# ROBUST DETECTION OF QRS COMPLEX USING KLAUDER WAVELETS

Philippe Ravier<sup>1</sup>, Olivier Buttelli<sup>2</sup>

Laboratoire d'Électronique, Signaux, Images, Orléans University, 12, rue de Blois, BP 6744 Orléans Cedex 2, France phone: +33(0)28494863, fax: +33(0)238417245, email: Philippe.Ravier@univ-orleans.fr

Laboratoire de la Performance Motrice, AMCO group, Orléans University, rue de Vendôme, BP 6237 Orléans Cedex 2, France phone: +33(0)28494863, fax: +33(0)238417260, email: Olivier.Butelli@univ-orleans.fr

# **ABSTRACT**

The level of performance as well as the timing accuracy in the detection of QRS complex may be crucial for further nonlinear biomedical signal processing applied on the derived RR time series. By studying the fatigue process during exercise, a highly non-stationary noise is expected as well as time-varying QRS complex morphology. In difficult recording conditions, the QRS complex detectors have to be extremely robust but also have to meet high performance requirements. To achieve this aim, a "smoothed" matched filtering is realized on specific wavelet coefficient patterns in the time-scale plane. The detection is enhanced using the family of Klauder wavelets which demonstrate similarities to ECG waveforms. The algorithm only needs to capture a correct QRS complex snapshot before launching the detection.

#### 1. CONTEXT OF DETECTION

In recent years, many devices have been proposed for real time detection of the R-R interval in the electrocardiogram (ECG). When recording conditions are too difficult, the detection of the typical QRS complexes in ECG signals becomes unreliable and the instrument fails to give correct R-R sequences for further analysis. This is particularly true when the experimental protocols are related to dynamic physical exercises and when recording conditions are realized during intensive sport practice in the open air. Moreover, the most difficult recording conditions often coincide with stressful physical situations, just when the data could be very useful for monitoring and for making relevant medical interpretations. Evaluation of physical fatigue can be quantified through Heart Rate Variability (HRV) which supposes a good timing accuracy in QRS detection.

Many real-time QRS detectors have been realized using the prefiltering/matched filter/thresholding detection scheme [1,2,3]. However, these operations are subject to the time varying morphology of the QRS complex in ECG signals.

The best results in the QRS detection are obtained with a neural networks approach [4]. The technique is constraining and very difficult to implement because of a long patient dependent learning phase. From a practical point of view, indicators must be rapidly delivered, if possible at a real-time rate during the experiments. The learning phase must then be prohibited whereas the detector must be adapted to

any kind of noise and any morphological change in the cardiac electrical waveforms.

In this paper, we consider these difficulties by enhancing both performance and robustness.

- Performance in a detection problem essentially depends on the amount and quality of the a priori information that is injected in the construction of the detector. In the present case, QRS waveforms are quite well known which means that the concept of matched filtering must be preserved to get optimal performance. Since ECG waveforms may vary considerably depending on the patient, the kind of experiment and the experimental conditions, it is therefore necessary to capture a "QRS signature" at the beginning of each recording from which the detection will be realized.
- Considering a QRS model brings performance to the detriment of robustness. We achieve robustness by making the concept of matched filtering more flexible, i.e. able to deal with the QRS complex time-varying morphology. The method used is based on the Frisch & Messer [5] approach.
- We propose to build the detector after linear transform, as described in [5,6]. Wavelet transform is a particular class of linear transform that allows the bad noise<sup>1</sup> properties to be locally attenuated. This point is important to fit with the constraining noise hypotheses imposed by the matched filter.

# 2. WAVELETS FOR DETECTION

For many years now, wavelets have proved their efficiency in various detection problems [7]. In QRS detection, the authors essentially used the multiscale feature of wavelet transform to distinguish QRS complex from high P or T waves, noise, baseline drift and artifacts [8, 9]. Detectors are based on the Mallat and Hwang's approach [10] where the idea is to measure a degree of singularity and regularity to detect and classify the peaks. The singularity degree of a signal can be measured through its wavelet coefficients producing localized maxima across several consecutive scales when a peak is present and the R-peak validity is estimated from the decay of the wavelet coefficients giving a regular-

<sup>&</sup>lt;sup>1</sup> The noise is not permanent and is highly nonstationary. The nature of the noise varies: impulsive, wide band, spectral line, baseline drift, abrupt dc change...

ity measure. In practice, detectors are made data-dependant because empirical rules as well as many experimental thresholds are used. Signals are decomposed on dyadic grids that are not flexible since the grid points are predetermined in the time-scale plane. In this work, we propose to calculate the time-scale grid adapting it to the time frequency QRS characteristics. We will show that the QRS time frequency characteristics will permit to choose the mother wavelet and also induce the time-scale discretization.

#### 2.1 Choice of the wavelet

For a detection purpose, the key point is to consider a wavelet which looks as "similar" as possible to the signal to be detected and if possible orthogonal to any kind of disturbance. The idea behind this natural choice is that the information will be highly concentrated thus carried by few large wavelet coefficients allowing an easy discrimination with the noise coefficients. The latter will spread over the entire time-scale plane. Finally, the ECG waveform will be coded with only a few strong wavelet coefficients, permitting the medical signal to be distinguished from the ground. To that aim, we chose the Klauder complex wavelet whose real and imaginary parts closely resemble an ECG waveform (see Fig 1).

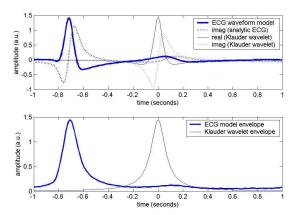


Figure 1 ECG waveform and its associated Klauder wavelet (for r=0.07, m=0.9) - (Top) Real and Imaginary parts - (Bottom) Envelope of the waveforms. This panel shows the similarities between the two waves.

Note that the similarity is enforced by a possible asymmetry of the Klauder waveform.

### 2.2 The family of Klauder wavelets

# 2.2.1 Definition

The family of Klauder wavelets is defined according to the pair of parameters  $r \in \Box^+$ ,  $m \in [-1/2; +\infty[$  as [11]:

$$\psi^{r,m}(t) = \frac{(2r)^{m+1/2} \Gamma(m+1)}{\sqrt{2\pi \Gamma(2m+1)}} \frac{1}{(r-it)^{m+1}}$$

where 
$$\Gamma(m) = (m-1)! = \int_{0}^{+\infty} t^{m-1} e^{-t} dt$$
 with  $m > 0$ .

The Fourier Transform of the Klauder wavelets has the algebraic expression:

$$\Psi^{r,m}(f) = \frac{(2r)^{m+1/2}\sqrt{2\pi}}{\sqrt{\Gamma(2m+1)}} (2\pi f)^m e^{-2\pi r f} U(f)$$

where U(f) stands for the unitary step function.

Note that  $\Psi^{r,m}(0) = 0 = \int \psi^{r,m}(t) dt$  which says that

 $\psi^{r,m}(t)$  is an admissible wavelet. This means that the original signal may be errorless reconstructed from the continuous wavelet coefficients<sup>2</sup>. The energy is preserved when transforming the temporal domain to the wavelet coefficients domain. This property makes it possible to reconstruct any time-scale area of the time-scale plane in the temporal domain for filtering purposes.

# 2.2.2 Time and frequency statistics

With the above definition, the Klauder wavelets have unitary energy both in time or frequency, relative to the  $L^2$  norm. Mean values and uncertainties can be calculated. The mean value of the function F(x) is classically obtained by:

$$x_0 = \frac{\int\limits_0^{+\infty} x |F(x)|^2 dx}{\|F(x)\|}$$

The uncertainty of the function F(x) is calculated according to:

$$Dx = \frac{\left\| \left( x - x_0 \right) F(x) \right\|}{\left\| F(x) \right\|}$$

The mean frequency reads  $f_0(r,m) = \frac{2m+1}{4\pi r}$  Hz.

The frequency uncertainty reads  $Df_{(r,m)} = \frac{\sqrt{2m+1}}{4\pi r}$  and

the temporal uncertainty reads  $Dt_{(r,m)} = \frac{r}{\sqrt{2m-1}}$ . Both

parameters r and m influence the central frequency as well as the time and frequency uncertainties. Actually, the parameter m is directly linked to the number of oscillations in the envelope (this is easily verified by evaluating the quality factor

$$Q = \frac{f_0}{D_f}$$
 that only depends on  $m$ ).

Note that  $Dt_{(r,m)}Df_{(r,m)} = \frac{1}{4\pi}\sqrt{\frac{2m+1}{2m-1}}$ . This means

that the Gabor-Heisenberg lowest time-frequency uncertainty

<sup>&</sup>lt;sup>2</sup> On the contrary, another well known family of complex wavelets – the Morlet wavelets, as defined in [14] - which are parameterized by the center frequency and frequency bandwidth do not respect this property.

bound  $1/4\pi$  is rapidly attained with the wavelet order. The Klauder wavelets are thus very well localized in time and frequency, bringing the advantage that wavelet coefficients are able to support most of the information concentrated in the smallest area around their position in the discretized time-scale plane.

Finally, a simple procedure for choosing the appropriate mother wavelet consists in selecting the parameters r and m that equalize the mean frequency and the time and frequency uncertainties between an ECG pattern and the desired wavelet. This procedure supplies in a simple way a means to automatically choose a wavelet which evolves like the ECG pattern.

Other parameterized wavelets [12] have been tested, for which we calculated  $f_0$  and  $D_f$  analytically (table II).

Table II Relations between the wavelet parameters  $f_c$ ;  $f_b$  and the  $f_0$ ;  $D_f$  frequency characteristics.

Wavelet	Klauder	Morlet	B-spline 1	B-spline 2
$f_0$	$\frac{2m+1}{4\pi r}$	$f_c$	$f_c$	$f_c$
$D_f$	$\frac{\sqrt{2m+1}}{4\pi r}$	$rac{1}{2\pi\sqrt{f_b}}$	$\frac{f_b}{\sqrt{12}}$	$\frac{f_b}{\sqrt{40}}$

Table III gives a quantitative comparison, in a mean square fit error sense, between real QRS waveforms and the matched wavelet.

Table III Mean square fit error between the QRS waveform and the matched wavelet, for different wavelets (average over the whole QRS snapshots of record #100 in the MIT database [13] - 2273 waves)

Wavelet	Klauder	Morlet	B-spline 1	B-spline 2
L <sup>2</sup> norm	0.28	0.41	0.33	0.40

## 3. THE PROCEDURE OF DETECTION

In order to take into account time-frequency uncertainties, the comparison is realized through a set of 4 neighbored wavelet coefficients, as proposed in [5], see Fig. 2. The idea is to cover the wavelet uncertainty area with a 4-point pattern of the sampling time-scale plane. The uncertainty area can completely be determined through the theoretical time and frequency uncertainties of the wavelet. The 4-point pattern is chosen to introduce some redundancy both in the time and scale directions and to limit the computation cost. Considering the continuous wavelet transform with discrete coefficients as

$$WT_x(j,n) = \int_{-\infty}^{+\infty} x(t) a_0^{\frac{j}{2}} \psi(a_0^{-j}t - nb_0)^* dt$$
, it is therefore possible to take  $D_t = 3b_0$  and  $D_f = 2(a_0 - 1)f_0$ 

such that the elementary pattern covers the wavelet time-frequency uncertainty (i.e. with a little redundancy in time and scale). Parameters  $a_0$  and  $b_0$  are thus automatically deduced from the estimated  $f_0$ .

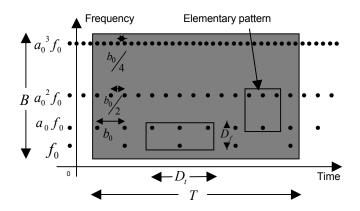


Figure 2 Time-scale grid, D domain and elementary pattern.

However, the QRS morphological variability has the effect of making the QRS pattern evolve slightly in a localized area along the time and frequency axis. These variations are solved by the Generalized Likelihood Ratio Test (GLRT) which consists of searching the maximum LRT over a time-frequency area to be defined. More precisely, defining a 4-coefficients pattern moving in the t/f plane, assumes a constant quality factor Q which is intrinsic in the wavelet transform construction (this is the effect of the scales structure). Cardiac waveform cycles always show a constant number of oscillations due to polarization mechanisms, except for pathological cases. This corresponds to a constant quality factor which is coherent with a wavelet behavior.

Applying the GLRT, the detector finally takes the maximum of the LRT when the pattern comparison is swept over a determined BT plane (Fig. 2). The bandwidth B and duration T of searching is determined such that the energy concentrated in the elementary pattern is uniformly distributed in this rectangular area ( $B = \sqrt{12}D_f$  and  $T = \sqrt{24}D_t$  according to [5]).

Finally, performance is obtained by matched filtering in the time-scale plane. Robustness is guaranteed by the GLRT. Note  $\mathbf{x}$ ,  $\mathbf{s}$  and  $\mathbf{\eta}$  the 4-wavelet coefficients elementary pattern vectors respectively of the signal, the model and the noise. Considering a possible set of K models  $S_k$ , the GLRT writes:

$$\Lambda_{GLRT}(x) = \max_{k=1}^{K} \left[ \max_{j=1}^{J} (\mathbf{x}_{j}^{H} \mathbf{s}_{j,k} + \mathbf{s}_{j,k}^{H} \mathbf{x}_{j} - \mathbf{s}_{j,k}^{H} \mathbf{s}_{j,k}) \right]$$

J stands for the number of elementary patterns in the domain BT

In practice, it is difficult to consider that the noise is white and Gaussian. Noise distribution and spectra should be estimated. Models of noise sources can be used but their statistical properties may be very different making it difficult to unify the noises in common and realistic distributions. However, this point is not so crucial since these bad properties are mitigated thanks to the effects of the wavelet transform. The wavelet transform tends to make the input samples Gaussian and to decorrelate them according to the degree of smoothness of the chosen wavelet.

### 4. RESULTS

We used the MIT/BIH arrhythmia database [13] to evaluate our detector. The database consists of 48 thirty-minute records. Medical annotations give reference for statistical detection performance evaluations.

We tested our algorithm on the first 10 free available records. Comparing the obtained results (Table II) with other algorithms [1,2,8] tested on the same database proves the efficiency of our method. On this beginning of the database, the algorithm produces 99.68 % of good detection.

No particular rules for the detection have been investigated, contrary to [8] where many rules have been developed, especially fitting the studied database. We only apply a 2 Hz high pass filter to remove the dc component on the raw signals and consider a refractory physiological delay of 200 ms after detection.

Note the rather good results obtained for the record #105 which is considerably noisy. Only the neural-network-based supervised method achieves better results. This method however requires a constraining learning phase.

Table II Results of the Klauder wavelet based QRS detection algorithm for the MIT/BIH database

Tape (#)	Total (beats)	FP (beats)	FN (beats)	Failed detection	Failed detection
				(FP+FN)	(%)
100	2273	0	0	0	0
101	1865	2	2	4	0.22
102	2187	0	1	1	0.05
103	2084	0	0	0	0
104	2229	2	10	12	0.54
105	2572	39	12	51	1.98
106	2027	0	1	1	0.05
107	2137	0	1	1	0.05
118	2278	0	0	0	0
119	1987	0	0	0	0
TOTAL	21639	43	27	70	0.32

## 5. CONCLUSION AND PERSPECTIVE

We have introduced a new QRS waveform detector. The detector guarantees performance and robustness. Performance is obtained thanks to an ECG pattern that gives the shape of the matched filter. Robustness is obtained by a time-frequency local filtering under a GLRT strategy. The interest of our detector principally relies on its automatic parameterisation: an ECG extracted pattern permits to select the mother wavelet and the time-scale discretisation. Furthermore, the good properties of the Klauder wavelet ensure good time location accuracy which is necessary for HRV analysis.

Unlike neural networks techniques, which require long learning periods before running, our detector rapidly adapts the filter for each recording.

Since the detector is based on a model, the technique can be used for QRS waveform classification. In particular, we will compare the classification rates with those obtained with the method proposed in [14], where the most adapted time scale representation is selected. The selection is made on the parameters defining the mother wavelet, leading to a minimum bad-classified rate.

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