Y(N)^H B^H = 1, \end{aligned} \quad (6)$$

in which  $N$  is the number of samples  $Y(n)$ . Eq. (6) can be rewritten as

$$\begin{pmatrix} Y(1)^T \otimes Y(1)^H \\ Y(2)^T \otimes Y(2)^H \\ \vdots \\ Y(N)^T \otimes Y(N)^H \end{pmatrix} (B^T \otimes B^H) = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}. \quad (7)$$

By multiplying both sides of Eq. (7) with a Householder or a discrete Fourier transform matrix, we obtain a set of equations of the form

$$\begin{pmatrix} \tilde{\mathbf{M}}_1 \\ \mathbf{M} \end{pmatrix} (B^T \otimes B^H) = \begin{pmatrix} \sqrt{N} \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad (8)$$

in which  $\tilde{\mathbf{M}}_1 \in \mathbb{C}^{1 \times (LK)^2}$  and  $\mathbf{M} \in \mathbb{C}^{(N-1) \times (LK)^2}$ . The first equation is only a normalization constraint. Every row  $B_k$  of  $\mathbf{B}$  leads to a vector in the kernel of  $\mathbf{M}$ . Generically, for  $N$  big enough, the kernel of  $\mathbf{M}$  is of dimension  $K$ . (Note that if  $L' > L$  vectors are stacked in  $Y(n)$ , the kernel is of dimension  $(L' - L + 1)K$  (number of possible time shifts induced by the equalizer  $\times$  number of source signals). In this way the equalizer length may be estimated.) Hence, the problem consists of (i) computing the kernel of  $\mathbf{M}$  (by means of an SVD), and (ii) looking for vectors in the kernel subspace that have a Kronecker structure.

The fact that the kernel of  $\mathbf{M}$  is spanned by the vectors  $\{B_k^T \otimes B_k^H\}$ , can be reformulated as

$$\begin{aligned} \mathbf{F}_1 &= \mathbf{B}^T \cdot \mathbf{D}_1 \cdot \mathbf{B}^* \\ &\vdots \\ \mathbf{F}_K &= \mathbf{B}^T \cdot \mathbf{D}_K \cdot \mathbf{B}^*, \end{aligned} \quad (9)$$

in which  $\mathbf{F}_1, \dots, \mathbf{F}_K$  are  $(LK \times LK)$  matrix representations of  $K$  basis vectors of the kernel, and in which the  $(K \times K)$  matrices  $\mathbf{D}_1, \dots, \mathbf{D}_K$  are diagonal. One can always choose kernel vectors that correspond to real-valued  $\mathbf{D}_k$ , which leads to Hermitean matrices  $\mathbf{F}_k$ . Consider a third-order tensor  $\mathcal{F} \in \mathbb{C}^{LR \times LR \times K}$ , in which the matrices  $\mathbf{F}_1, \dots, \mathbf{F}_K$  are stacked. Eq. (9) represents a Canonical Decomposition of  $\mathcal{F}$ , with orthonormality constraints on  $\mathbf{B}$  [7, 13].  $\mathbf{B}$  may be found from this decomposition.

Now we will derive one particular computation scheme. We can reduce the dimensionality of the problem by noticing that  $\mathbf{B}^T$  spans the column space of every matrix  $\mathbf{F}_k$ . Hence, in the absence of noise, the column space of  $\mathbf{B}^T$  can be determined as the subspace corresponding to the  $K$  dominant left singular vectors of the matrix  $\mathbf{F} = [\mathbf{F}_1 \dots \mathbf{F}_K]$ . In the presence of noise, this approach is (slightly) suboptimal. A technique for the computation of the best subspace in least-squares sense is explained in [5, 6]. Let the columns of  $\mathbf{U} \in \mathbb{C}^{LK \times K}$  form an orthonormal basis of this subspace. Then the remaining problem is of the form

$$\begin{aligned} \mathbf{G}_1 &= \mathbf{E} \cdot \mathbf{D}_1 \cdot \mathbf{E}^H \\ &\vdots \\ \mathbf{G}_K &= \mathbf{E} \cdot \mathbf{D}_K \cdot \mathbf{E}^H, \end{aligned} \quad (10)$$

in which  $\mathbf{G}_k \in \mathbb{C}^{K \times K}$ ,  $1 \leq k \leq K$ , are defined by

$$\mathbf{G}_k = \mathbf{U}^H \cdot \mathbf{F}_k \cdot \mathbf{U}$$

and in which the unknown factor  $\mathbf{E} \in \mathbb{C}^{K \times K}$ , equal to  $\mathbf{U}^H \cdot \mathbf{B}^T$ , is unitary.  $\mathbf{E}$  may very efficiently be computed from (10) by means of the JADE algorithm [2].

Let us associate to  $\mathbf{U}^T$  a filter  $\mathbf{U}^T(z)$  of which the matrix coefficients are the subsequent  $(K \times K)$  submatrices of  $\mathbf{U}^T$ . Note that the computation of  $\mathbf{U}$  in fact reduced the equalization problem to the blind separation of an instantaneous linear mixture: after passing  $\hat{Y}(n)$  through  $\mathbf{U}^T(z)$ , the remaining mixture is instantaneous since we have  $\mathbf{B} = \mathbf{E}^T \cdot \mathbf{U}^T$ .

### 3. THE SISO CASE

Contrary to paraunitary channels, SISO FIR channels cannot be exactly equalized by means of an FIR equalizer. In this section we will look for the optimal equalizer of a given length  $L$ .

Let  $B \in \mathbb{C}^{1 \times L}$  represent an (approximate) equalizer of a channel with transfer function  $a(z)$ . Subsequent observations are stacked in a vector  $Y(n) \in \mathbb{C}^L$ . We compute a matrix  $\mathbf{M}$  in the same way as in the previous section. We now have to determine the minimum of the cost function

$$f(B) = \|\mathbf{M} \cdot (B^T \otimes B^H)\|^2, \quad (11)$$

under the normalization constraint  $\|B\| = 1$ . Let the singular values of  $\mathbf{M}$  be given by  $\sigma_p$  and let the right singular vectors be stacked in  $(L \times L)$  matrices  $\mathbf{V}_p$ ,  $1 \leq p \leq L^2$ . Then  $f$  is given by

$$f(B) = \sum_{ijkl} \sum_{p=1}^{L^2} \sigma_p^2 (\mathbf{V}_p)_{ij} (\mathbf{V}_p)_{kl}^* b_i b_j^* b_k^* b_l. \quad (12)$$

Minimization of this cost function is equivalent to maximization of

$$g(B) = \sigma_1^2 - \sum_{ijkl} \sum_{p=1}^{L^2} \sigma_p^2 (\mathbf{V}_p)_{ij} (\mathbf{V}_p)_{kl}^* b_i b_j^* b_k^* b_l. \quad (13)$$

This function is nonnegative because  $B$  is unit-norm and  $\{\mathbf{V}_p\}$  correspond to orthonormal vectors. Let  $\mathcal{T} \in \mathbb{C}^{L \times L \times L \times L}$  be the supersymmetric tensor with entries

$$t_{ijkl} = \sigma_1^2 \delta_{ik} \delta_{jl} - \sum_{p=1}^{L^2} \sigma_p^2 (\mathbf{V}_p)_{ij} (\mathbf{V}_p)_{kl}^*,$$

in which  $\delta_{rs}$  is the Kronecker delta ( $\delta_{rs} = 1$  if  $r = s$  and 0 otherwise). Then we have

$$g(B) = \sum_{ijkl} t_{ijkl} b_i b_j^* b_k^* b_l. \quad (14)$$

This means that maximization of  $g(B)$  corresponds to finding the best supersymmetric rank-1 approximation of  $\mathcal{T}$ . Indeed, if we want to minimize

$$\begin{aligned} h(\lambda, B) &= \|\mathcal{T} - \lambda B^T \circ B^H \circ B^H \circ B^T\|^2 \\ &= \sum_{ijkl} |t_{ijkl} - \lambda b_i b_j^* b_k^* b_l|^2 \end{aligned}$$

over  $\lambda \in \mathbb{R}$  and unit-norm  $B$ , then the optimum corresponds to the optimal equalizer and  $\lambda_{\text{opt}}$  is the global maximum of  $g$ . We refer to [5, 9, 19] for background and algorithms.

#### 4. OVERSAMPLING

A common way to ensure that an FIR equalizer exists, is temporal and/or spatial oversampling [11, 16]. Let  $K_2$  be the product of the number of antennas and the temporal oversampling rate. Then we suppose that  $K_2 > K_1$ .

A difficulty is that equalizers are not unique, like in Section 2. Let  $\alpha$  be the smallest integer such that  $K_2(\alpha + 1) \geq (L_s + \alpha)K_1$ . Then  $L_s + \alpha$  equalizers may be obtained for each source signal, each containing  $K_2(\alpha + 1)$  coefficients. These equalizers result in different time shifts. In [17] this constraint is combined with the CM constraint. This leads to a set of  $K_1$  matrices  $\mathbf{F}_1, \dots, \mathbf{F}_{K_1}$  of dimension  $(K_2(\alpha + 1)(L_s + \alpha) \times K_2(\alpha + 1)(L_s + \alpha))$ , satisfying

$$\begin{aligned} \mathbf{F}_1 &= \mathbf{B}^T \cdot \mathbf{D}_1 \cdot \mathbf{B}^* \\ &\vdots \\ \mathbf{F}_{K_1} &= \mathbf{B}^T \cdot \mathbf{D}_{K_1} \cdot \mathbf{B}^*, \end{aligned} \quad (15)$$

in which  $\{\mathbf{D}_k\}$  are  $(K_1 \times K_1)$  real diagonal and in which  $\mathbf{B} \in \mathbb{C}^{K_1 \times K_2(\alpha+1)(L_s+\alpha)}$ . In a row of  $\mathbf{B}$  the different equalizers for one source are stacked. Eq. (15) can again be considered as the Canonical Decomposition of the third-order tensor in which the matrices  $\{\mathbf{F}_k\}$  are stacked. The difference with Eq. (9) is that now matrix  $\mathbf{B}$  is not subject to orthonormality constraints.

To reduce the computational complexity, we propose to work in analogy with Section 2. First we compute the  $K_1$ -dimensional subspace that best spans the column spaces of  $\mathbf{F}_1, \dots, \mathbf{F}_{K_1}$  [5, 6]. Let the columns of  $\mathbf{U} \in \mathbb{C}^{K_2(\alpha+1)(L_s+\alpha) \times K_1}$  form an orthonormal basis of this subspace. The problem then reduces to

$$\begin{aligned} \mathbf{G}_1 &= \mathbf{E} \cdot \mathbf{D}_1 \cdot \mathbf{E}^H \\ &\vdots \\ \mathbf{G}_{K_1} &= \mathbf{E} \cdot \mathbf{D}_{K_1} \cdot \mathbf{E}^H, \end{aligned} \quad (16)$$

in which  $\mathbf{G}_k$ ,  $1 \leq k \leq K_1$ , defined by

$$\mathbf{G}_k = \mathbf{U}^H \cdot \mathbf{F}_k \cdot \mathbf{U},$$

are only of dimension  $(K_1 \times K_1)$ . The unknown factor  $\mathbf{E} \in \mathbb{C}^{K_1 \times K_1}$ , equal to  $\mathbf{U}^H \cdot \mathbf{B}^T$ , is now non-unitary. Algorithms for non-unitary simultaneous diagonalization may be found in [7, 13, 15, 18] and the references therein.

#### 5. SIMULATIONS

A first simulation illustrates the technique derived in Section 2, however with one modification. Practice shows that, in the presence of noise, the right singular vectors of the matrix  $\mathbf{M}$  in Eq. (8), corresponding to the smallest singular values, occasionally yield multiple equalizers for the same source. These equalizers correspond to different time shifts. In such a case, we increase the number of singular vectors that are retained. Although the matrix  $\mathbf{B}$  in Eq. (9) is now not row-wise orthonormal anymore, we may still proceed as in Section 2. The only difference is that the factor  $\mathbf{E}$  in Eq. (10) is square non-unitary, like in Section 4.

In our simulation, channel and equalizer are  $(2 \times 2)$  paraunitary of length  $L = 4$ . The matrices  $\mathbf{Q}_l$  in Eq. (2) are of the form

$$\mathbf{Q}_l = \begin{pmatrix} \cos \theta_l & \sin \theta_l e^{j\phi_l} \\ -\sin \theta_l e^{-j\phi_l} & \cos \theta_l \end{pmatrix},$$

in which the parameters  $\theta_l$  and  $\phi_l$ , for  $0 \leq l \leq L - 1$ , are drawn from a uniform distribution over  $[0, 2\pi)$ . Sources are QAM4. A data block consists of 200 samples. The channel outputs are contaminated by i.i.d. zero-mean Gaussian noise of variance  $\sigma_N^2$ . The experiment consists of 300 Monte Carlo runs. In each run, new realizations of channel, sources and noise are generated. The average

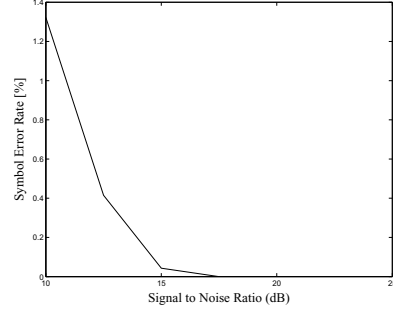


Figure 1: SER as a function of SNR in the first experiment.

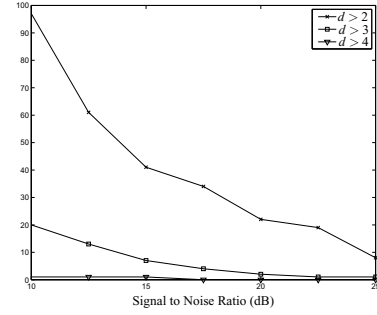


Figure 2: Number of cases in which a kernel of dimension  $d > 2$  was used (first experiment).

Symbol Error Rate (SER) is plotted as a function of the Signal-to-Noise Ratio (SNR) in Fig. 1. All symbols were estimated correctly when the SNR  $\geq 17.5$  dB. Fig. 2 shows in how many cases an extended kernel was used.

A second simulation concerns the SISO case. The channel is taken equal to an FIR filter of length  $L_s = 4$ , with zeros  $0.1 - 0.2i$ ,  $0.3 + 0.1i$  and  $0.1 - 0.4i$ . We try to equalize this channel by means of an FIR filter of length  $L_e = 2$ . Like in the previous simulation, sources are QAM4, data blocks consist of 200 samples and 300 Monte Carlo runs are carried out. In each run, new realizations of sources and noise are generated. For the best rank-1 approximation we used the algorithm described in [5, §3.3], initialized with the starting value proposed in [9, §6]. The average SER is plotted as a function of the SNR in Fig. 3.

Next, we consider the computation scheme proposed in Section 4. We consider the case where 2 source signals are mixed by a channel of length  $L_s = 2$ . Furthermore,  $K_2 = 4$ . The channel coefficients are drawn from a unit-variance zero-mean complex Gaussian distribution. Sources are QAM4. A data block consists of only

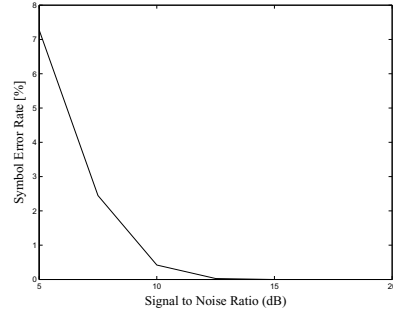


Figure 3: SER as a function of SNR in the second experiment.

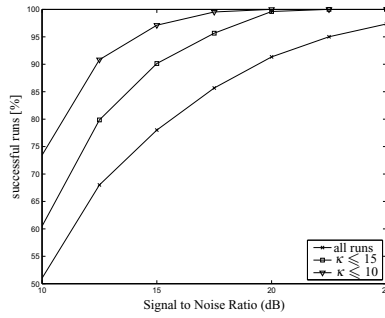


Figure 4: Percentage of successful runs as a function of SNR in the third experiment.

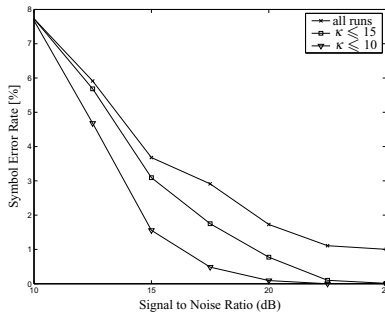


Figure 5: SER as a function of SNR in the third experiment.

100 samples. The channel outputs are contaminated by i.i.d. zero-mean Gaussian noise of variance  $\sigma_N^2$ . The experiment consists of 300 Monte Carlo runs. In this case, the matrices  $\{\mathbf{F}_k\}$  are of dimension  $(8 \times 8)$ . On the other hand, the matrices  $\{\mathbf{G}_k\}$  are only of dimension  $(2 \times 2)$  and the solution may be obtained by a simple Eigenvalue Decomposition of a  $(2 \times 2)$  matrix [10]. For the computation of  $\{\mathbf{F}_k\}$ , we used the third algorithm proposed in [17].

The estimation of the equalizer was considered as a failure if the SER was over 30 %. The percentage of successful runs, as a function of the SNR, is plotted in Fig. 4. The corresponding SER values are shown in Fig. 5.

Of course, when the channel is ill conditioned, this makes the equalization task much more difficult. In our simulation, the condition number  $\kappa$  varied between 2.3 and 64. Figs. 4 and 5 show curves for the overall average, for  $\kappa \leq 15$  (253 runs) and for  $\kappa \leq 10$  (207 runs).

## 6. CONCLUSION

In this paper we have derived algebraic algorithms for the blind deconvolution of CM sources. Data blocks may be short. The techniques can also be used to extract CM sources from mixtures that also contain non-CM components.

## REFERENCES

- [1] J. Benesty and P. Duhamel, "A fast constant modulus adaptive algorithm," *IEE Proc. F*, vol. 138, pp. 379–387, April 1991.
- [2] J.-F. Cardoso, A. Souloumiac, "Blind beamforming for non-Gaussian signals," *IEE Proc.-F*, vol. 140, pp. 362–370, 1994.
- [3] P. Comon, L. Rota, "Blind separation of independent sources from convolutive mixtures," *IEICE Trans. on Fundamentals of Elec. Com. Comput. Sciences*, vol. E86-A(3):pp. 550–563, March 2003.
- [4] L. De Lathauwer, B. De Moor, J. Vandewalle, "An algebraic approach to the blind identification of paraunitary filters," in

*Proc. IEEE Wireless Communications and Networking Conference*, Chicago, USA, Sept. 2000.

- [5] L. De Lathauwer, B. De Moor, J. Vandewalle, "On the best rank-1 and rank- $(R_1, R_2, \dots, R_N)$  approximation of higher-order tensors," *SIAM J. Matrix Anal. Appl.*, vol. 21 (4), pp. 1324–1342, April 2000.
- [6] L. De Lathauwer, J. Vandewalle, "Dimensionality reduction in higher-order signal processing and rank- $(R_1, R_2, \dots, R_N)$  reduction in multilinear algebra," *Lin. Alg. Appl.*, to appear.
- [7] L. De Lathauwer, B. De Moor, J. Vandewalle, "Computation of the canonical decomposition by means of a simultaneous generalized Schur decomposition," *SIAM J. Matrix Anal. Appl.*, to appear.
- [8] I. Fijalkow, A. Touzni, J. R. Treichler, "Fractionally spaced equalization using CMA: robustness to channel noise and lack of disparity," *IEEE Trans. on Signal Processing*, vol. 45 (1), pp. 56–66, Jan. 1997.
- [9] E. Kofidis, P. A. Regalia, "On the best rank-1 approximation of higher-order super-symmetric tensors," *SIAM J. Matrix Anal. Appl.*, vol. 23 (3), pp. 863–884, 2002.
- [10] S.E. Leurgans, R.T. Ross, R.B. Abel, "A decomposition for three-way arrays," *SIAM J. Matrix Anal. Appl.*, vol. 14, pp. 1064–1083, 1993.
- [11] E. Moulines, P. Duhamel, J.-F. Cardoso, S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," *IEEE Trans. on Signal Processing*, vol. 43 (2), pp. 516–525, Feb. 1995.
- [12] L. Rota, P. Comon, S. Icart, "Blind equalization of MIMO channels," in *Proc. IEEE Workshop on Signal Processing Advances in Wireless Communications*, Rome, Italy, June 2003.
- [13] N. Sidiropoulos, G. Giannakis, R. Bro, "Blind PARAFAC receivers for DS-CDMA systems," *IEEE Trans. on Signal Processing*, vol. 48, pp. 810–823, 2000.
- [14] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*. London: Prentice-Hall, 1993.
- [15] A.J. van der Veen, A. Paulraj, "An analytical constant modulus algorithm," *IEEE Trans. Signal Processing*, vol. 44 (5), pp. 1136–1155, May 1996.
- [16] A.J. van der Veen, S. Talwar, A. Paulraj, "A subspace approach to blind space-time signal processing for wireless communication systems," *IEEE Trans. Signal Processing*, vol. 45, pp. 173–190, Jan. 1997.
- [17] A.J. van der Veen, A. Trindade, "Combining blind equalization with constant modulus properties," in *Proc. Asilomar Conf. on Signals, Systems, and Computers*, Pacific Grove (CA), Oct. 2000.
- [18] A.J. van der Veen, "Joint diagonalization via subspace fitting techniques," in *Proc. IEEE ICASSP*, Salt Lake City (UT), May 2001.
- [19] T. Zhang, G. Golub, "Rank-one approximation to high order tensors," *SIAM J. Matrix Anal. Appl.*, vol. 23, pp. 534–550, 2001.