SEQUENTIAL LMS FOR LOW-RESOURCE SUBBAND ADAPTIVE FILTERING: OVERSAMPLED IMPLEMENTATION AND POLYPHASE ANALYSIS

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ABSTRACT

The periodic and sequential partial update normalized LMS (P-NLMS and S-NLMS) algorithms and their variants are often used to reduce the computation cost of NLMS. In this paper, S-NLMS is employed in a low-resource subband adaptive filter implemented on an oversampled DFT filterbank. To analyze the system performance, we present a polyphase filterbank model for the P-NLMS and S-NLMS algorithms. It is shown that implicitly both algorithms employ perfect reconstruction delay chain analysis/synthesis filterbanks. As a result, the decimation involved in partial filter update does not introduce any steady-state performance degradation. The presented model can be employed to further predict and justify the convergence behavior of the partial update algorithms more accurately. Implementation of the S-NLMS algorithm on subband adaptive filters employing oversampled filterbanks is next described. Evaluation of the adaptive system performance shows that for stationary inputs, the S-NLMS algorithm (with proper step-size scaling) performs very similarly for moderate decimation factors of the S-NLMS.

1. . INTRODUCTION

The normalized least mean square (NLMS) algorithm has been widely used for adaptive noise and echo cancellation. However, in many practical applications a large number of adaptive taps must be employed, leading to prohibitively large computational (memory and CPU) requirements for real-time implementation on low-resource platforms. Alleviation of this problem often comes in the form of various partial update NLMS algorithms. In this paper we focus our attention on periodic and sequential partial update NLMS (P-NLMS and S-NLMS) and sequential block update NLMS (SB-NLMS) [1,2,3]. While much work has been done on examining the convergence properties and the computation cost reduction of the sequential update algorithms, a multirate analysis of the methods has not yet been presented.

Motivated by the research presented in [4], we present a signal processing view of the partial update algorithms, based on multirate filterbanks. Specifically, we show that the sequential update algorithms can be modelled by subband adaptive filters employing perfect reconstruction (PR) delay lines as their analysis and synthesis filters, and polyphase components of the adaptive filter as subband adaptive filters. This new perspective on the sequential update algorithms helps us to better understand, theorize and justify their convergence and steady-state behavior.

We propose and implement the S-NLMS algorithm for subband adaptive filters implemented on oversampled DFT filterbanks. The evaluations demonstrate that while the method is computationally more efficient than the NLMS, it performs very similar to the NLMS for stationary inputs.

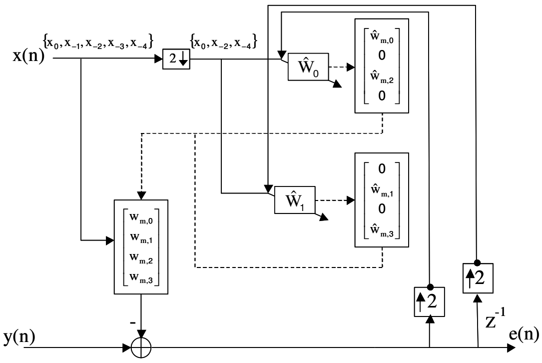

Figure 1: Block diagram of the S-NLMS for $D = 2$.

In Section 2, we briefly introduce the basic S-NLMS and P-NLMS methods. Next in Section 3 a polyphase filterbank view of the partial update algorithms is presented. Section 4 details an implementation of S-NLMS on an oversampled filterbank platform, and presents the evaluation results. Conclusions of this work are finally presented in Section 5.

2. THE S-NLMS ALGORITHM

Given the adaptive filter $\boldsymbol{W}n = [w_{0,n}, w_{1,n}, \dots, w_{L-1,n}]$ (of length L) at time n, input signal $x(n)$ with variance σ_x^2, vector $\boldsymbol{X}n = [x(n), x(n-1), \dots, x(n-L+1)]$, desired signal $y(n)$, and error signal $e(n)$, the filter update equation in S-NLMS (a simple extension of S-LMS in [1]) is given by

$$w_{i,n+1} = \begin{cases} w_{i,n} + \dfrac{\mu e(n) x(n-i)}{(L/D)\sigma_x^2} & if\,(n-i) \bmod D = 0 \\ w_{i,n} & otherwise \end{cases} \qquad (1)$$

$$e(n) = y(n) - \boldsymbol{W}_n^T \boldsymbol{X}_n .$$

For $D = 1$, this reduces to NLMS. As Fig. 1 illustrates for $D = 2$, this algorithm uses a decimated version of the input signal to update only one of the D adaptive sub-filters at a time. Note that each sub-filter is of length L/D . In SB-NLMS the adaptive filter is divided into D blocks, each containing L/D (assumed to be an integer) consecutive samples of the adaptive filter [3].

Eq. (2) characterizes the P-NLMS (based on P-LMS in [1]) where instead of the input signal, the error signal is decimated. In Eq. (2), $j = D\lfloor n/D \rfloor$, where $\lfloor \cdot \rfloor$ represents integer truncation.

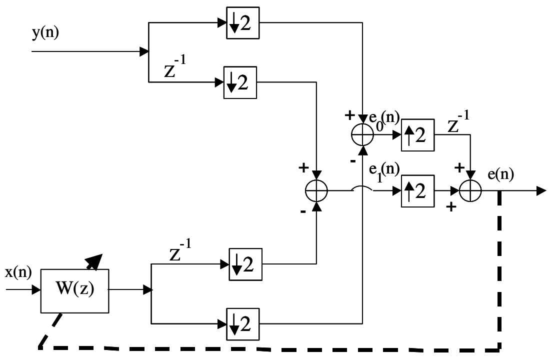

Figure 2: Adaptive filter using a PR delay chain filterbank.

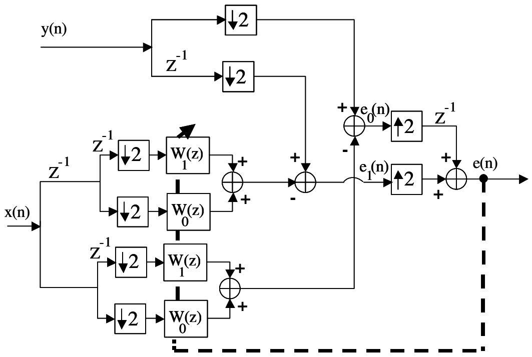

Figure 3: PR delay chain with polyphase filters adapted jointly.

$$w_{i,n+1} = \begin{cases} w_{i,n} + \dfrac{\mu e(j)x(j-i)}{(L/D)\sigma_x^{2}} & \textit{if } (n+i+1) \bmod D = 0 \\ w_{i,n} & \textit{otherwise} \end{cases} \qquad (2)$$

$$e(n) = y(n) - \mathbf{W}_n^T \mathbf{X}_n$$

3. POLYPHASE FILTERBANK MODELS FOR PARTIAL UPDATE NLMS

From Fig. 1, it can be easily seen that the sub-filters are indeed the polyphase components of the adaptive filter. We will now show that the whole S-NLMS system employs PR analysis/synthesis filterbanks.

It is proposed in [4] that a time-domain adaptive filter can be implemented in subbands using a PR analysis-synthesis filter bank. Accordingly, by interchanging the analysis filter and decimation operations, one can adapt the polyphase components of the adaptive filter. To model S-NLMS, we choose the simple case of the delay chain PR system. The descriptions and figures in this section depict the approach for the $D=2$ case; extension to higher values of D is straightforward. Shown in Fig. 2 is the adaptive filter cascaded with a PR delay chain. Fig. 3 depicts the equivalent polyphase model where the adaptive polyphase components should be jointly adapted as suggested in [4]. In the next step depicted in Fig. 4, each of the two error signals ($e_0(n)$ and $e_1(n)$) are employed sequentially to adapt the polyphase components for all input samples. Finally, if only even samples of $x(n)$ are used for adaptation (i.e. the thin dashed lines in Fig. 4 are disconnected), we obtain the S-NLMS system as presented in Fig. 1.

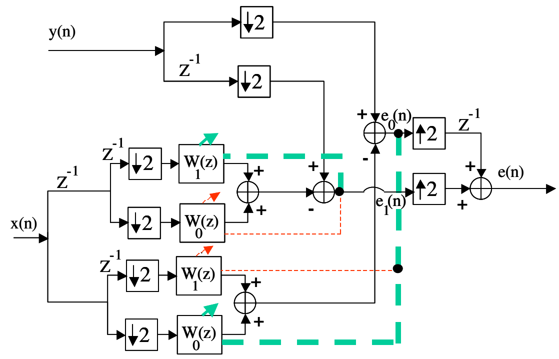

Figure 4: Polyphase adaptive filter representation using a delay chain. For S-NLMS, ignore the thin dashed lines

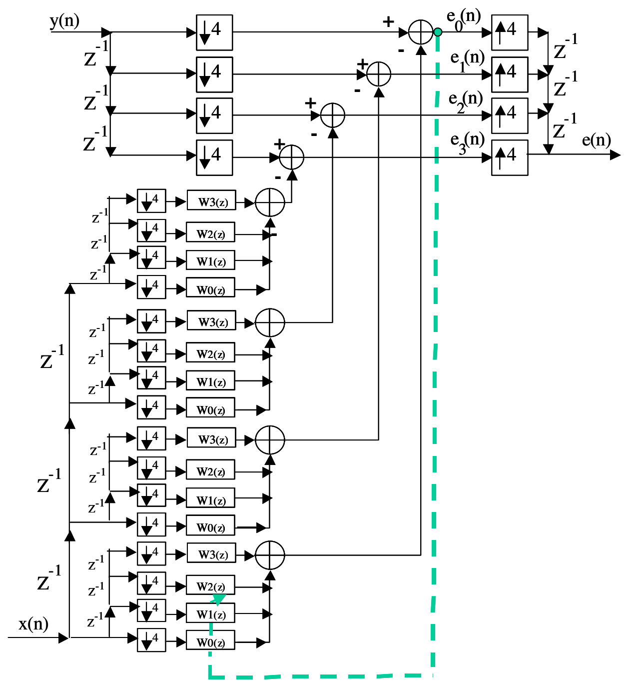

Fig. 5: Polyphase model for SB-NLMS, for a filter length of four and two sub-filters (only one adaptation branch shown).

As shown, S-NLMS is still using a PR polyphase system and thus the decimation of the input signal does not introduce any aliasing. The only difference between S-NLMS and the polyphase adaptive system of [4] is in the adaptation strategy: rather than updating all the polyphase components for all time samples, S-NLMS updates only one (out of D) polyphase components at a time. This may, however, slow down the convergence as reported in the literature [1-4].

With slight modifications, the proposed polyphase filterbank accurately models the P-NLMS algorithm. Specifically, if the error signal $e_1(n)$ (in Fig. 4) is not employed for adaptation, and both polyphase components are adapted using only $e_0(n)$, we end up with the P-NLMS algorithm.

Next, the polyphase model for SB-NLMS is described considerin the case of a filter of length $L=4$, with two sub-filters $\mathbf{W}_{s0}=[w_{0,n}, w_{1,n}]$ and $\mathbf{W}_{s1}=[w_{2,n}, w_{3,n}]$. The PR polyphase filterbank model is depicted in Fig. 5. Since $D=L$, each polyphase filter

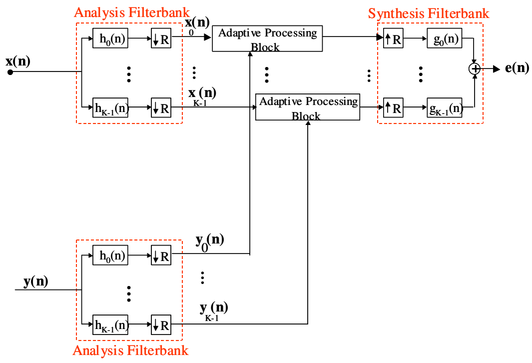

Figure 6: Block diagram of the OS-SAF system.

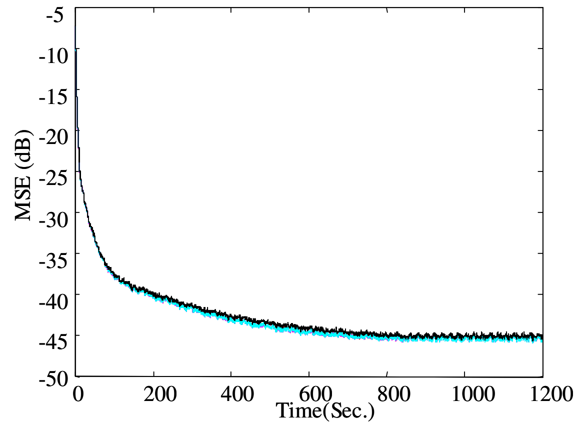

Figure 7: MSE convergence of the S-NLMS algorithm on the OS-SAF platform for $D = 1,4,8,12$ and 24, and stationary input signals.

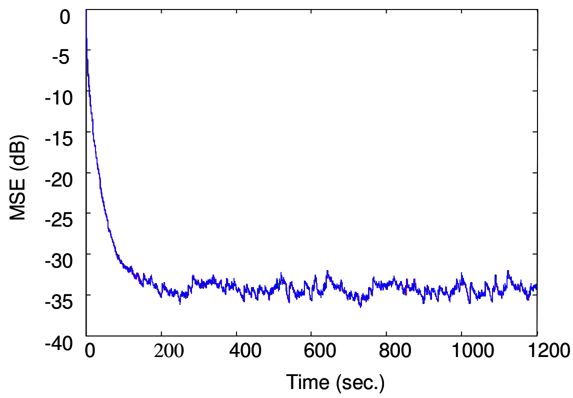

Figure 8: MSE convergence of the S-NLMS algorithm in time-domain for $D = 1,4,8,12$ and 24, and stationary input signals.

contains only one filter tap. The error signals [$e_0(n)$, $e_2(n)$] and [$e_1(n)$, $e_3(n)$] adapt the sub-filters $\boldsymbol{W}_{s0}$ and $\boldsymbol{W}_{s1}$, respectively. At time n, $e_0(n)$ adapts $\boldsymbol{W}_{s0}$ (including $W_0(z)$ and $W_1(z)$ components) as illustrated in Fig. 5. Then at time $n+1$, $e_1(n)$ adapts (not shown in the Figure) a copy of $\boldsymbol{W}_{s1}$ (including $W_2(z)$ and $W_3(z)$ components) located in its associated polyphase set (the set generating $e_1(n)$). Similarly, the sequential block adaptation employs $e_2(n)$ and $e_3(n)$ at times $n+2$ and $n+3$, respectively. Some researchers have proposed selective partial update variants to SB-NLMS to increase the performance [3]. In the polyphase model

Table 1: Maximum step-size (μ_{max}) for various decimation rates, using the OS-SAF system.

Decimation Rate D	**1**	**4**	**8**	**12**
μ_{max}	0.65	2.7	2.0	2.0
μ_{max}/D	0.65	0.675	0.25	0.17

Table 2: Maximum step-size (μ_{max}) for various decimation rates, using the time-domain S-NLMS.

Decimation Rate D	**1**	**4**	**8**	**12**
μ_{max}	1.2	4.0	2.2	1.5
μ_{max}/D	1.2	1	0.275	0.125

of Fig. 5, this is simply represented by optimally choosing one of the error signals (and its associated sub-filter) at any time instant. Based on the proposed model, various sequential and non-sequential adaptation and blocking strategies are possible.

4. S-NLMS EMPLOYED IN A SUBBAND ADAPTIVE FILTER

We employ the S-NLMS algorithm in an oversampled subband adaptive filter (OS-SAF) [5] for acoustic echo cancellation. The OS-SAF, depicted in Fig. 6, includes a near-PR oversampled DFT filterbank, with adaptive filters in subbands consisting of $L = 32$ taps. The oversampling factor is $OS = K/R$, where R is the subband decimation factor (analysis hop-size) and K is number of complex subbands. The filterbank selected for this system was a weighted overlap-add (WOLA) filterbank used for many low-power tasks [5]. It was configured for an analysis window length of 128 points, a synthesis window length of 32 points, $k = 32$ complex bands (of which 16 are unique because of Hermitian symmetry), $R = 4$, and $OS = 8$ as a result. The WOLA filterbank has various associated advantages (see [5]) and of particular note is its capability for high over-sampling factors permitting the use of a relatively short and low-delay prototype filter with a long frequency transition region. Each adaptive processing block in Fig. 6 is an adaptive filter employing the S-NLMS using Eq. (1) (extended for complex subband signals).

The reference signal used in simulations was white noise sampled at 8 kHz. The echo was generated using the eighth plant (128 samples long at 8 kHz) from ITU-T Recommendation G.168 [7], and normalized for an echo return loss (ERL) of 10 dB. There was no near-end disturbance in the primary signal.

We have to comment on the μ setting in S-NLMS and P-NLMS as compared to the NLMS. Some researchers have used identical μ values for all methods (see [1,2]). However, others (for example [4]) have optimized their μ values to obtain the same steady-state performance. Since a subband adaptive structure similar to [4] is employed in this research, we choose the optimized μ approach. Our observations (described below) show that using a μ proportional to D (through the term L/D in the denominator of Eq. (1)) leads to almost identical (short-term and asymptotic) S-NLMS performances for various decimation rates. Also, considering that each time-step for filter update is D times larger in S-NLMS compare to NLMS, the proposed scaling imposes the same time-constant for filter adaptation for both S-NLMS and NLMS. Thus, we have adopted this step-size scaling strategy throughout this research for S-NLMS, and P-NLMS.

Fig. 7 depicts the fullband residual error (echo) of the echo canceller against time for $D = 1,4,8,12$ and 24. As the mean-squared error (MSE) shows, both the short-term convergence and the

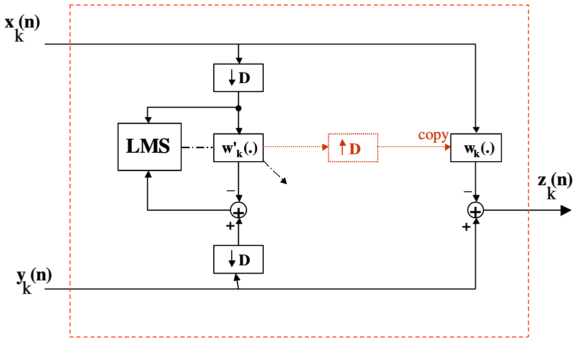

Figure 9: Whitening by decimation of the subband signals.

asymptotic performances are almost identical for various decimation rates. There is no considerable misadjustment due to μ scaling for higher values of D. Intentionally, a high over-sampling of $OS = 8$ was used in the OS-SAF to prevent aliasing errors in the filterbank analysis stage from interfering with the adaptation (see [6]). Inevitably, this leads to a slow asymptotic convergence as depicted in Fig. 7.

To investigate the effect of the proposed step-size scaling on the stability of the S-NLMS algorithm, we performed a series of experiments for various D values with the same system set-up to find the maximum step-size (μ_{max}) for a stable performance in each case. Table 1 lists the results for $D = 1,4,8,12$. As shown, the value of μ_{max}/D does not linearly decline with D at least up to $D = 8$. This shows that for moderate decimation rates ($D << L$), the proposed step-size scaling does not render the system unstable. In practice, much smaller step-sizes are employed in NLMS to avoid artifacts. Thus the employed step-size scaling is very practical for moderate decimation rates of S-NLMS.

To compare the performance of the S-NLMS on the OS-SAF system with time-domain S-NLMS, we simulated a time-domain S-NLMS system with an adaptive filter length of $L = 128$ (the same as the echo plant length). The primary input signal was white noise filtered through a low-pass filter to obtain the same spectral shape as those of subband signals in the OS-SAF. Using the same step-size scaling strategy, we achieved identical convergence performance for $D = 1,4,8,12$ and 24 as depicted in Fig. 8. Also, as Table 2 shows, the maximum step-size for the time-domain S-NLMS follows trends very similar to those in Table 1 for the OS-SAF system.

The performance of S-NLMS in practical applications (like acoustic echo cancellation), where the input (often speech and audio) signals and the unknown plant are both nonstationary, will degrade more severely by increasing the decimation rate D [1-3]. This is partially due to the fact that the plant might change before all the polyphase components are updated.

4.1 Whitening by Decimation and S-NLMS

We have already proposed a method of whitening by decimation (WBD) of the OS-SAF subband inputs [5] as shown in Fig. 8. The idea is to use a decimated version of the bandlimited subband signals for adaptation. Then the decimated signals would have wider bandwidths, leading to faster convergence. To limit the aliasing errors, the decimation rate is limited to $D \le OS/2$ [5,6]. In light of the analysis presented in Section 3, it is evident that in a polyphase model of WBD only one out of the D adaptive polyphase components is used, and the other polyphase components are set to zero. Obviously this leads to aliasing and performance limitation as D increases [6]. The direct effect of aliasing is only on the adaptive side branch (in Fig. 9) and the adaptation process. For small values of D ($D \le OS/2$), the introduced aliasing limits the asymptotic performance of the OS-SAF system but the degradation is negligible for many practical applications [6].

5. CONCLUSION

We have presented a new perspective on the family of partial update adaptive algorithms, including S-NLMS, P-NLMS, and SB-NLMS. The presented models demonstrate that partial update algorithms are indeed subband adaptive systems, employing a PR delay chain analysis/synthesis filterbank, with polyphase components of the adaptive filter as SAFs. While S-NLMS decimates the input signals, P-NLMS decimates the error signal. However, due to the PR property of their employed filterbanks, no aliasing will occur. It is also shown that the difference of various partial update algorithms is only in the way the subband error signals are related to (and used to adapt) the polyphase components of the adaptive filter. We employed the S-NLMS for a low-resource OS-SAF system. Performance evaluations confirm that with proper step-size scaling, it can achieve the same performance as the NLMS with stationary signals.

Another important observation is that the S-NLMS (and other partial update algorithms reviewed) is not strictly following the NLMS method for each polyphase component. Notice in Fig. 4 that although the error signal $e_0(n)$ is used to update only the polyphase filter $W_0(z)$, the sub-filter $W_1(z)$ is also contributing to the error signal $e_0(n)$. The same conclusion may be drawn from Fig. 5, as well as Equations (1) and (2). As a result, their mis-adjustment and convergence properties might not strictly follow the NLMS patterns. WBD on the other hand is strictly following the NLMS strategy since there is only one adaptive polyphase filter contributing to the error signal.

For future work, we will try to gain more insight into the transient and steady-state behavior of the partial update LMS and affine projection algorithms, based on the proposed models.

6. REFERENCES

[1] S. C. Douglas, “Adaptive filters employing partial updates” *IEEE Trans. on Circuits and Systems-II,* Vol. 44, No. 3, pp. 209-216, Mar. 1997.
[2] T. Aboulnasr, and K. Mayya, “Selective coefficient update of gradient-based adaptive algorithms,”, *Proc. of the ICASSP’97,* pp. 1929-1932, Munich, Germany, Apr. 1997.
[3] K. Dugancay and O. Tanrikulu, “Selective-partial-update NLMS and affine projection algorithms for acoustic echo cancellation”, *Proc. of the ICASSP 2000,*.Vol. 1, pp. 448-451, 5-9 Jun. 2000.
[4] S. Sandeep Pradhan and V. U. Reddy, “A new approach to subband adaptive filtering”, *IEEE Trans. on Signal Processing,* Vol. 47, No. 3, pp. 655-664, Mar. 1999.
[5] H. R. Abutalebi, H. Sheikhzadeh, R. L. Brennan, and G. H. Freeman, “Convergence Improvement for Oversampled Subband Adaptive Noise and Echo Cancellation”, *Proc. of Eurospeech*, 2003.
[6] H. Sheikhzadeh, H. R. Abutalebi, R. L. Brennan, and J. Sollazzo, “Performance Limitations of a New Subband Adaptive System for Noise and Echo Reduction”, in *Proc. of IEEE ICECS,* 2003.
[7] ITU-T G.168 Recommendation, *Digital Network Echo Cancellers*, Int’l Telecommunication Union, 2000.